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Performance-based Seismic Design of Structures

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Dr. Naveed Anwar


Dr. Pennung Warnitchai

- Some other references of this training material include the following.
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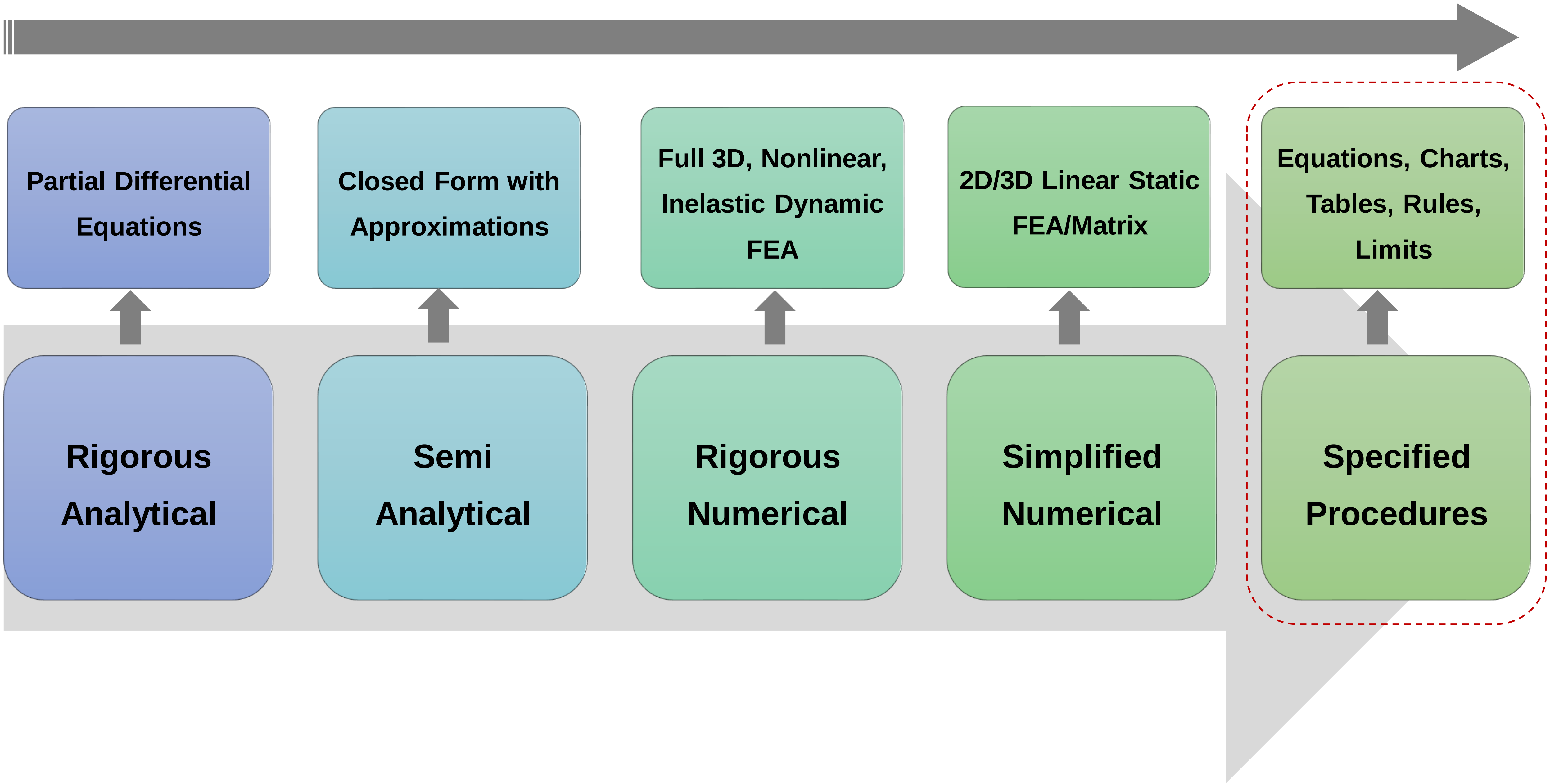
    - Various International Codes and Guidelines
    - Online Training Material from US Geological Survey (USGS)
    - Lectures of Dr. Punchet Thammarak at Asian Institute of Technology (AIT), Thailand
    - Online Educational Resources from IRIS (www.iris.edu)
    - Class Notes of Prof. Dr. Worsak Kanok-Nukulchai at Asian Institute of Technology (AIT), Thailand

- The material is taken solely for educational purposes. **All sources are duly acknowledged.**

Code-based Seismic Design of Structures

- The Concept of Vibration Modes of a Structure
- The Response Spectrum Analysis Procedure
    - The Concept of Response Spectrum
    - The Elastic Response Spectrum Analysis (RSA) Procedure
    - The Concept of Inelastic Response Spectra and Design Spectra
    - The Code-based Response Spectrum Analysis (RSA) Procedure
    - The RSA Procedure in BCP (2007)
- The RSA Procedure as prescribed in ASCE 7-16 – Software Demonstration


Partial Differential Equations
Closed Form with Approximations
Full 3D, Nonlinear, Inelastic Dynamic FEA
2D/3D Linear Static FEA/Matrix
Equations, Charts, Tables, Rules, Limits
Rigorous Analytical
Semi Analytical
Rigorous Numerical
Simplified Numerical
Specified Procedures

Structural Analysis vs. Structural Design

Structural Analysis

- Fairly General, Unified (FEM, BEM ...)
- **Input:** Structural Geometry, Element/Member Cross-sections, Reinforcement details
- **Output:** Element/ Member Actions, Displacements ... etc.

Structural Design

- Structural Material (RC, PSC, HRS, CFS, timber ...)
- Design Code (ACI, BS Codes, EuroCode, JIS ...)
- Design Approach (working stress, ultimate strength, limit state ...)
- Structural Members (beams, columns, slabs, footings ...)
- Local Construction Techniques and Practices
- **Input:** Element/ Member Actions, Displacements ... etc.
- **Output:** Structural Geometry, Element/Member Cross-sections, Reinforcement details

Seismic Analysis Procedures

Structural Model / Seismic Loading	Linear E, A, I, L, G etc. = Constant, K= Constant	Nonlinear E ≠ Constant, EI ≠ Constant, K≠ Constant
Static	**1.** Equivalent Lateral Force (ELF) Procedure **2.** Response Spectrum Analysis (RSA) Procedure (or Mode Spectral Analysis)	**5.** Several Pushover Analysis Methods or Nonlinear Static Procedures (NSPs)
Dynamic	**3.** Modal Response History (or Time History) Analysis Procedure (Modal RHA/THA) **4.** Linear Response History (or Time History) Analysis Procedure (Direct Integration Linear RHA /LTHA)	**6.** Nonlinear Modal Response History Analysis or Fast Nonlinear Analysis (FNA) **7.** Nonlinear Response History (or Time History) Analysis Procedure (Direct Integration Nonlinear RHA/THA)

SEISMIC DESIGN CODES

There are several design methods suggested in seismic codes:

- **Equivalent static analysis (ESF) method (or ELF method)**
- **Response spectrum analysis (RSA) method**
- **Time history analysis method**

Equivalent static analysis method is commonly used for the seismic design of ordinary and "regular" buildings and structures.

SEISMIC DESIGN CODES

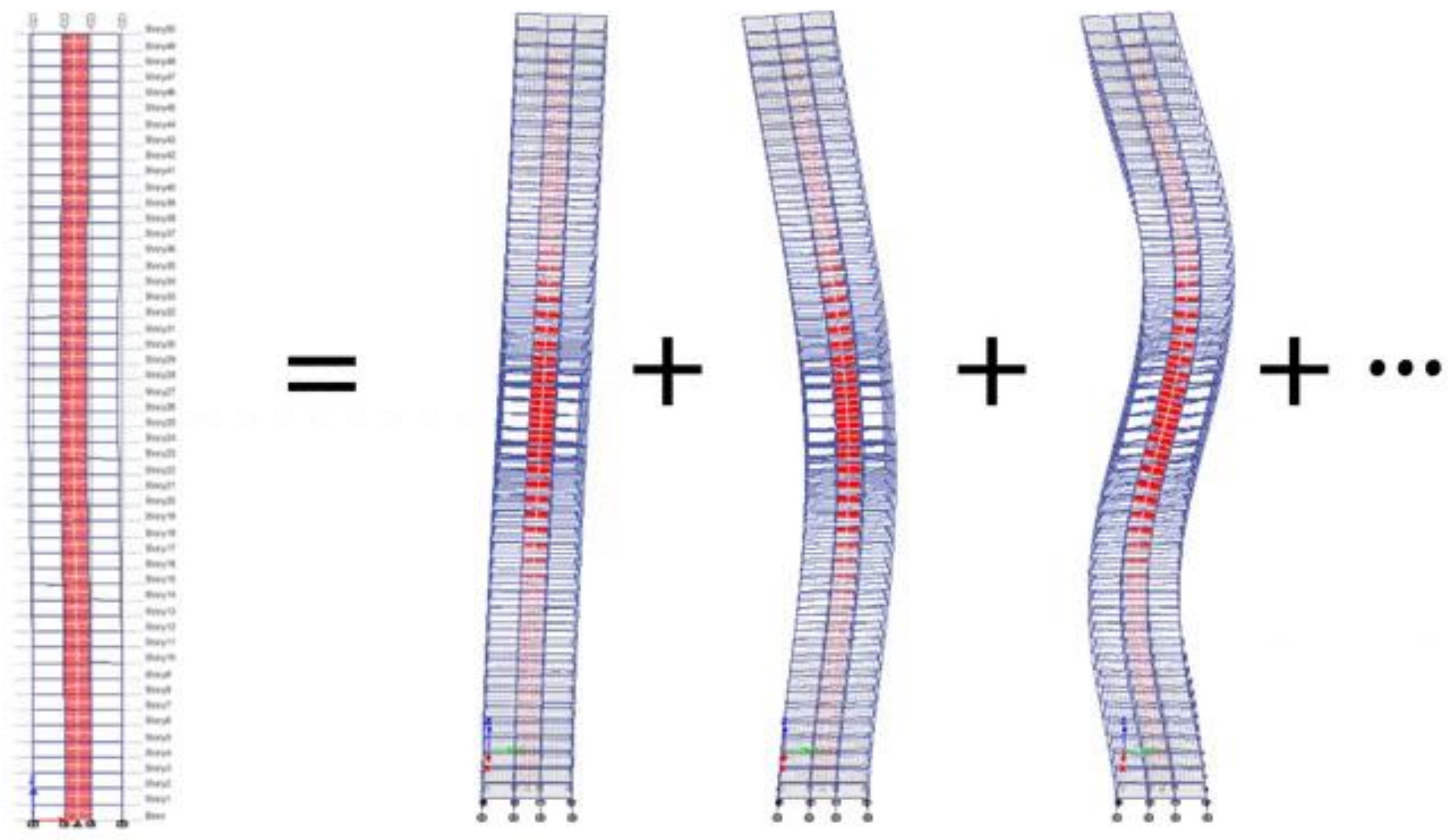
Dynamic analysis methods (Response spectrum and Time history) are required for "irregular" buildings and structures, very important structures, and structures that seismic response is not dominated by the fundamental vibration mode. ... more tedious, more difficult.

Irregularity = Irregular geometry, non-uniform distribution of mass or stiffness, structural discontinuity, etc.

- Vertical Structural Irregularities
- Plan Structural Irregularities

The Concept of Vibration Modes of a Structure

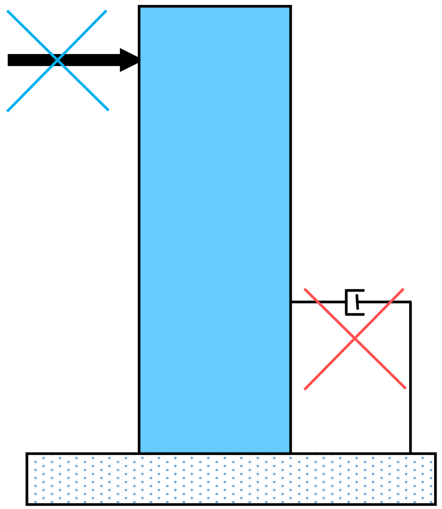
The Basic Concept



Free Vibration Analysis

- Definition
  - Natural vibration of a structure released from initial condition and subjected to no external load or damping
- Main governing equation -Eigenvalue Problem

$$\boldsymbol{M}\,\ddot{\boldsymbol{U}}(t) + \cancel{\boldsymbol{C}\,\dot{\boldsymbol{U}}(t)} + \boldsymbol{K}\,\boldsymbol{U}(t) = \cancel{\boldsymbol{P}(t)}$$



- Solution gives
  - Natural Frequencies
  - Associated mode shapes
  - An insight into the dynamic behavior and response of the structure

Natural Periods or Frequency

- The **heartbeat** of the structure
- Indicates the **“stiffness” and “mass” relationship**
- Basis for **damping, resonance and amplification effects**
- Many relationships for tall buildings (0.1 N, with Height etc.)

Mode Shapes

- A mode shape is a set of **relative (not absolute) nodal displacements** for a particular mode of free vibration for a specific natural frequency

- There are as many modes as there are DOF in the system

- **Not** all of the modes are significant

- **Local modes** may disrupt the modal mass participation

Modal Analysis

- The modal analysis determines the **inherent** natural frequencies of vibration
- Each natural frequency is related to a time period and a mode shape
- Time Period is the time it takes to complete one cycle of vibration
- The **Mode Shape** is **normalized deformation pattern**
- The number of Modes is typically equal to the number of Degrees of Freedom
- The Time Period and Mode Shapes are inherent properties of the structure and **do not depend on the applied loads**

Modal Analysis

- The Modal Analysis should be run **before applying loads** any other analysis to check the model and to understand the response of the structure.

- Modal analysis is **precursor** to most types of analysis including Gravity Load Analysis, Response Spectrum Analysis, Time History Analysis, Push-over Analysis, etc.

- Modal analysis is a useful tool even if full Dynamic Analysis is not performed.

- Modal analysis is easy to run and is **fun to watch** when animated.

Applications of Modal Analysis

- **The Time Period and Mode Shapes, together with animation immediately exhibit the strengths and weaknesses of the structure.**

- **Modal analysis can be used to check the accuracy of the structural model**
  - The Time Period should be within reasonable range,
  - The disconnected members are identified
  - Local modes are identified that may need suppression

- **The symmetry of the structure can be determined**
  - For doubly symmetrical buildings, generally the first two modes are translational and the third mode is rotational
  - If the first mode is rotational, the structural is un-symmetrical

- **The resonance with the applied loads or excitation can be avoided**
  - The natural frequency of the structure should not be close to excitation frequency

What modal analysis used for :-

a) Finding loose components :

If the structure is under-constrained, analysis will result a 0-hz (i-e static) mode for each unconstrained direction. They will not be exactly 0-hz for two reasons i-e round off or eigen-value analysis is an iterative process. But f will be very low, like 0.001 hz. These modes are also reffered to as "rigid-body modes" or "strain-less modes".

This is because the structure (or part of structure) translates or rotates in such a way that no stress results, i-e moves in some direction as it was rigid.

So the displacement shape of these modes provide enough information about which component may be loose or which constraints are missing.

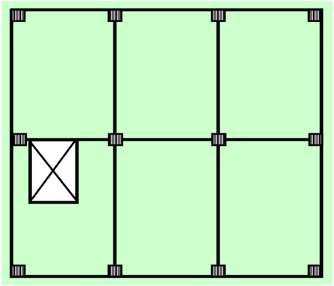
b) Deciding which rotational speeds are dangerous. rotor dynamics

c) Where to constrain or load a structure?
The role of nodes etc.

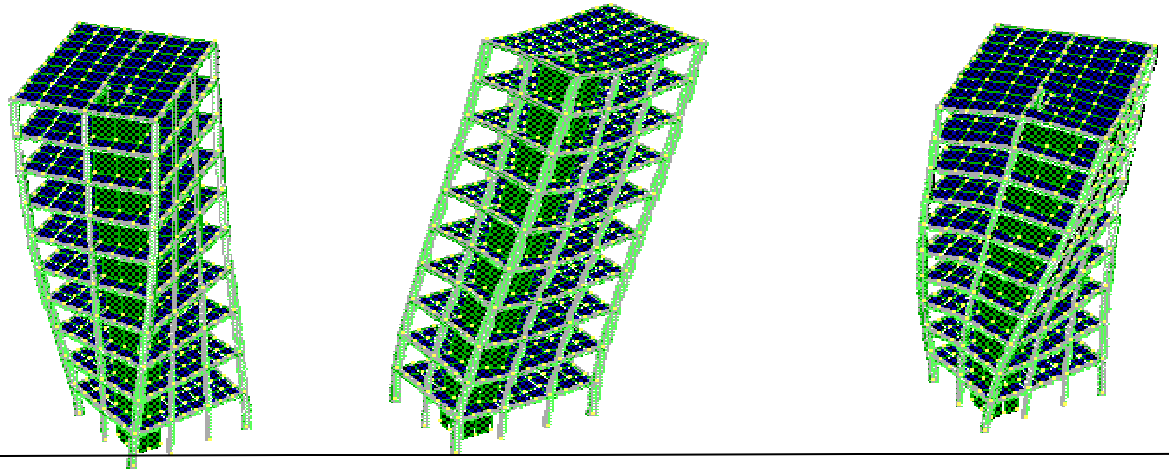
d) Finding out how to move a mode $T = 2\pi\sqrt{\frac{m}{K}}$.

Eccentric and Concentric Response

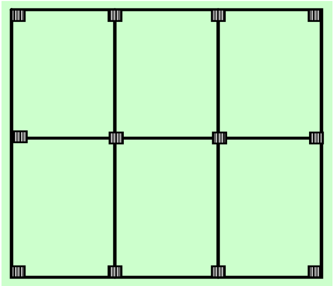
Unsymmetrical Mass and Stiffness

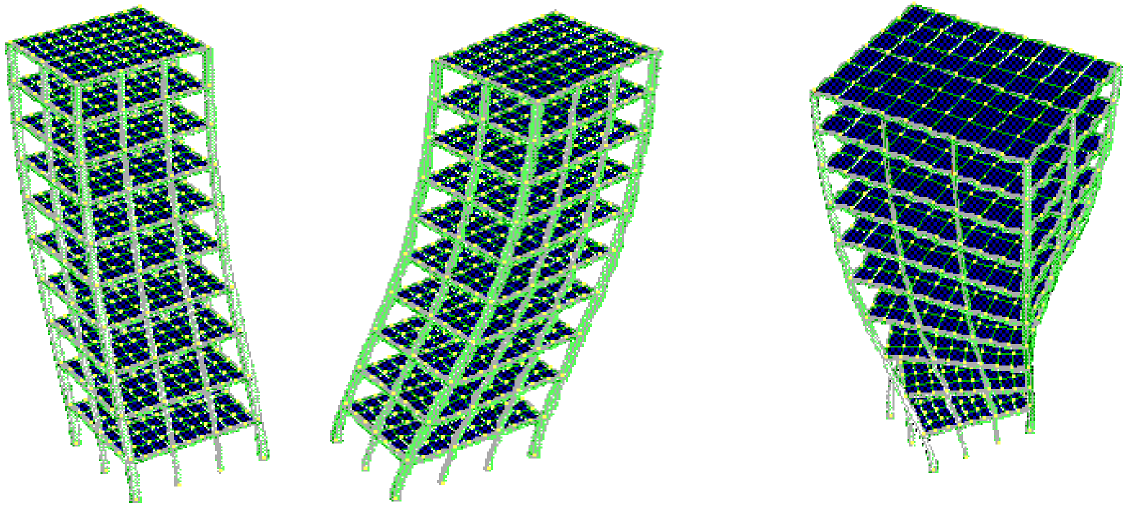


Mode-1 Mode-2 Mode-3

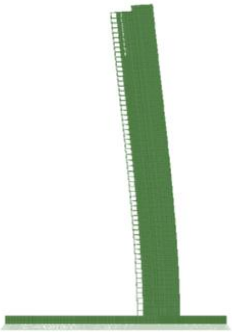
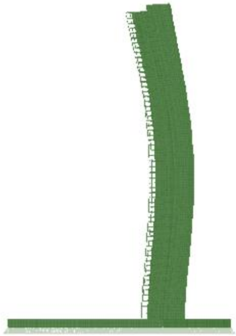
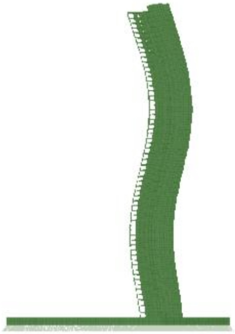
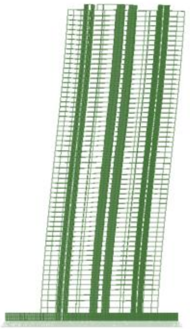
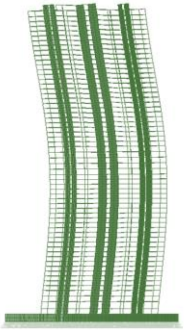
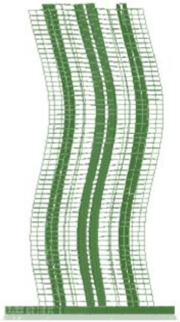
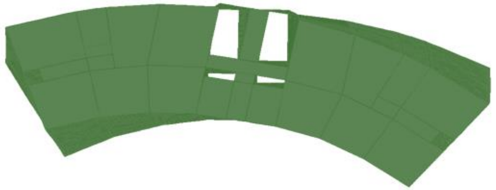
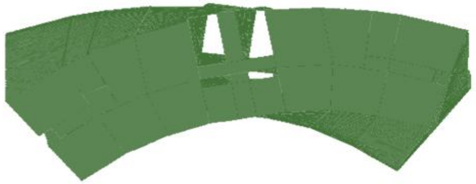


Symmetrical Mass and Stiffness





Modal Analysis Results

Translation in Minor	 • T_1 = 5.32 sec • 60% in Minor direction	 • T_6 = 1.28 sec • 18% in Minor direction	 • T_9 = 0.75 sec • 6.5% in Minor direction
Translation in Major	 • T_2 = 4.96 sec • 66% in Major direction	 • T_4 = 1.56 sec • 15% in Major direction	 • T_7 = 0.81 sec • 5.2% in Major direction
Torsional	T_3 = 4.12 sec 	T_5 = 1.30 sec 	T_8 = 0.65sec 

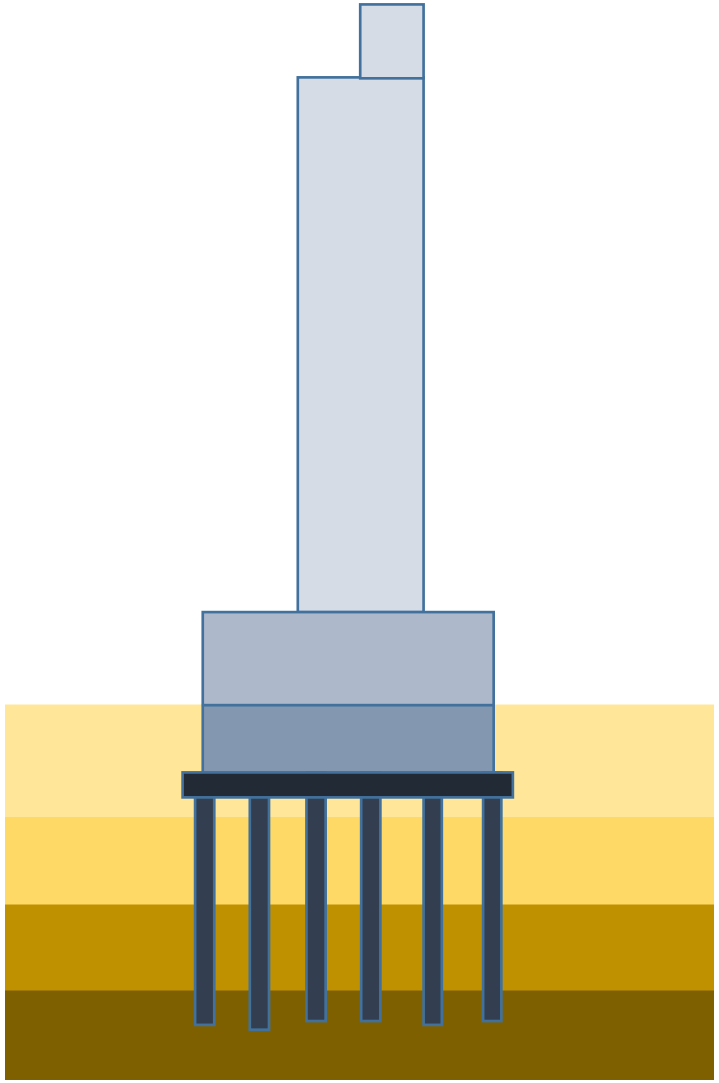
Modal Response Influenced by

Mass

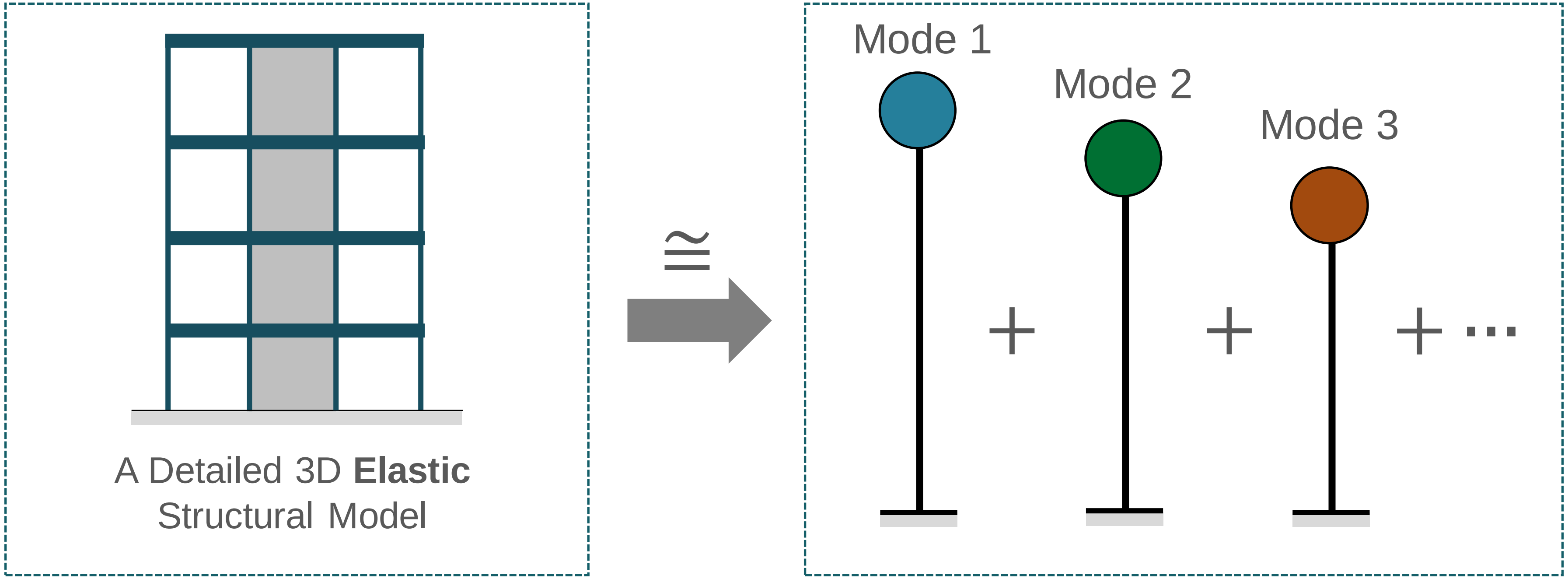
- Structure
- Attachments
- Occupants

Stiffness

- Tower
- Podium
- Basement
- Foundation
- Soil



The Classical Modal Analysis Procedure for Forced Vibrations



Modal Analysis

- To determine vibration modes of building
- To understand behaviour of building in schematic design stage
  - Adequacy of lateral stiffness
  - Minimize torsional response under earthquake
  - Tune to structure to be dynamically regular
  - Determine the principal directions of building

- Mass source

1.0 DL + 1.0 LL MEP + 0.25 LL STO

  - DL = Dead load
  - LL MEP = MEP and other permanent equipment live load
  - LL STO = Storage live load

The Response Spectrum Analysis (RSA) Procedure

The Concept of Elastic Response Spectrum

The governing equation of motion of an SDF system subjected to a ground motion $\ddot{u}_g(t)$ can be written as follows.

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = -m\,\ddot{u}_g(t)$$

$$c = 2\,m\,\xi\,\omega \quad , \quad \omega^2 = \frac{k}{m}$$

$$\ddot{u}(t) + 2\,\xi\,\omega\,\dot{u}(t) + \omega^2 u(t) = -\ddot{u}_g(t)$$

Solution

$$u(t, T, \xi)$$
$$\dot{u}(t, T, \xi)$$
$$\ddot{u}(t, T, \xi)$$

Output of the above equation (u, $\dot{u}$, $\ddot{u}$) are the dynamic response to the ground motion for a structure considered as a single DOF

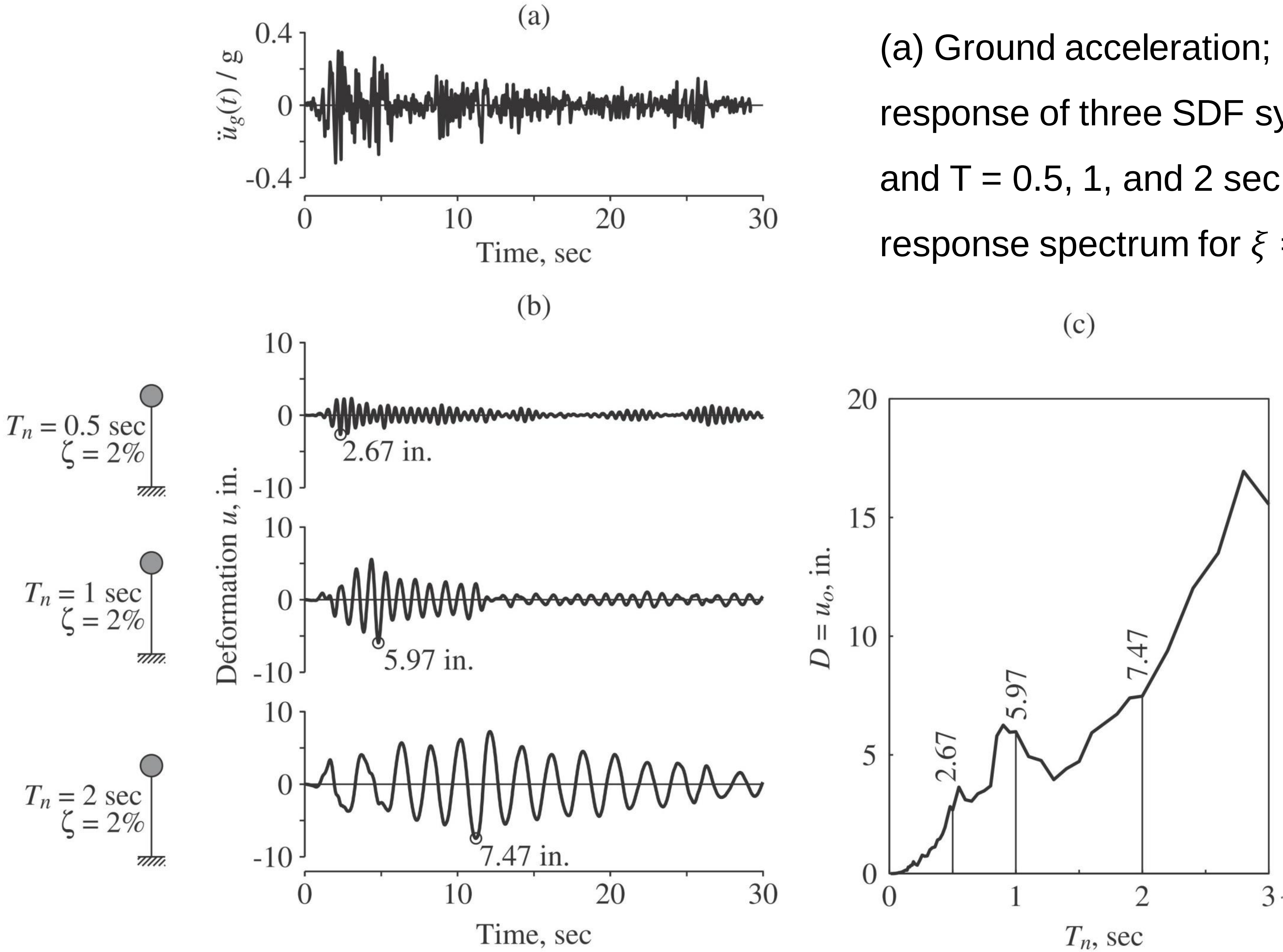
A plot of the "**maximum**" response for different ground motion history, different time period and damping ratio give the "**Spectrum of Response**"

$$u_o(T,\xi) = max|u(t,T,\xi)|$$

$$\dot{u}_o(T,\xi) = max|\dot{u}(t,T,\xi)|$$

$$\ddot{u}_o(T,\xi) = max|\ddot{u}(t,T,\xi)|$$

The deformation response spectrum is a plot of u_o against T for fixed ξ. A similar plot for $\dot{u}_o$ is the velocity response spectrum, and for $\ddot{u}_o$ is the acceleration response spectrum.



(a) Ground acceleration; (b) deformation response of three SDF systems with ξ = 2% and T = 0.5, 1, and 2 sec; (c) deformation response spectrum for ξ = 2%.

Elastic Response Spectra

If a record of ground acceleration $\ddot{u}_g(t)$ is known, then the deformation response of a linearly elastic SDOF system can be computed by the convolution integral (See Eq.(27)), and internal forces of interest to structural engineers such as bending moments, shears can be subsequently determined.

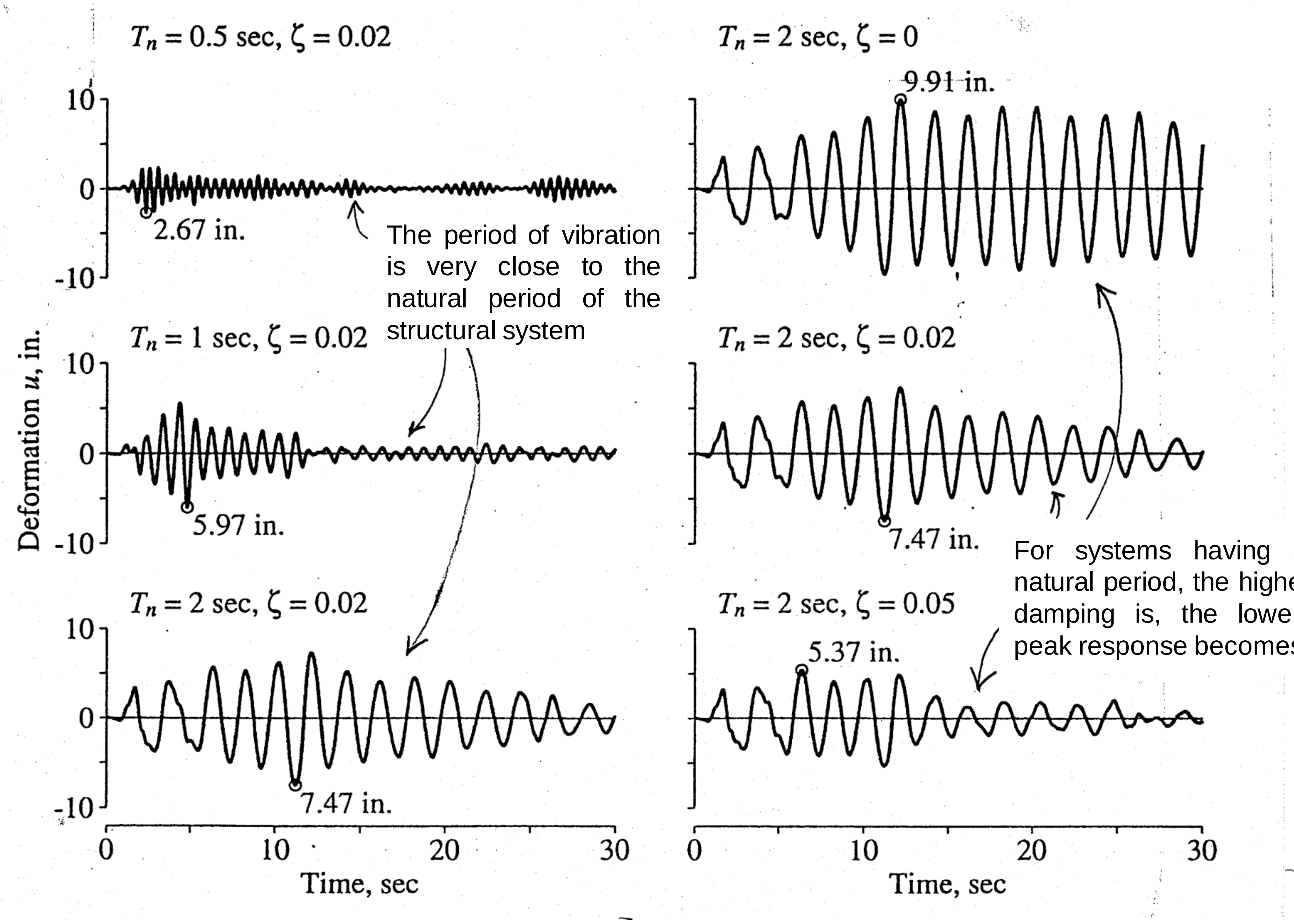
Equation of motion: $$m\,\ddot{u} + c\,\dot{u} + k\,u = -m\,\ddot{u}_g(t)$$

The equation can also be written in the form of

$$\ddot{u} + 2\,\xi\omega_n\,\dot{u} + \omega_n^2\,u = -\ddot{u}_g(t) \qquad (29)$$

It turns out that, for a given ground acceleration $\ddot{u}_g(t)$, the deformation response depends only on ω_n (or T_n) and ξ of the SDOF system.

Deformation Response of SDOF systems to the El Centro Ground Motion



The period of vibration is very close to the natural period of the structural system

For systems having same natural period, the higher the damping is, the lower the peak response becomes.

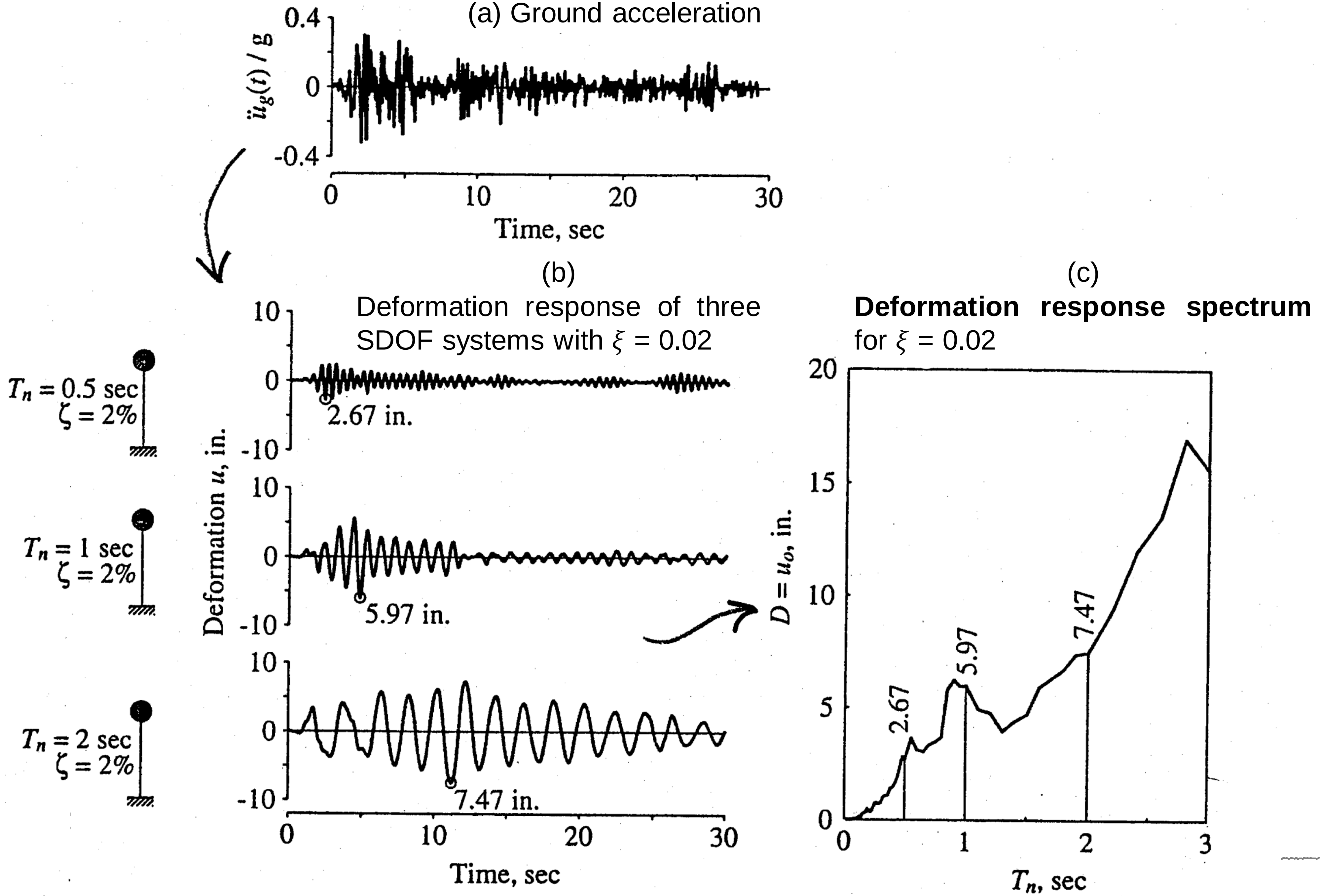
(a) Ground acceleration

(b)

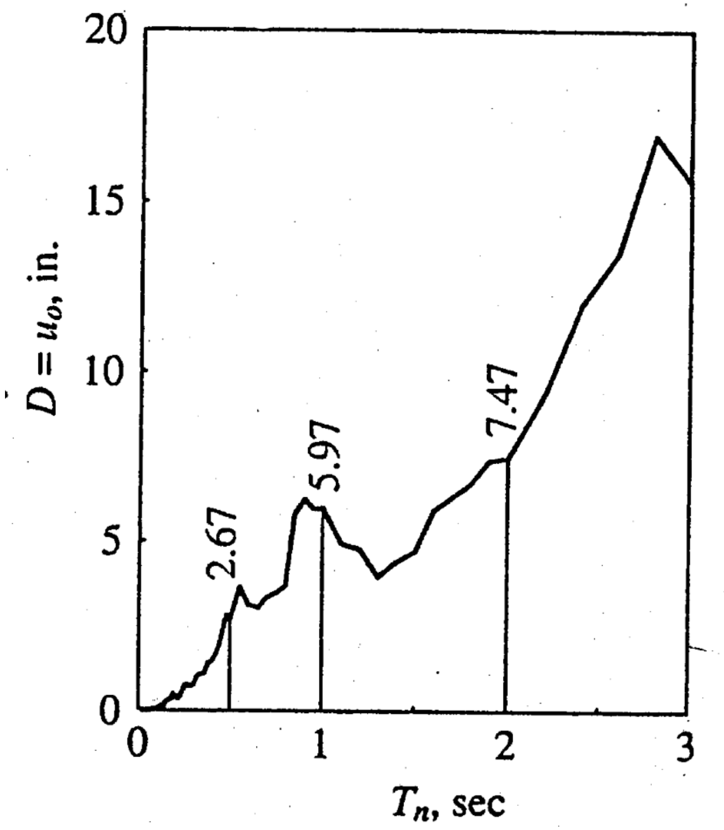
Deformation response of three SDOF systems with ξ = 0.02

(c)

Deformation response spectrum for ξ = 0.02



Deformation response spectrum for ξ = 0.02



Response Spectrum: A plot of the peak value of a response quantity as a function of the natural vibration period T_n of the system, or related parameter, is called the response spectrum for the quantity.

The response spectrum provides a convenient mean to summarize the peak response of all possible linear SDOF systems to a particular component of ground motion.

It also provides **a practical approach to apply the knowledge of structural dynamics to the design of structures** and development of lateral force requirements in building codes.

Let u_0 be the peak displacement of SDOF system,

Once u_0 is obtained from the deformation response spectrum, the corresponding peak internal forces f_{s0} can be determined by:

$$f_{s0} = k\, u_0$$

or

$$f_{s0} = m\, \omega_n^2\, u_0 = m\, A \qquad (30)$$

where

$$A = \omega_n^2\, u_0 \qquad (31)$$

Note that f_{s0} is $m \times A$ not $m \times the\ peak\ value\ of\ acceleration\ (\ddot{u}_t)_0$

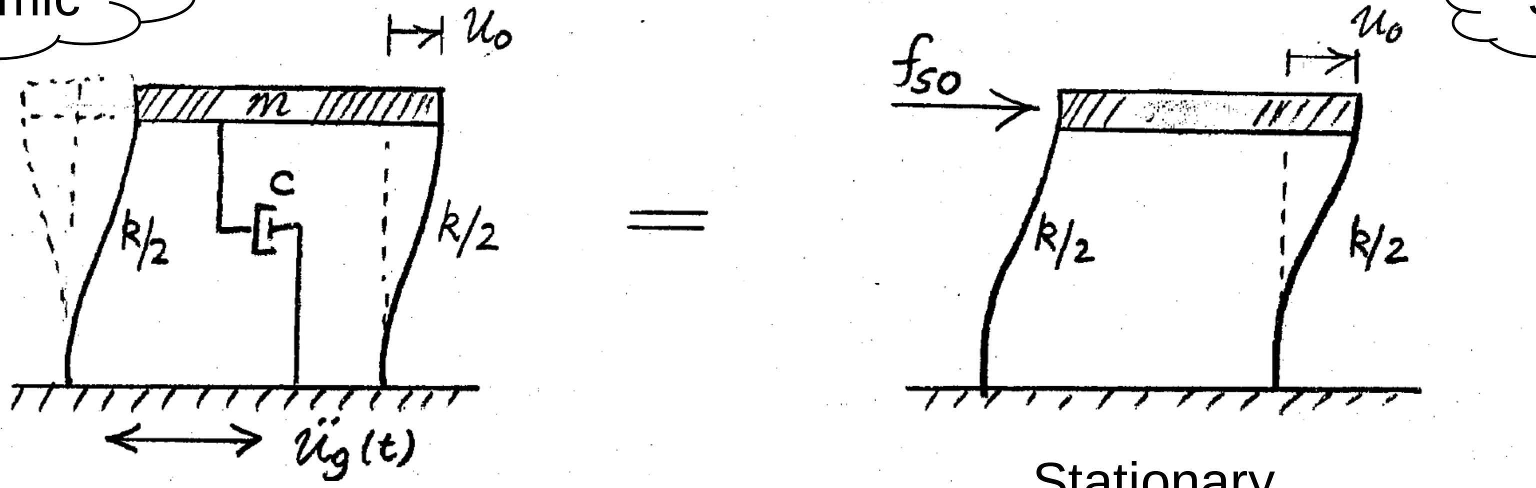
A is not the real peak acceleration response but it has units of acceleration.

A is called "Peak Pseudo-acceleration" or "Spectral Acceleration".

f_{s0} can also be considered as an "**equivalent static force**" because if the force f_{s0} is applied to the structure statically it will produce the equivalent amount of peak deformation response u_0.

Dynamic

Static



at the instant where $u(t) = u_0$

Stationary

Let V_{b0} be the peak value of base shear

$$V_{b0} = f_{s0} = m\,A \qquad (32)$$

It can be written in the form

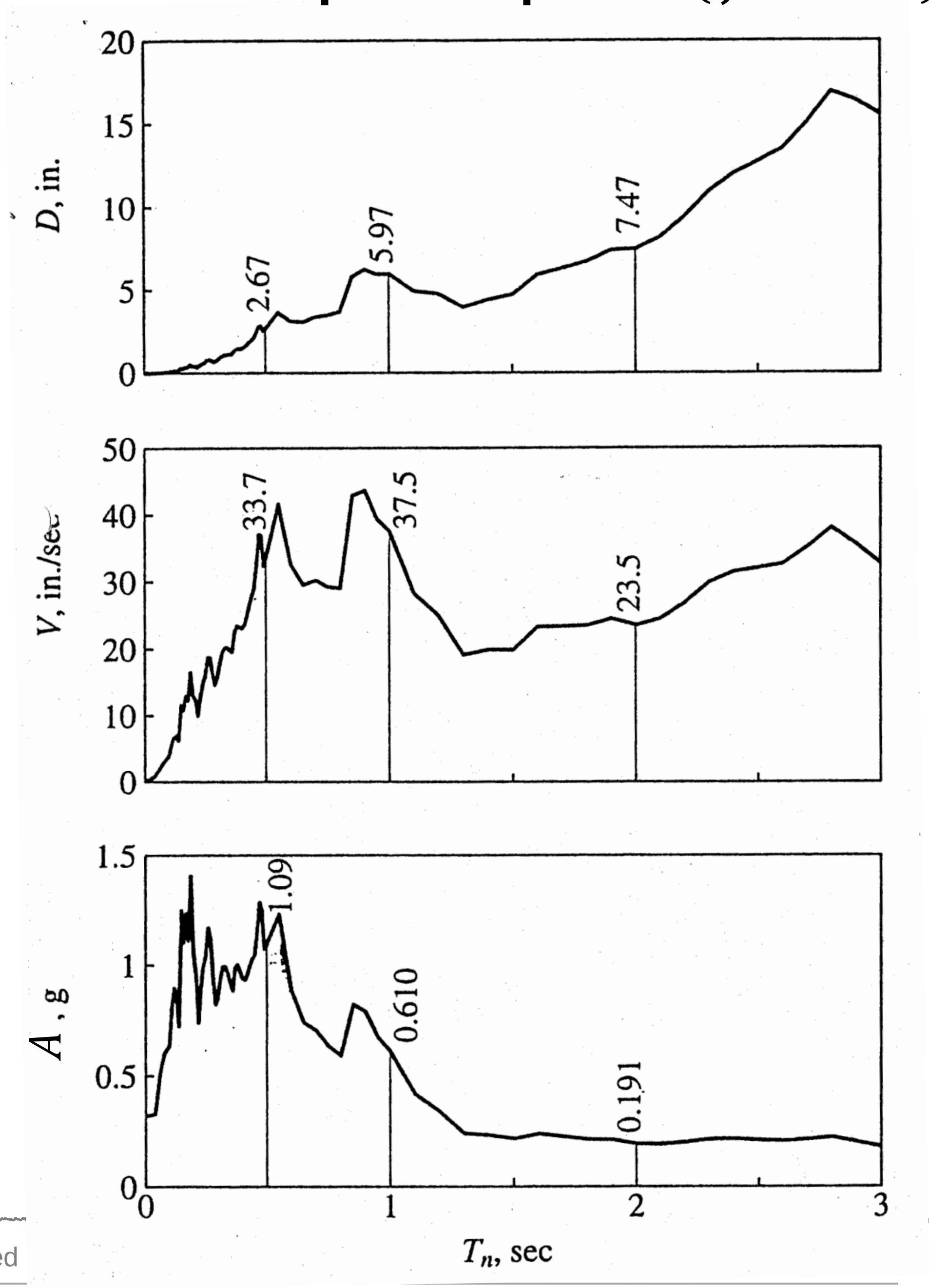
$$V_{b0} = \frac{A}{g}\,w \qquad (33)$$

Where w is the weight of the structure and g is the gravitational acceleration.

A/g may be interpreted as the base shear coefficient or lateral force coefficient*.

*It is used in the building codes to represent the coefficient by which the <u>structural weight is multiplied to obtain the base shear</u>.

Response Spectra $(\xi = 0.02)$ for El Centro Ground Motion



Deformation Response Spectrum

$$D = u_0$$

Pseudo-Velocity Response Spectrum

$$V = \omega_n D$$

Pseudo-Acceleration Response Spectrum

$$A = \omega_n^2 D$$

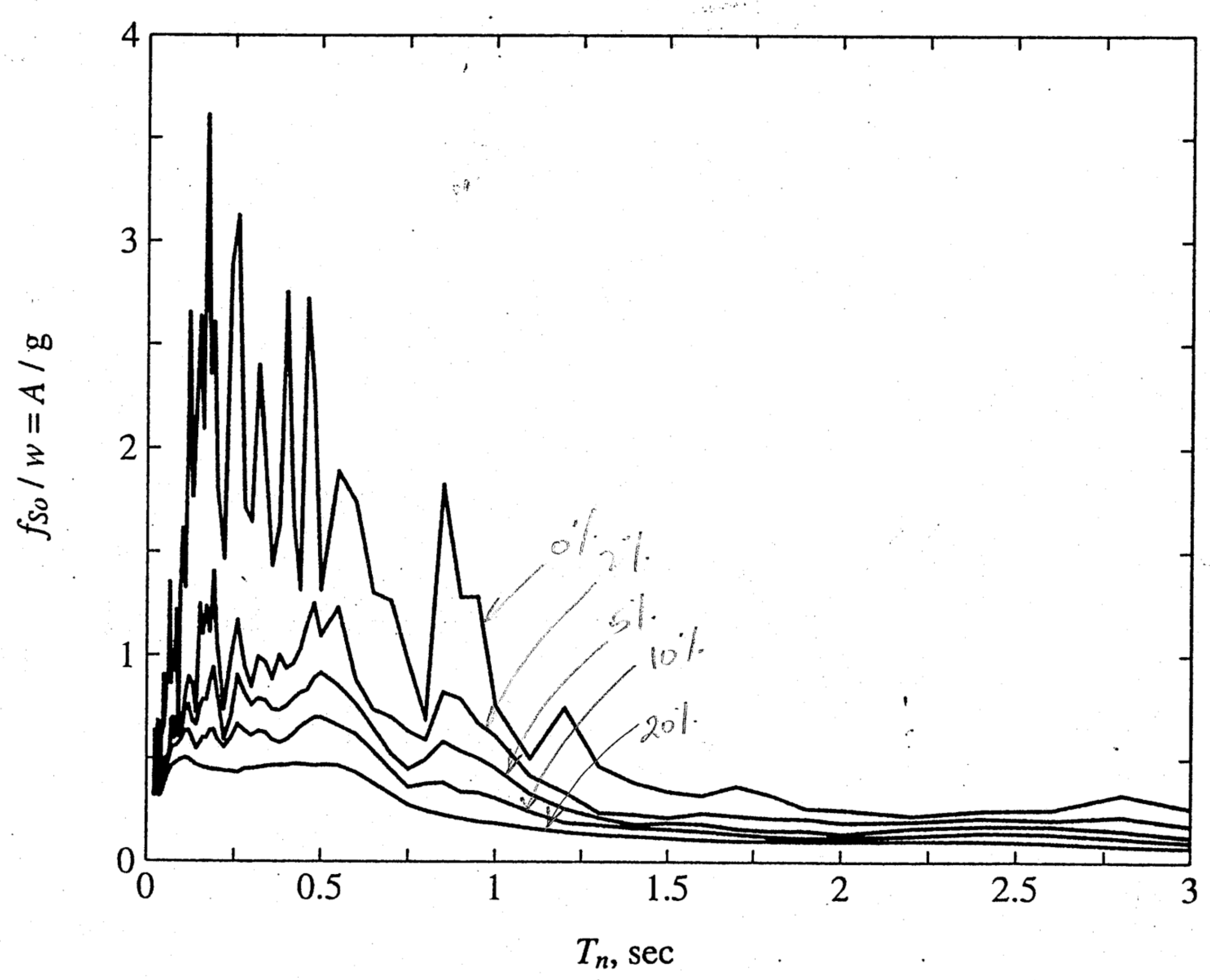
(This graph shows A/g)

The Pseudo velocity $\mathcal{V}$ is related to the peak values of the strain energy, E_{s0}, stored in the system:

$$E_{s0} = \frac{1}{2}ku^2{}_0 = \frac{1}{2}kD^2 = \frac{1}{2}m\omega^2{}_nD^2 = \underbrace{\frac{1}{2}m\mathcal{V}^2}_{} \qquad (34)$$

The kinetic energy of the structural mass m with velocity $\mathcal{V}$.

Normalized pseudo-acceleration, or base shear coefficient, response spectrum for El Centro ground motion; $\xi = 0, 0.02, 0.05, 0.1, 0.2$



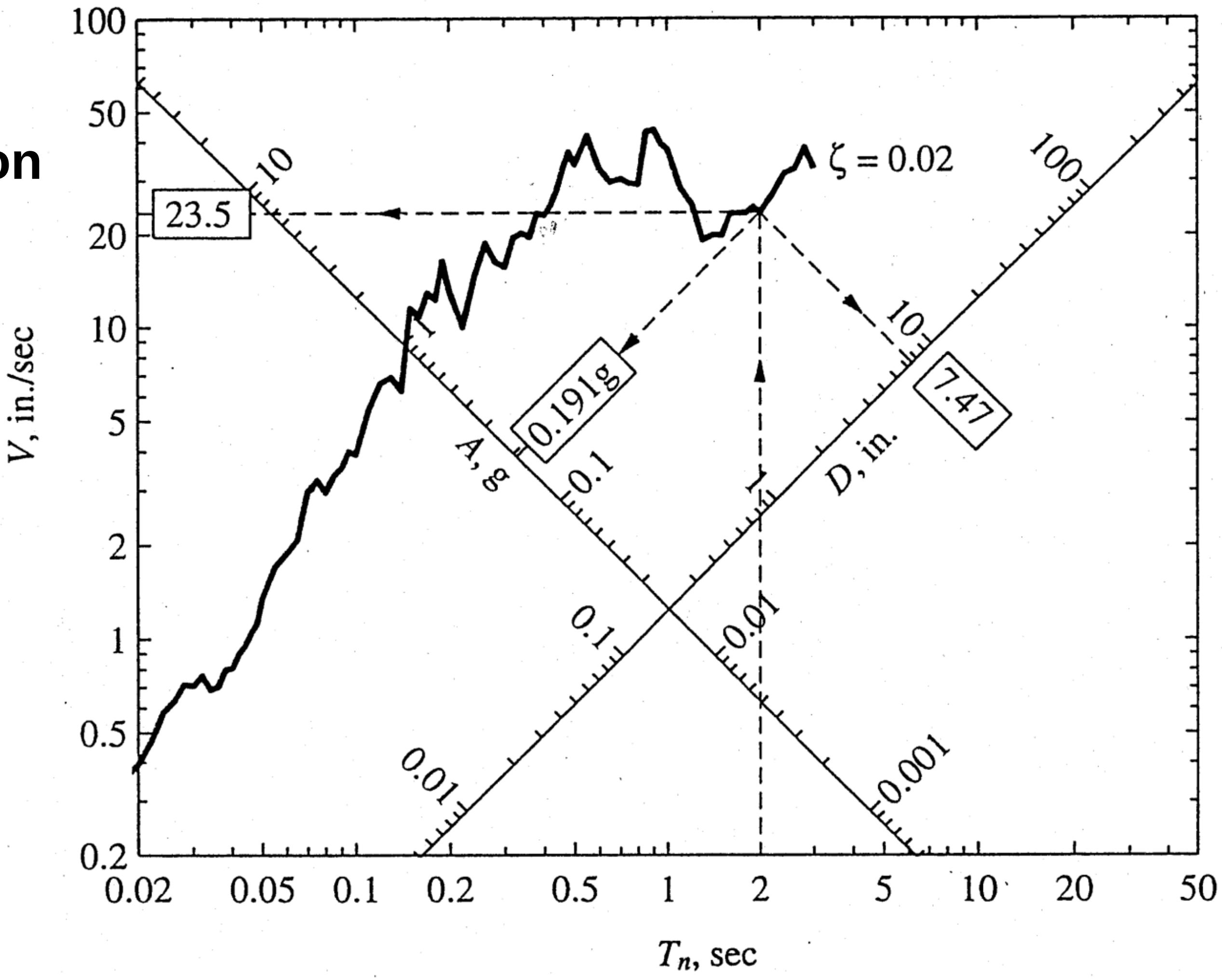
Combined D-V-A Spectrum

Each of the deformation, pseudo-velocity, and pseudo-acceleration response spectra for a given ground motion continue the same information-they are simply different ways of presenting the same information on structural response.

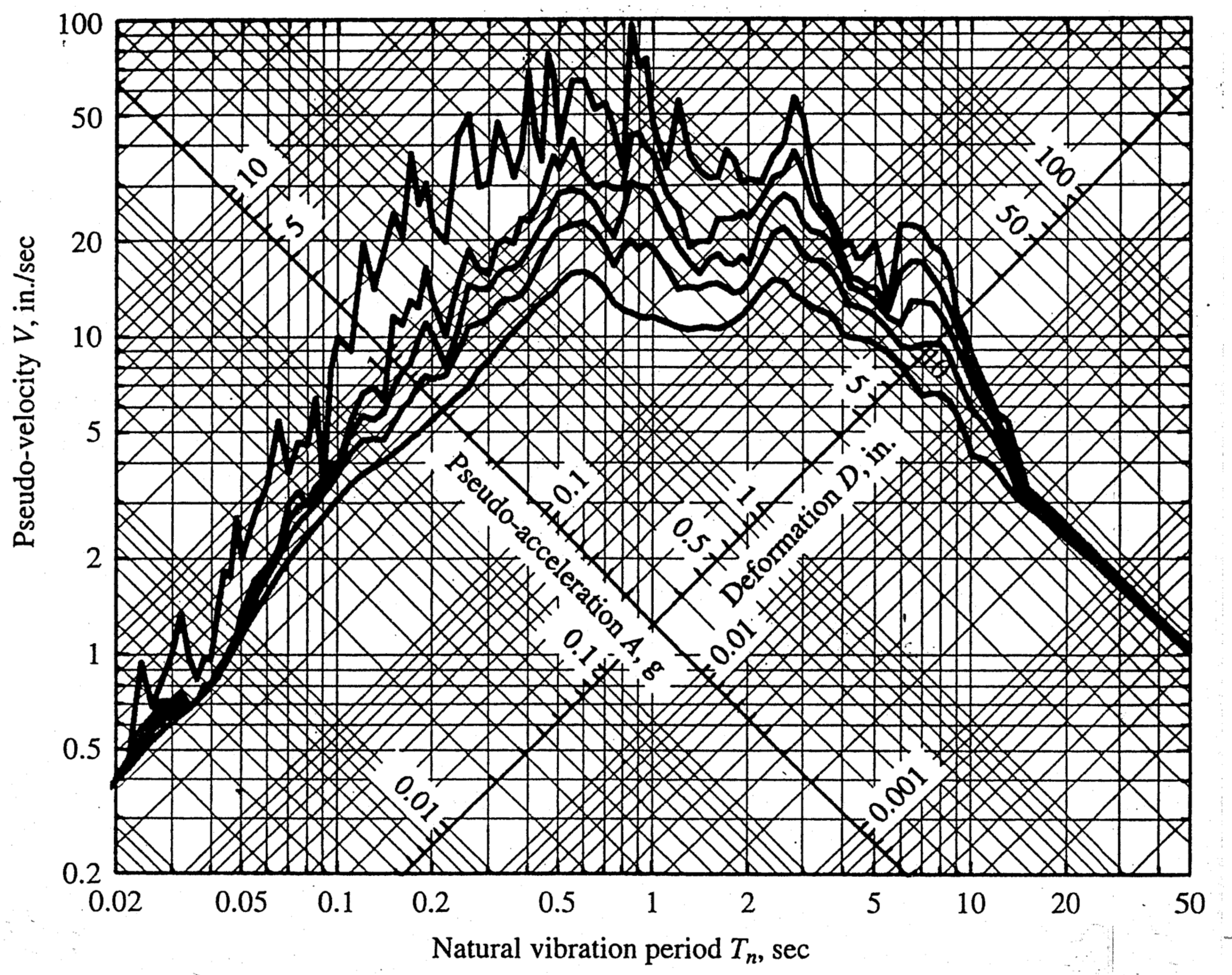
Combined D-V-A response spectrum $(\xi = 0.02)$ for El Centro ground motion

A single curve can simultaneously show three different quantities.

- *The peak deformation*
- *The peak pseudo-velocity which is related to the peak strain energy*
- *The peak pseudo-acceleration which is related to the peak value of equivalent static force (and base shear).*



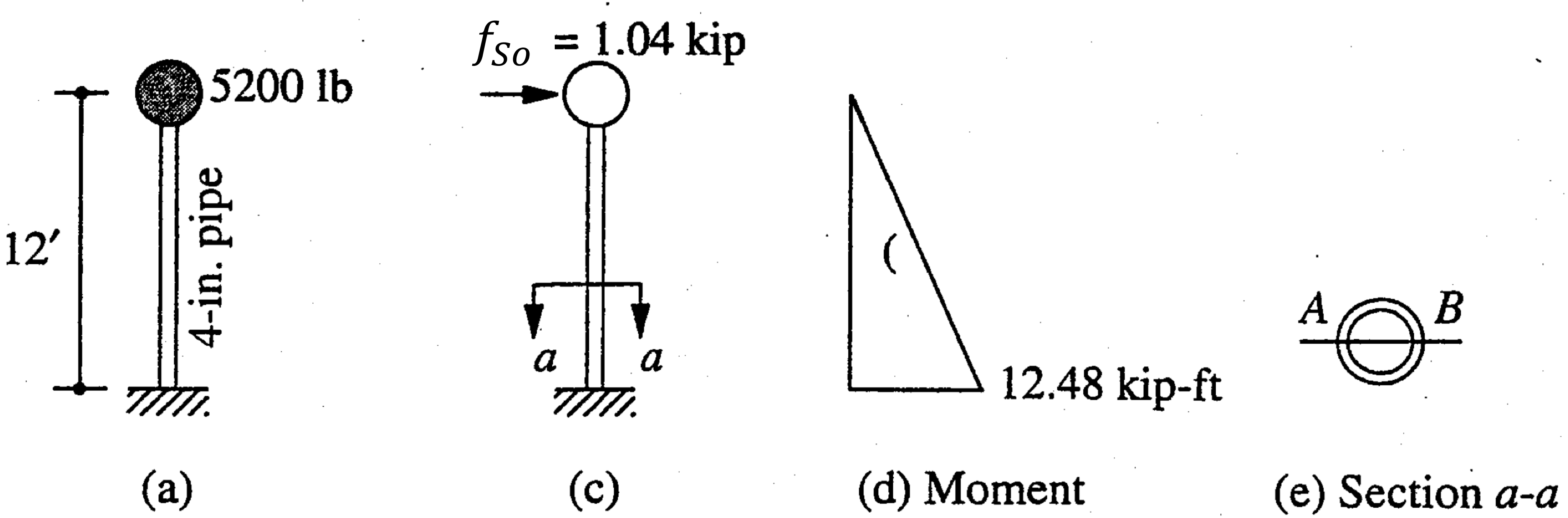
Combined D-V-A response spectrum for El Centro ground motion; $\xi = 0, 0.02, 0.05, 0.10\ and\ 0.20$



Example

A 12-ft-long vertical cantilever, a 4-in.-nominal-diameter standard steel pipe, supports a 5200-lb weight attached at the tip as shown in Fig. E6.2. The properties of the pipe are: outside diameter, d_o= 4.500 in., inside diameter d_i= 4.026 in., thickness t = 0.237 in., and second moment of cross-sectional area, I = 7.23 in^4, elastic modulus E = 29,000 ksi, and weight = 10.79 lb/foot length. Determine the peak deformation and bending stress in the cantilever due to the El Centro ground motion. Assume that ξ = 2%.

Solution



Note: The unit of **force** is $\boldsymbol{kip}$: 1 kip = 1000 lb

The unit of **mass** is therefore the unit of **force** divided by the unit of **acceleration**.

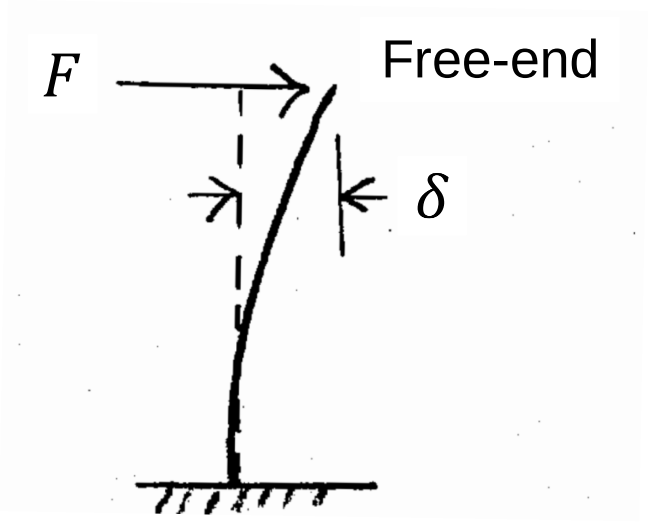
The unit of **acceleration** is $\boldsymbol{in/sec^2}$

$$1g = 9.81\, m/\sec^2 = 32.2\, ft/\sec^2 = 386\, in/\sec^2$$

The unit of $\boldsymbol{E}$ is the unit of **force** divided by the unit of **area**.

The lateral stiffness $\boldsymbol{K}$ in this case is determined from

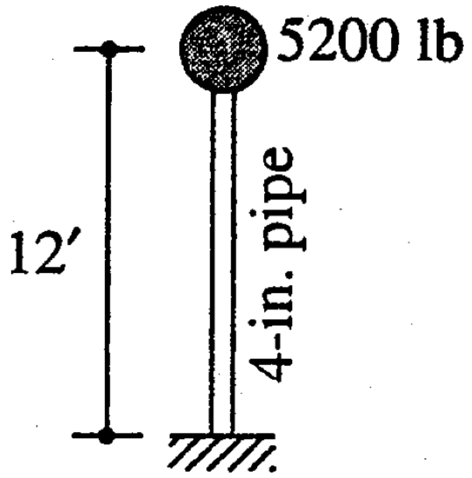
$$K = \frac{F}{\delta}$$



Fixed-End

The lateral stiffness of this SDF system is

$$k = \frac{3EI}{L^3} = \frac{3(29 \times 10^3)7.23}{(12 \times 12)^3} = 0.211\, kip/in.$$

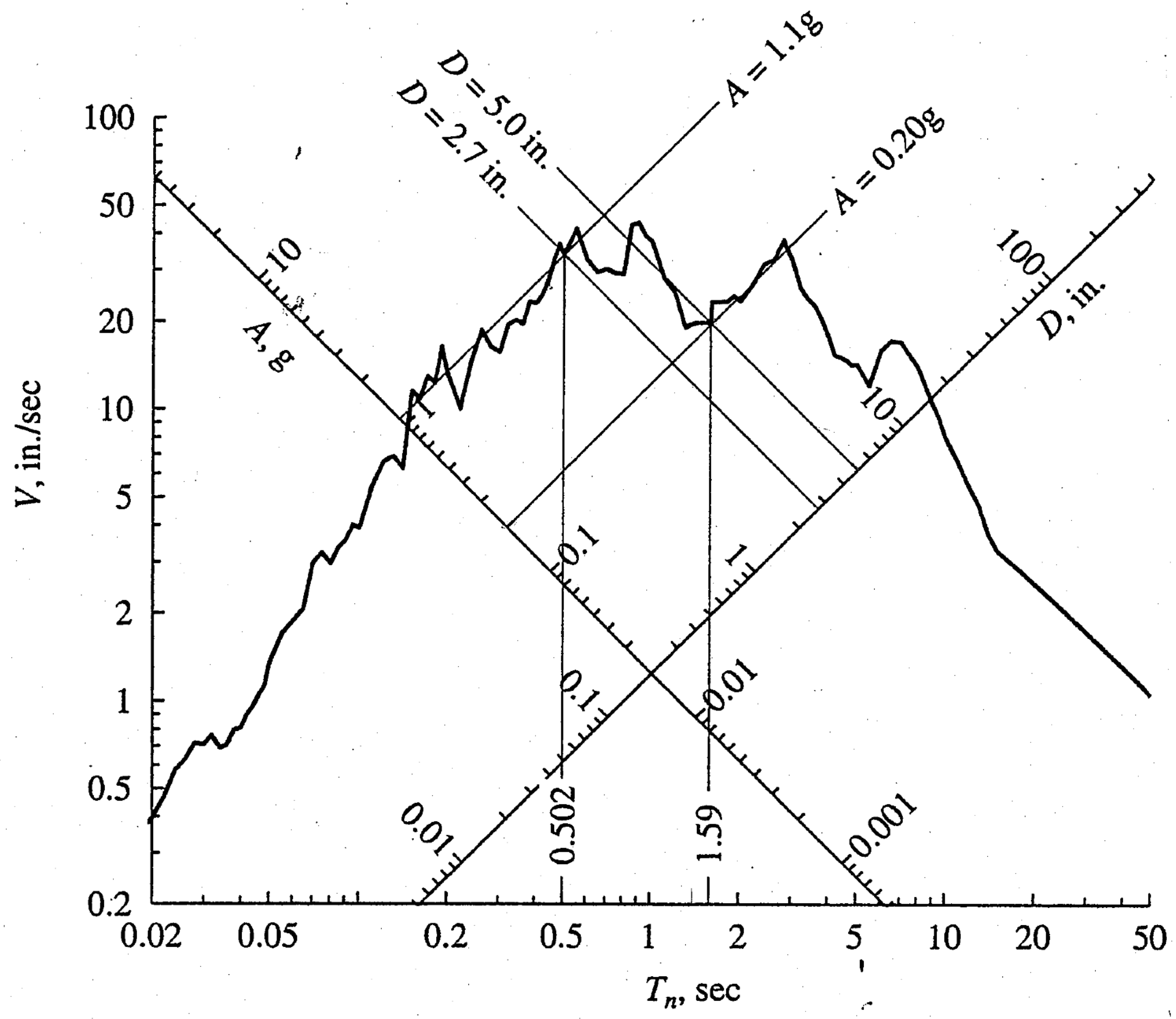


The total weight of the pipe is $10.79 \times 12 = 129.5\ lb$, which may be neglected relative to the lumped weight of $5200\ lb$. Thus

$$m = \frac{w}{g} = \frac{5.20}{386} = 0.01347\ kip - sec^2/in.$$

The natural vibration frequency and period of the system are

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.211}{0.01374}} = 3.958\ rad/sec \quad \Rightarrow \quad T_n = 1.59\, sec$$



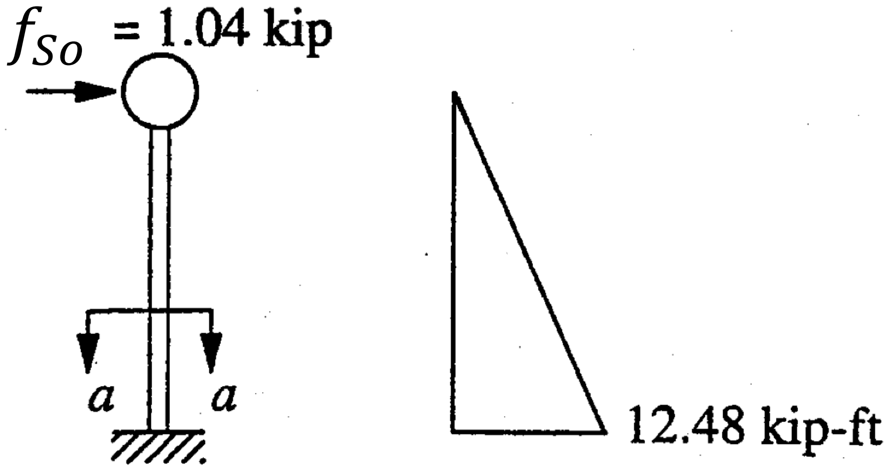
From the response spectrum curve for ξ = 2%, for T_n = 1.59 sec,

$$D = 5.0\ in. = u_o$$

$$A = 0.20g$$

The equivalent static force is

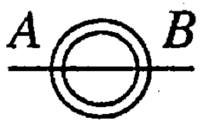
$$f_{So} = \frac{A}{g}w = 0.20 \times 5.2 = 1.04\ kips$$



(c) (d) Moment

The **bending moment diagram** is shown in Fig. (d) with the maximum moment at the base = 12.48 kip-ft. Points *A* and *B* shown in Fig. (e) are the locations of maximum bending stress

$$\sigma_{max} = \frac{Mc}{I} = \frac{(12.48 \times 12)(4.5/2)}{7.23} = 46.5\ ksi$$



(e) Section *a-a*

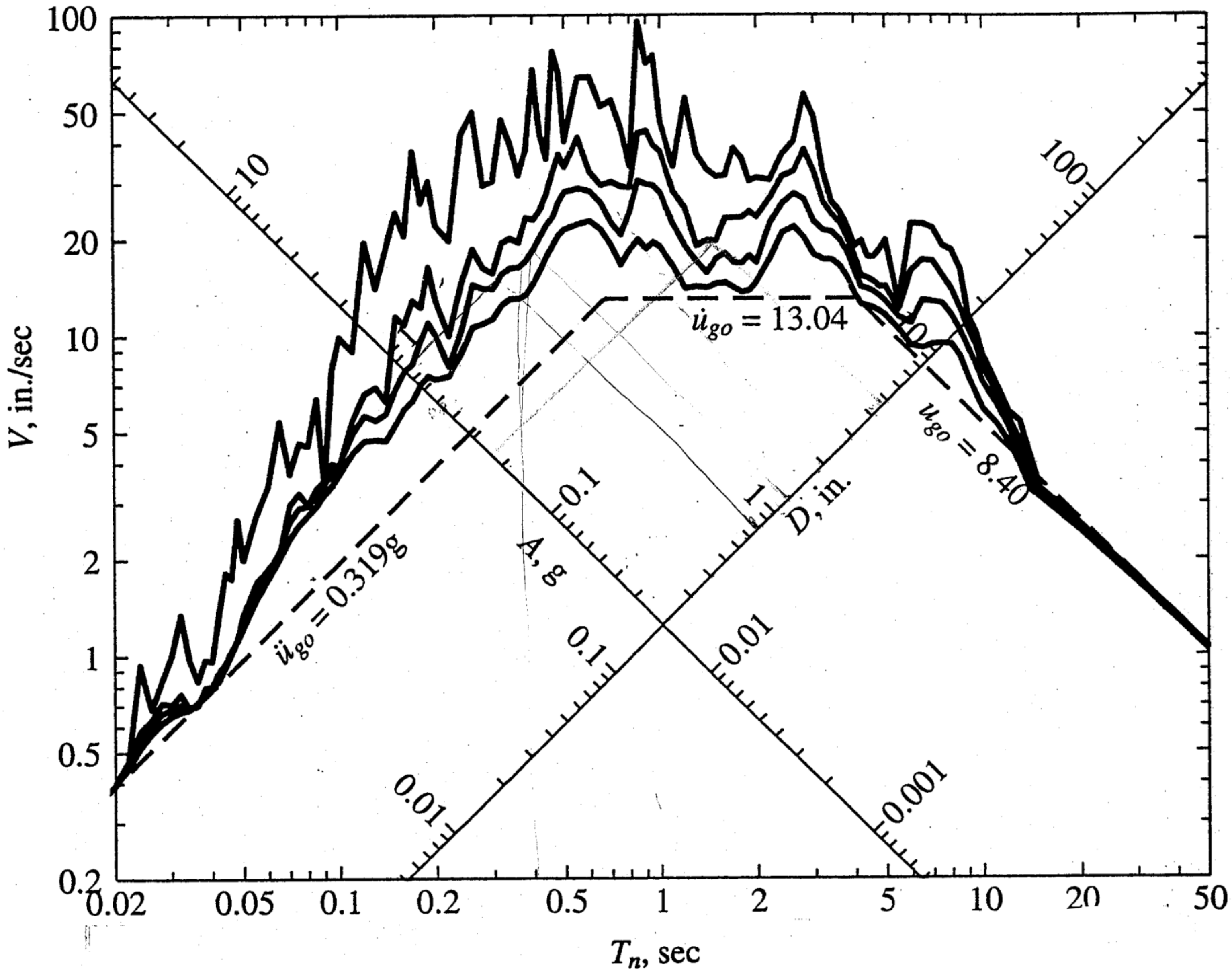
As shown, σ = +46.5 ksi at A and σ = −46.5 ksi at B, where + denotes tension. **The algebraic signs of these stresses are irrelevant** because the direction of the peak force is not known, as the pseudo-acceleration spectrum is, by definition, positive.

Response Spectrum Characteristics

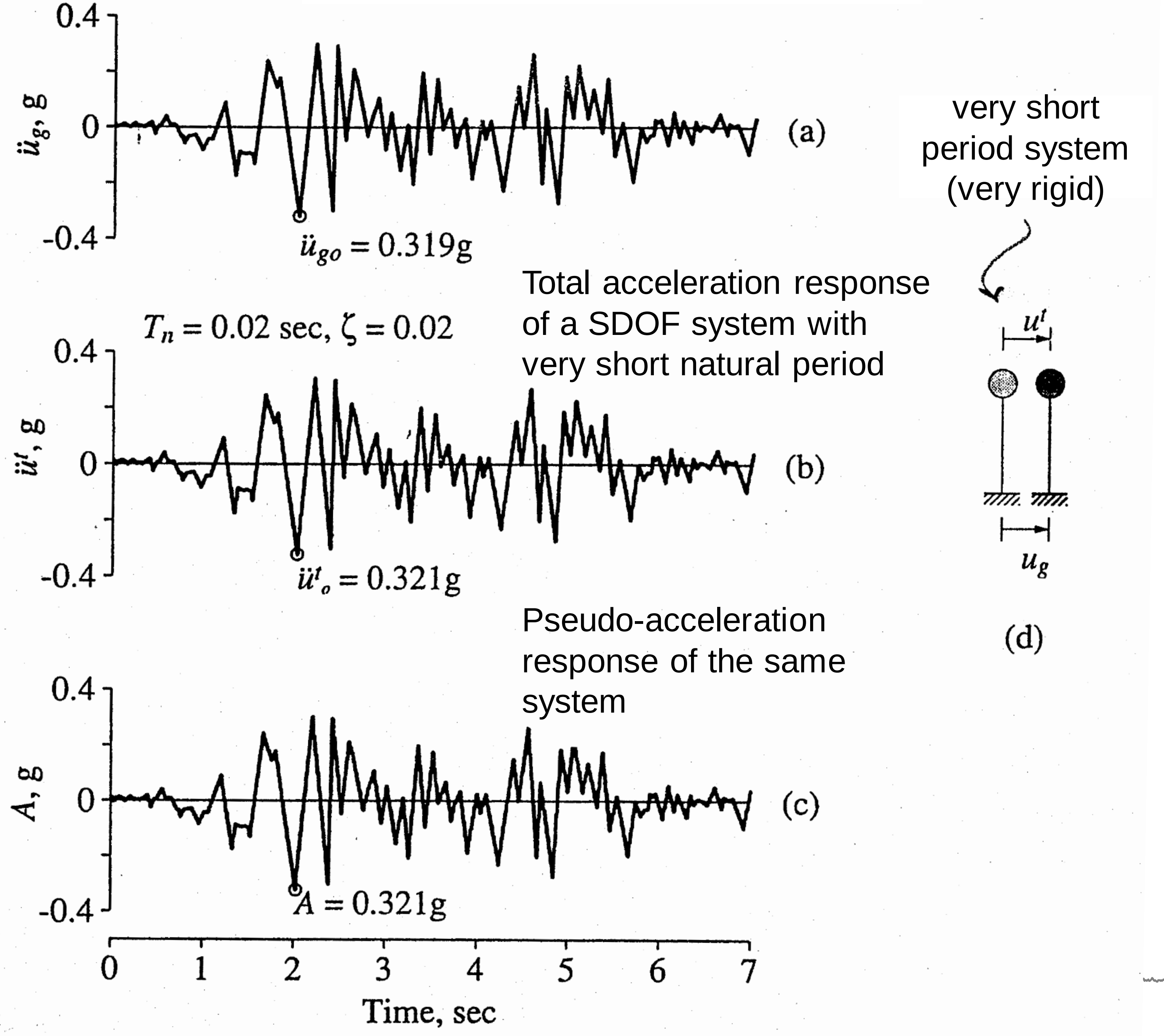
Combined D-V-A response spectrum ($\xi = 0$, 0.02, 0.05, 0.1) and peak values of ground acceleration, ground velocity, and ground displacement for El Centro ground motion

For systems with very short period, the pseudo-acceleration A for all damping values approach $\ddot{u}_{g0}$ and D is very small.

For systems with very long period, D for all the damping values approach u_{g0} and A is very small; thus the forces in the structures, which are related to the mA, would be very small.



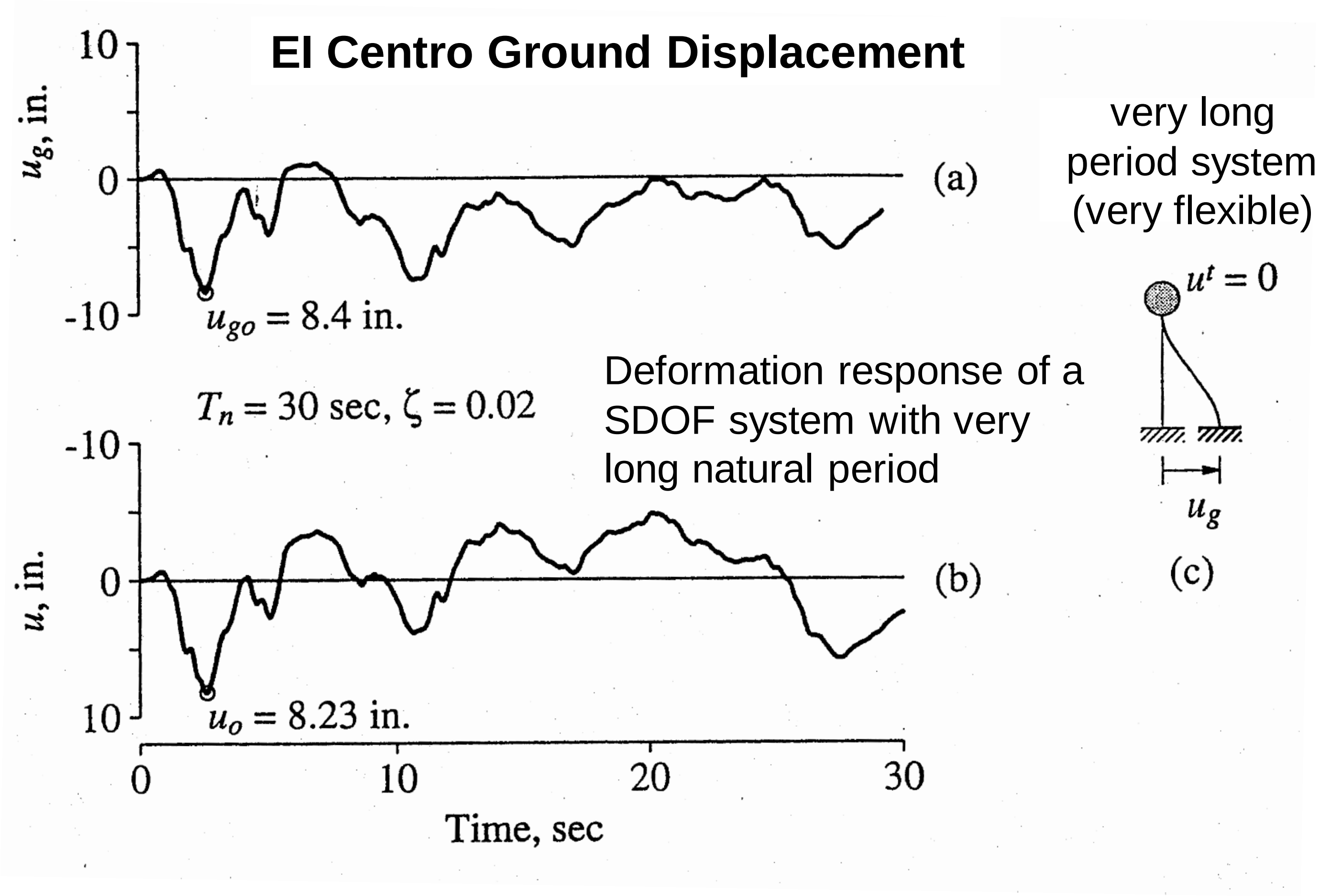
El Centro Ground Acceleration



A very short period system is extremely stiff and rigid. Its deformation response to the ground motion is very small. So its mass move rigidly with the ground and its peak structural acceleration should be approximately $\ddot{u}_{g0}$.

To drive the structural mass to move with acceleration of $\ddot{u}_{g0}$, it is necessary to have $f_{s0} \approx m\,\ddot{u}_{g0}$,

therefore, $A \approx \ddot{u}_{g0}$



A very long period system is extremely flexible. The mass would be expected to remain essentially stationary while the ground below moves.

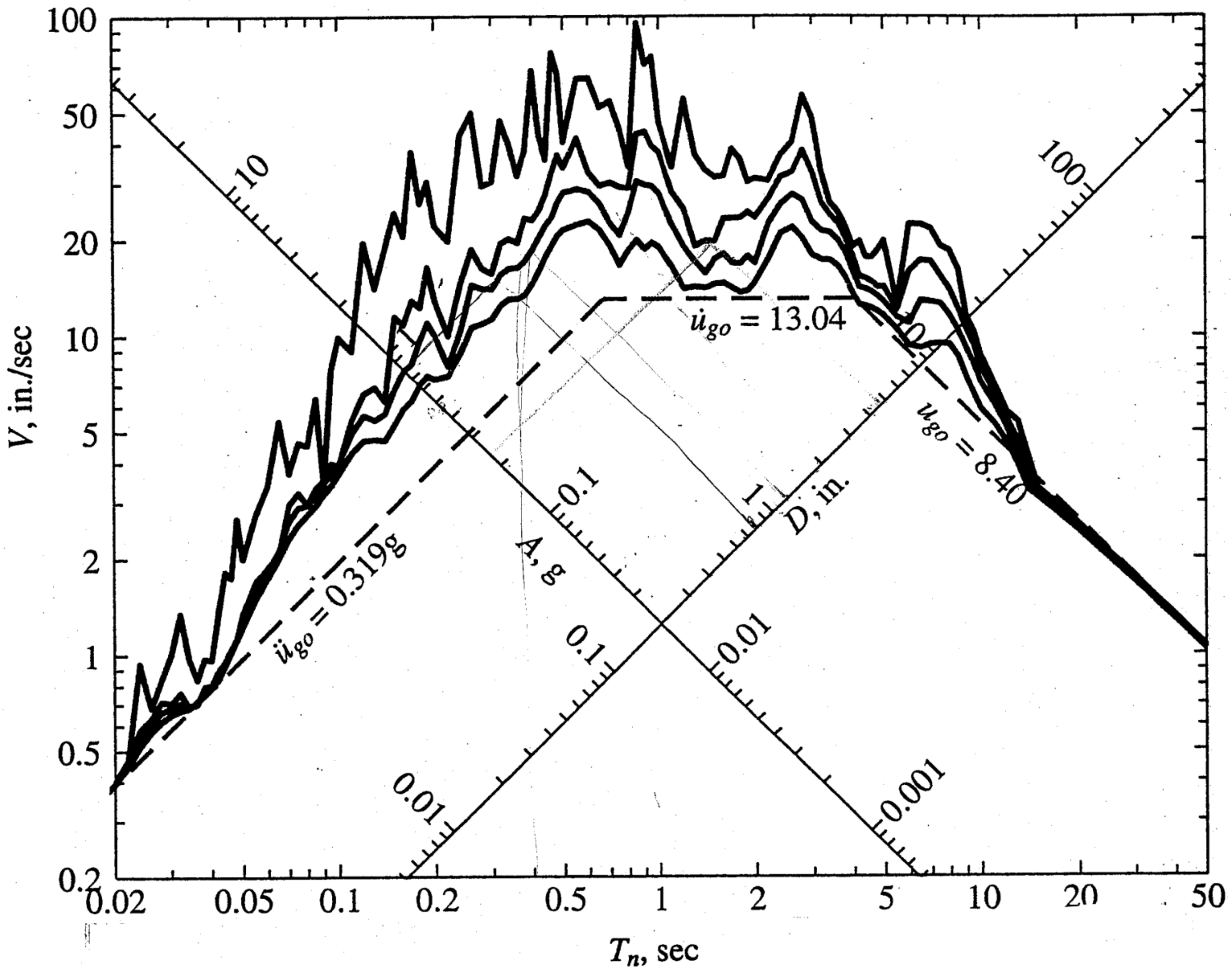
Thus $u(t) \cong -u_g(t)$ that is $D \cong u_{g0}$

Response Spectrum Characteristics

Combined D-V-A response spectrum ($\xi = 0$, 0.02, 0.05, 0.1) and peak values of ground acceleration, ground velocity, and ground displacement for El Centro ground motion

The reduction of response due to additional damping is different for the different spectral regions- greatest in the velocity-sensitive region.

The effectiveness of damping in reducing the structural response also depends on the ground motion characteristics.



- For Mexico city 1985 earthquake where ground motion is nearly harmonic over many cycles, the effect of damping would be large for a system near “resonance”.

- For Park Filed 66 earthquake where ground motion is very short and shock like, the effect of damping would be small, as in the case of half cycle sine pulse excitation.

Response Spectra

- The construction of response spectra plots requires the **solution of single degree of freedom systems for a sequence of natural frequency and of the damping ratio** in the range of interest.

- Every solution provides **only one point** (the maximum value) of the response spectrum.

- Since a large number of systems must be analyzed in order to fully plot each response spectrum, the task is lengthy and time consuming even with the use of computer.

- Once these curves are constructed and are available for the excitation of interests, the **analysis for the design of structures subjected to dynamic loading is reduced to a simple calculation of natural frequency of the system and the use of response spectra.**

Response Spectra

- Dynamic analysis of a system with n **degree of freedom** can be transformed to the problem of solving **n systems** in which each one is a **single degree of freedom system**

- The understanding and mastery of the concepts and methods of solutions for a single degree of freedom system is quit important.

Pseudo-velocity response spectrum

- Consider a quantity V for an SDF system with natural frequency ω related to its peak deformation $D \equiv u_o$ due to earthquake ground motion:

$$V = \omega D = \frac{2\pi}{T} D$$

- The quantity V has units of velocity. This is pseudo-velocity.

- Using the above expression, the displacement response spectrum can be converted to the Pseudo-velocity response spectrum.

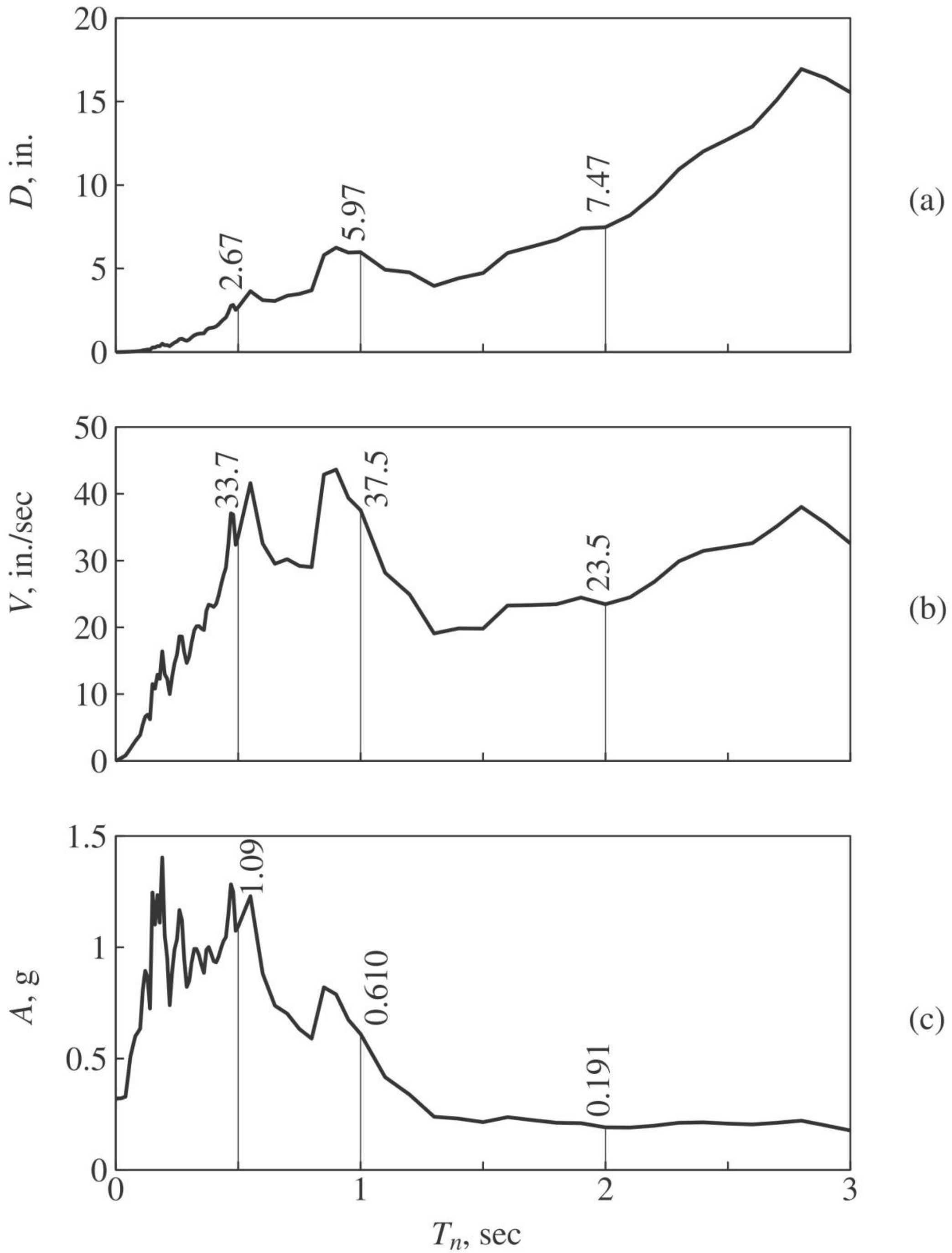
Pseudo-acceleration response spectrum

- Consider a quantity A for an SDF system with natural frequency ω related to its peak deformation $D \equiv u_o$ due to earthquake ground motion:

$$A = \omega^2 D = \left(\frac{2\pi}{T}\right)^2 D$$

- The quantity A has units of acceleration. This is pseudo-acceleration.

- Using the above expression, the displacement response spectrum can be converted to the Pseudo-acceleration response spectrum.



Response spectra ($\xi = 0.02$) for El Centro ground motion: (a) deformation response spectrum; (b) pseudo-velocity response spectrum; (c) pseudo-acceleration response spectrum.

Pseudo-acceleration response spectrum

- The quantity A is related to the peak value of base shear V_{bo} or the peak value of the equivalent static force f_{so}.

$$V_{bo} = f_{so} = mA$$

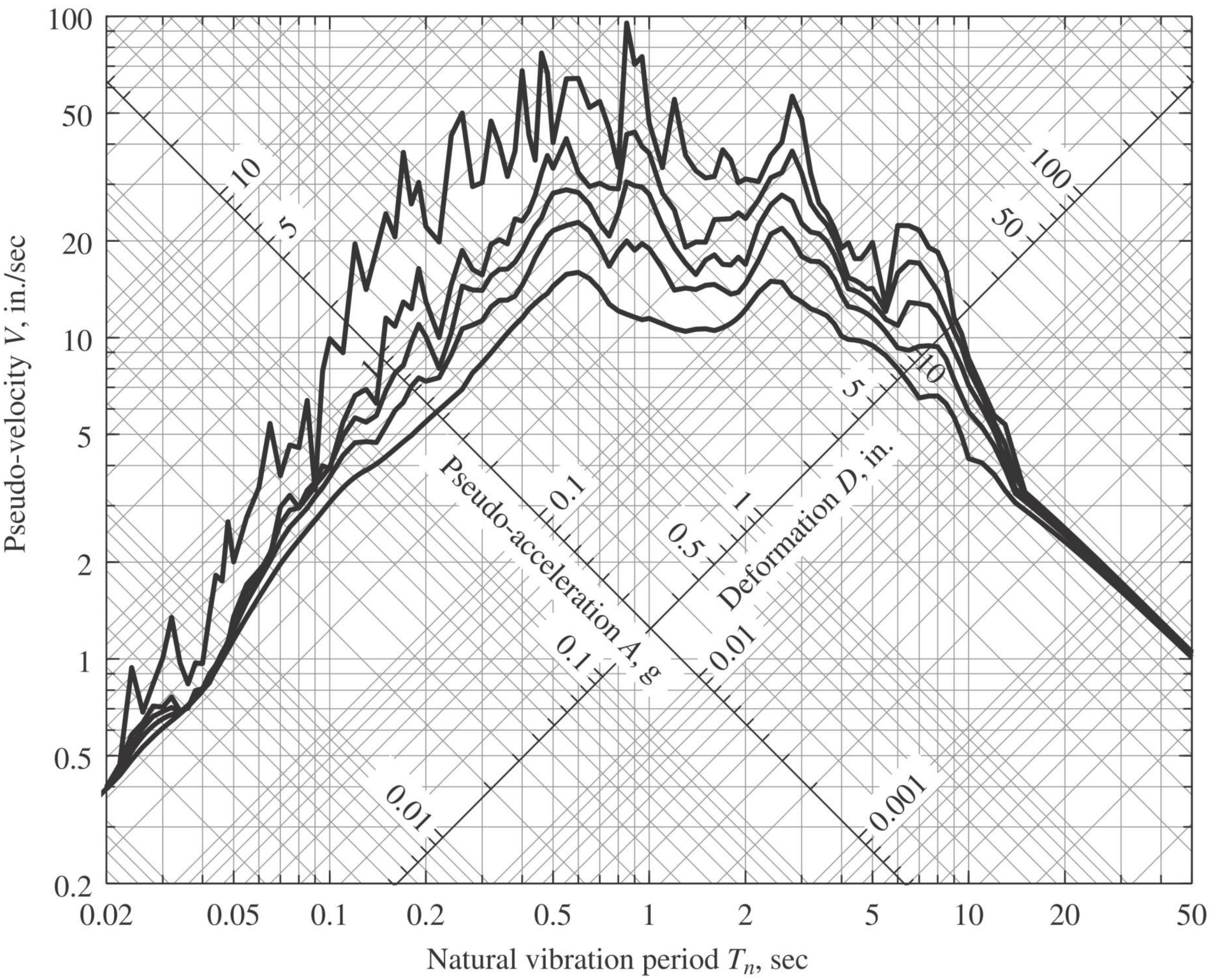
- The peak base shear can be written in the form

$$V_{bo} = \frac{A}{g}w \quad , \quad f_{so} = \frac{A}{g}w$$

Combined D-V-A response spectrum

- The three spectra (deformation, pseudo-velocity, and pseudo-acceleration) are simply different ways of presenting the same information on structural response for a given ground motion.

- Knowing one of the spectra, the other two can be obtained by algebraic operations mentioned earlier.



Combined D–V –A

response spectrum for El

Centro ground motion

ξ = 0, 2, 5, 10, and 20%.

Construction of Response Spectrum

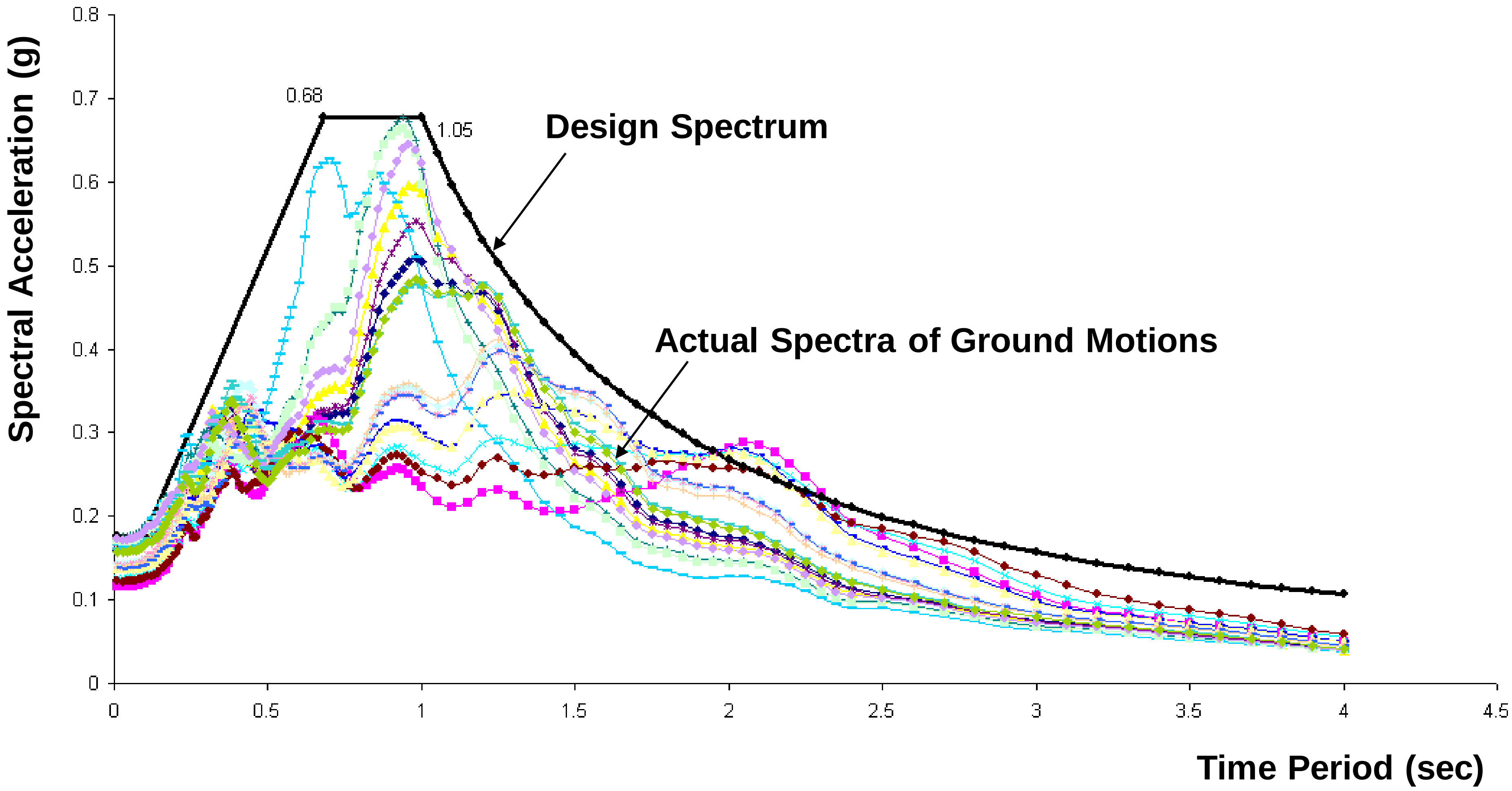
The response spectrum for a given ground motion component $\ddot{u}_g(t)$ can be developed by implementation of the following steps:

1) Numerically define the ground acceleration $\ddot{u}_g(t)$; typically, the ground motion ordinates are defined every 0.02 sec.
2) Select the natural vibration period T and damping ratio ξ of an SDF system.
3) Compute the deformation response $u(t)$ of this SDF system due to the ground motion $\ddot{u}_g(t)$ by any of the numerical methods.
4) Determine u_o, the peak value of $u(t)$.

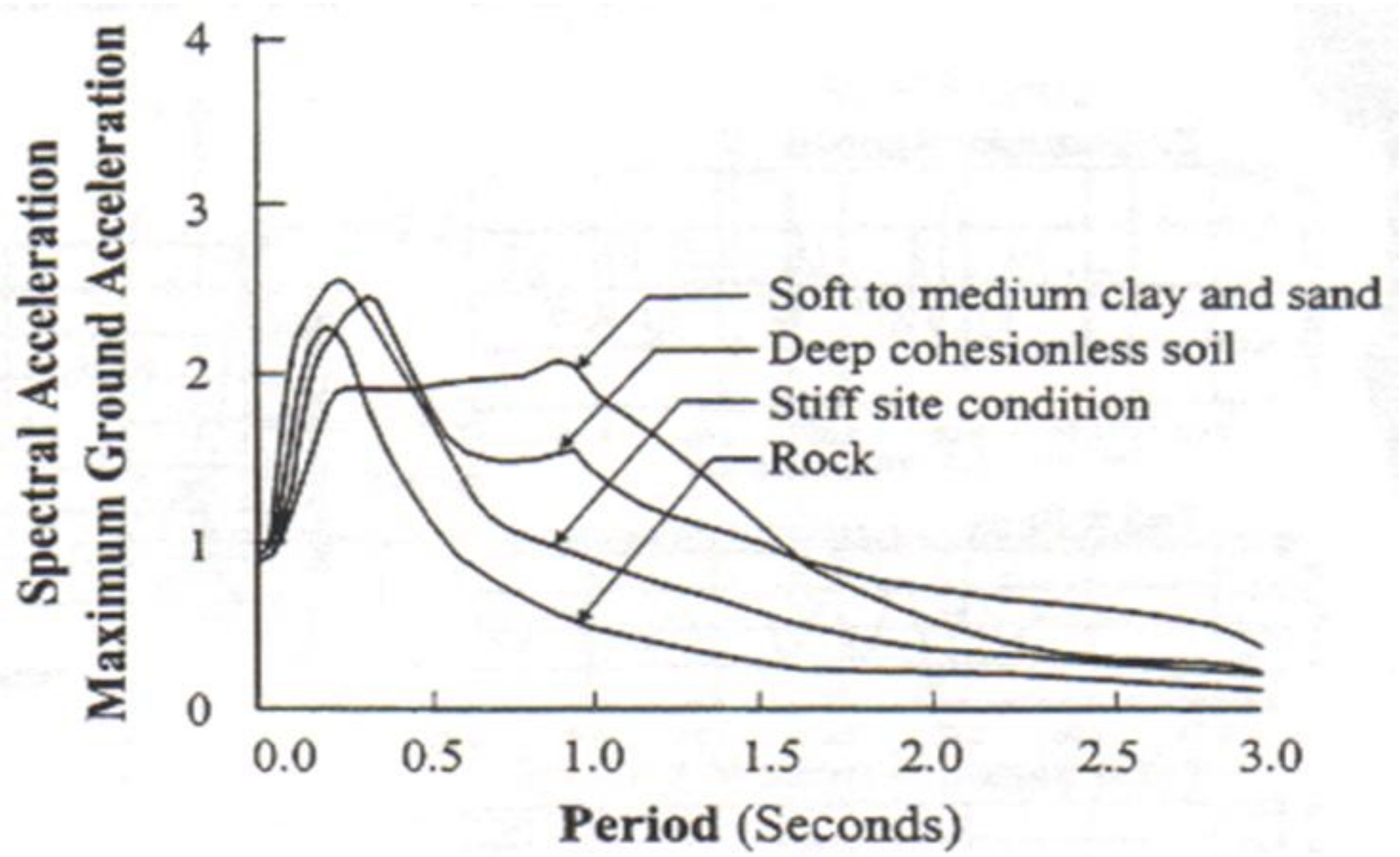
Construction of Response Spectrum

5) The spectral ordinates are $D = u_o$, $V = (2\pi/T)D$, and $A = (2\pi/T)^2 D$.
6) Repeat steps 2 to 5 for a range of T and ξ values covering all possible systems of engineering interest.
7) Present the results of steps 2 to 6 graphically to produce three separate spectra or a combined spectrum.

Sample Response Spectra



Spectra For Different Soils



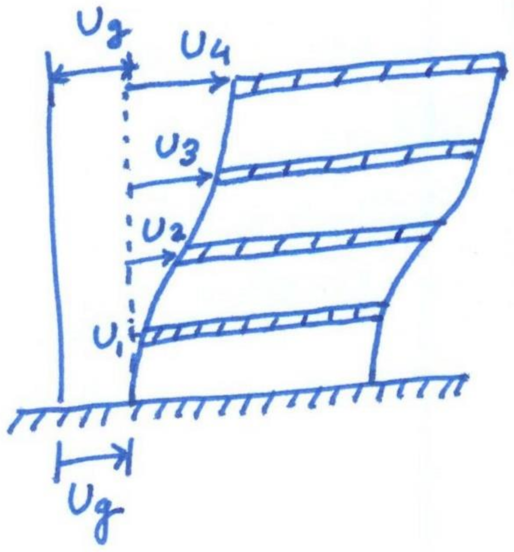
The Response Spectrum Analysis (RSA) Procedure for Buildings – Mathematical Formulation

The Elastic Response Spectrum Analysis (RSA) Procedure

Arturo Danusso (1880 - 1968) Theodore von Karman (1881 - 1963) Maurice Anthony Biot (1905 - 1985)

Earthquake Response of MDF Structures

A shear building subjected to ground displacement $u_g(t)$:



$$\mathbb{U}^t(t) = \begin{Bmatrix} u_g+u_4 \\ u_g+u_3 \\ u_g+u_2 \\ u_g+u_1 \end{Bmatrix} = \begin{Bmatrix} u_g \\ u_g \\ u_g \\ u_g \end{Bmatrix} + \begin{Bmatrix} u_4 \\ u_3 \\ u_2 \\ u_1 \end{Bmatrix}$$

$$= \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix} u_g + \mathbb{U}(t)$$

u_g : Ground displacement

u : Structural (relative) displacement

$$\mathbb{U}^t(t) = \mathbb{1}\, u_g(t) + \mathbb{U}(t)$$

$$\ddot{\mathbb{U}}^t(t) = \mathbb{1}\, \ddot{u}_g(t) + \ddot{\mathbb{U}}(t)$$

Earthquake Response of MDF Structures

Mathematical Formulation:

$$\mathbb{M}\ddot{\mathbb{U}}^t(t) + \mathbb{C}\dot{\mathbb{U}}(t) + \mathbb{K}\mathbb{U}(t) = \mathbb{O}$$

(Eq. of motion for MDF systems subjected to ground motion)

$$\ddot{\mathbb{U}}^t(t) = \mathbb{1}\ddot{u}_g(t) + \ddot{\mathbb{U}}(t)$$

$$\mathbb{M}\ddot{\mathbb{U}}(t) + \mathbb{C}\dot{\mathbb{U}}(t) + \mathbb{K}\mathbb{U}(t) = -\mathbb{M}\mathbb{1}\ddot{u}_g(t)$$

Transforming the above eq. into principal (modal) coordinates,

$$\mathbb{U}(t) = \sum_{i=1}^{4} \phi_i q_i(t) = \Phi \mathbb{q}$$

Where

$$\Phi = [\phi_1 \;\; \phi_2 \;\; \phi_3 \;\; \phi_4], \quad \mathbb{q} = \begin{Bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \end{Bmatrix}$$

ϕ_i = ith-mode vector

Using Modal Orthogonality:

$$\mathbb{M}\Phi\ddot{q}(t) + \mathbb{C}\Phi\dot{q}(t) + \mathbb{K}\Phi q(t) = -\mathbb{M}\mathbb{1}\ddot{u}_g(t)$$

multiplying with Φ^T, (and using modal orthogonality)

$$\Phi^T\mathbb{M}\Phi\ddot{q}(t) + \Phi^T\mathbb{C}\Phi\dot{q}(t) + \Phi^T\mathbb{K}\Phi q(t) = -\Phi^T\mathbb{M}\mathbb{1}\ddot{u}_g(t)$$

$$\begin{bmatrix}\mu_1 & 0 & 0 & 0\\ 0 & \mu_2 & 0 & 0\\ 0 & 0 & \mu_3 & 0\\ 0 & 0 & 0 & \mu_4\end{bmatrix}\ddot{q}(t) + \begin{bmatrix}2\xi_1\mu_1\omega_1 & & 0\\ & \ddots & \\ 0 & & 2\xi_4\mu_4\omega_4\end{bmatrix}\dot{q}(t) + \begin{bmatrix}\mu_1\omega_1^2 & & 0\\ & \ddots & \\ 0 & & \mu_4\omega_4^2\end{bmatrix}q(t)$$

$$= -\mathbb{L}\ddot{u}_g(t)$$

where $\mathbb{L} = \begin{bmatrix}L_1\\ L_2\\ L_3\\ L_4\end{bmatrix}$, $L_i = \phi_i^T . M . \mathbb{1}$

$$= \begin{bmatrix}\phi_{i4} & \phi_{i3} & \phi_{i2} & \phi_{i1}\end{bmatrix}\begin{bmatrix}M_4 & & & 0\\ & M_3 & & \\ & & M_2 & \\ 0 & & & M_1\end{bmatrix} \times \begin{bmatrix}1\\ 1\\ 1\\ 1\end{bmatrix}$$

$$L_i = \sum_{j=1}^{4} M_j \phi_{ij}$$

Uncoupled Equations of Motion:

The "ith-mode" equation is

$$\mu_i \ddot{q}_i(t) + 2\xi_i \mu_i \omega_i \dot{q}_i(t) + \mu_i \omega_i^2 q_i(t) = -L_i \ddot{u}_g(t)$$

$$\mu_i = \Phi_i^T \mathbb{M} \Phi_i = \begin{bmatrix} \phi_{i4} & \phi_{i3} & \phi_{i2} & \phi_{i1} \end{bmatrix} \begin{bmatrix} M_4 & & & 0 \\ & M_3 & & \\ & & M_2 & \\ 0 & & & M_1 \end{bmatrix} \begin{bmatrix} \phi_{i4} \\ \phi_{i3} \\ \phi_{i2} \\ \phi_{i1} \end{bmatrix}$$

$$\mu_i = \sum_{j=1}^{4} M_j \phi_{ij}^2$$

Modal Participation Factor:

$$\Gamma_i = \frac{L_i}{\mu_i} = \frac{\Phi_i^T \cdot \mathbb{M} \cdot \mathbb{1}}{\Phi_i^T \cdot \mathbb{M} \cdot \Phi_i}$$

(How a particular ith mode responds to ground vibration)

So the ith-mode equation becomes,

$$\ddot{q}_i(t) + 2\xi_i \omega_i \dot{q}_i(t) + \omega_i^2 q(t) = -\Gamma_i \ddot{u}_g(t)$$

Uncoupled Equations of Motion

Elastic Modal Response History Analysis (RHA) Procedure

Solve the uncoupled equation of motion for each significant mode (represented by an SDF system) and sum the dynamic responses to get the dynamic response of MDF system

Response Spectrum Analysis (RSA) Procedure

Directly pick the peak response of each significant mode (represented by an SDF system) and approximately combine those peak responses to get the peak response of MDF system

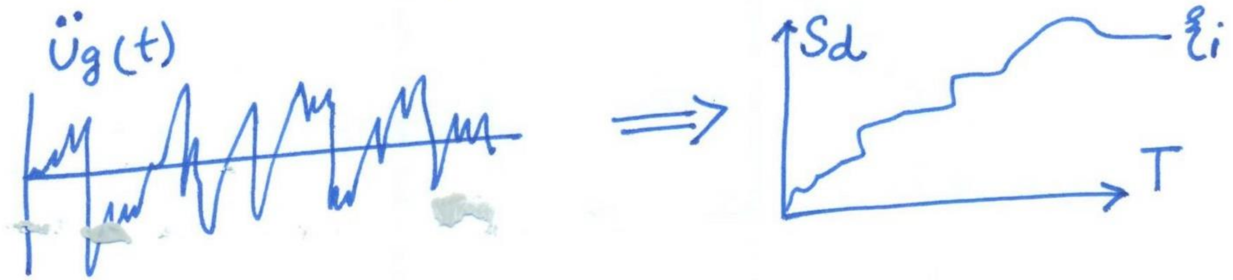
The "Elastic" Response

Spectrum Analysis Procedure

So the ith-mode equation becomes,

$$\ddot{q}_i(t) + 2\xi_i\omega_i\dot{q}_i(t) + \omega_i^2 q_i(t) = -\Gamma_i \ddot{u}_g(t)$$

The maximum deformation in this ith mode is $q_{i,max} = \Gamma_i S_{di}$



$$u_{i,max} = \Gamma_i \Phi_i S_{di}$$

$S_{di} \rightarrow$ determined from elastic response spectrum at T_i and ξ_i.

$$IDR_{i,max} = \Gamma_i(\phi_{ji} - \phi_{j-1,i})S_{di}$$

(at any jth floor)

The “Elastic” Response

Spectrum Analysis Procedure

The equivalent static forces corresponding to ith-mode: (i.e. the forces which will deform structure in $u_{i,max}$ shape and magnitude):

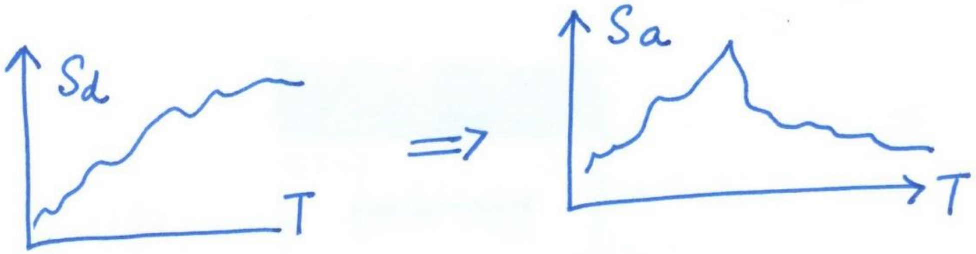
$$
\begin{aligned}
f_{si} &= \mathbb{K}\, u_{i,max} \\
&= \Gamma_i \mathbb{K} \Phi_i S_{di} \\
&= \Gamma_i \omega_i^2 \mathbb{M} \Phi_i S_{di} \\
&= \Gamma_i \cdot \omega_i^2 \cdot S_{di} \begin{bmatrix} M_4 & & 0 \\ & \ddots & \\ 0 & & M_1 \end{bmatrix} \begin{bmatrix} \phi_{i4} \\ \vdots \\ \phi_{i1} \end{bmatrix}
\end{aligned}
$$

From eigen-value equation of ith-mode:

$$\mathbb{K}\Phi_i = \omega_i^2 \mathbb{M} \Phi_i$$

The “Equivalent Static Forces” for each Mode

$$S_a = \omega^2 S_d$$



Therefore,

$$\boxed{f_{si} = \Gamma_i \mathbb{M} \Phi_i S_{ai}}$$

$S_{ai} \rightarrow$ determined from elastic response spectrum at T_i and ξ_i.

The "Elastic" Response Spectrum Analysis Procedure

Modal Combination:

Modal responses attain peaks at different time intervals → So how to combine them? ↵ approximate rules

a) SRSS: $\gamma_{max} \approx \sqrt{\sum_{i=1}^{N} \gamma_{i,max}^2}$ (most widely used)

$\gamma_{i,max}$ → Peak modal response

N → No. of modes considered in analysis

b) Absolute Sum: $\gamma_{max} \simeq \sum_{i=1}^{N} \gamma_{i,max}$

(upper-bound, too conservative not popular)

SRSS combination developed → Rosenblueth's PhD Thesis (1951)

↳ good for structures with well-separated natural frequencies. (closely spaced → not good)

c) Complete Quadratic Combination (CQC):

(Chopra, Chapter 13)

The "Elastic" Response Spectrum Analysis Procedure

Summary:

1) Determine $\mathbb{M}$, $\mathbb{K}$ matrices and ξ_i
2) T_n, Φ_n (eigen-value analysis)
3) Compute peak modal responses of each significant mode. For each ith-mode,

   (i) For T_i and $\xi_i \rightarrow$ Pick S_{ai} and S_{di} from spectrum

   (ii) Compute story disp and story IDRS

   $$u_{i,max} = \Gamma_i \Phi_i S_{di}$$

   $$IDR_{i,max} = \Gamma_i (\phi_{j,i} - \phi_{j-1,i}) S_{di}$$

   at jth floor

   (iii) Compute equivalent static forces

   $$f_{si} = \Gamma_i \mathbb{M} \Phi_i S_{ai}$$

   (iv) Compute all responses by static analysis of structure under f_{si}

4) Estimate Peak respose by combining peaks of individual modal Responses. Usually lower modes contribute significantly to overall response so may be only first few modes can be included in analysis.

The "Elastic" Response

Spectrum Analysis Procedure

Notes:

a) RSA — reduces the dynamic problem into a static problem, infact a series of static analysis under f_{si} (equivalent static forces for few modes).

RSA still retains the features of dynamic analysis — T_n, ξ_i, ω_i, ϕ_i

The "dynamic part of the problem" is already done while developing spectra (S_{di} vs. T and S_{ai} vs. T).

The "Elastic" Response Spectrum Analysis Procedure

Notes:

b) RSA vs. RHA

(i) $U_{i,max} = \Gamma_i \Phi_i S_{di}$ = Same as peak disp from RHA because the spectra is already based on RHA.

but

U_{max} from modal combination rule $\neq$ Peak disp extracted from RHA results (no trend)

c) The direction of f_{si} is controlled by algebraic sign of Φ_i.  , -----

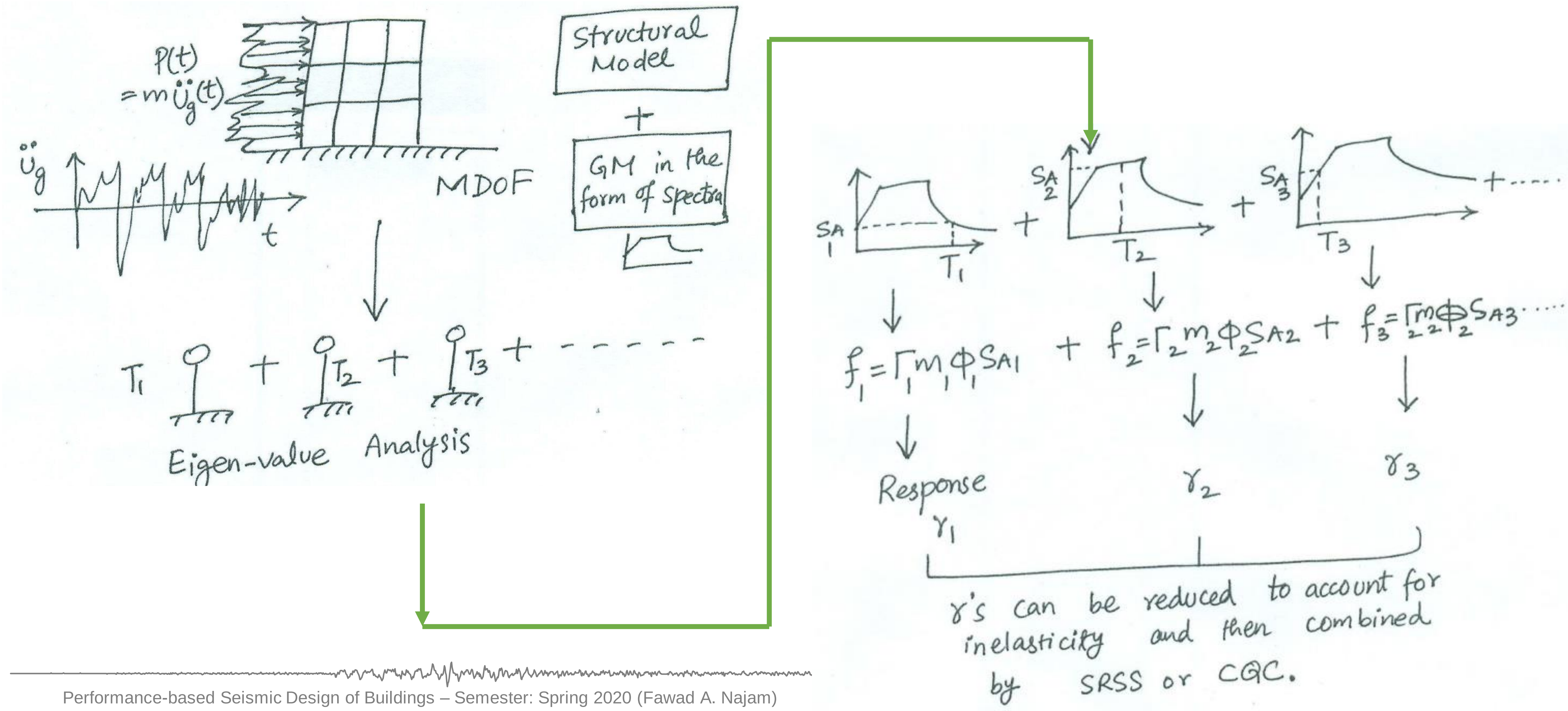
d) Avoid a pitfall: In RSA, calculating one response from combined peak of other response is WRONG. e.g

IDR_{max} at any floor $\neq U_{max\,upper} - U_{max\,lower}$ (U_{max} is combined from all modes)

or

Story shear at any floor $\neq \sum_{i=1}^{N} f_{si} \Rightarrow$ combined equivalent forces for all modes)

The "Elastic" Response Spectrum Analysis Procedure



How to Use Response Spectra?

1) For each mode of free vibration, corresponding **Time Period** is obtained.
2) For each Time Period and specified damping ratio, the specified Response Spectrum is read to obtain the corresponding **Acceleration**.
3) For each Spectral Acceleration, corresponding **velocity and displacements** response for the particular degree of freedom is obtained.
4) The displacement response is then used to obtain the corresponding **stress resultants**.
5) The stress resultants for each mode are then **added using some combination rule** to obtain the final response envelop.

ELF vs. Response Spectrum Analysis

- Static methods specified in building codes are based on single mode response and appropriate for simple and regular structures
- Dynamic analysis should be used for complex buildings to determine significant response characteristics
  - Effects of structure's dynamic characteristics on vertical distribution of lateral forces
  - Increase in dynamic loads due to torsional motions
  - Influence of higher modes, resulting in an increase in story shear and deformations

The Concepts of Inelastic Response Spectra and Design Spectra

(Evolution of Seismic Design Factors (R, Ω and C_d) in Building Codes)

(Evolution of the Concept of Ductility in Building Codes)

Basic Design Objective of Seismic Resistant Design:

- *To protect the life safety of the building occupants and the general public.*
- *To control the severity of damage in small or moderate earthquakes.*

Expected Seismic Performance of Buildings:

- *Resist a **minor level of earthquake** ground motion **without damage***
- *Resist a **moderate level of earthquake** ground motion without structural damage, but possibly experience **some non-structural damage***

❑ *Elastic or lightly inelastic response*
❑ *Drift control*

Need **Elastic Response Spectrum**

- *Acceleration spectra for strength design*
- *Displacement spectra for stiffness (Drift) design*

- *Resist a **major level of earthquake** ground motion having an intensity equal to the strongest either experienced or forecast for the building site, **without collapse**, but possibly with some structural as well as nonstructural damage*

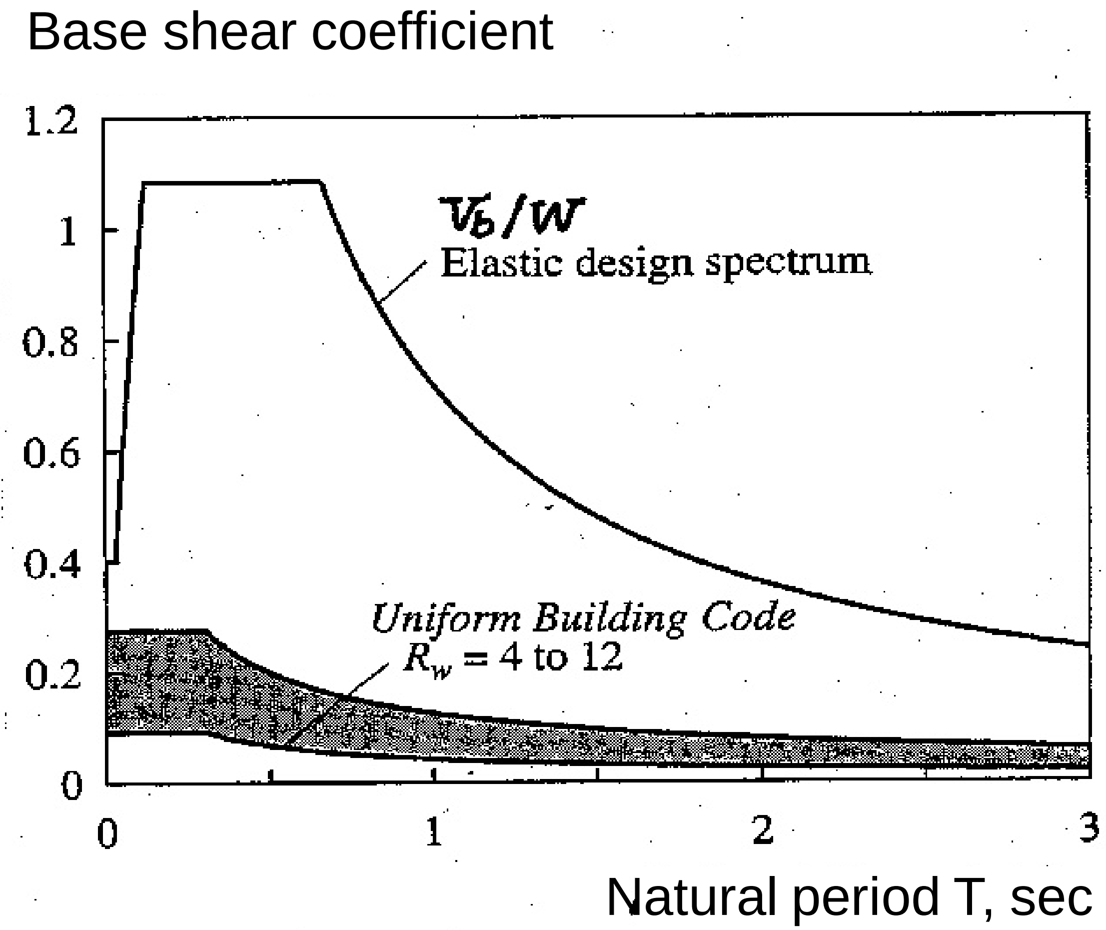
❑ *Inelastic response*
❑ *Control of inelastic deformation*

Need **Inelastic Response Spectrum**

- *Inelastic strength Demand spectra for strength design*

UNIFORM BUILDING CODE

Comparison of base shear coefficient from elastic design spectrum and Uniform Building Code (a US. code) :



V_b: *Peak base shear induced in a linear elastic system by strong ground motion (PGA=0.4g)*

W: *The weight of the system*

Most buildings are designed for base shear much smaller than the elastic base shear *associated with the strongest shaking that can occur at the site.*

Buildings design by the code forces will be deformed ***beyond the limit of linearly elastic*** *behavior when subjected to ground motions represented by the 0.4g design spectrum. Thus it should not be surprising that buildings suffer damage during intense ground shaking.*

The design should be based on Inelastic Response Spectrum.

The challenge to the engineer is to design the structure so that the damage is controlled to an acceptable degree.

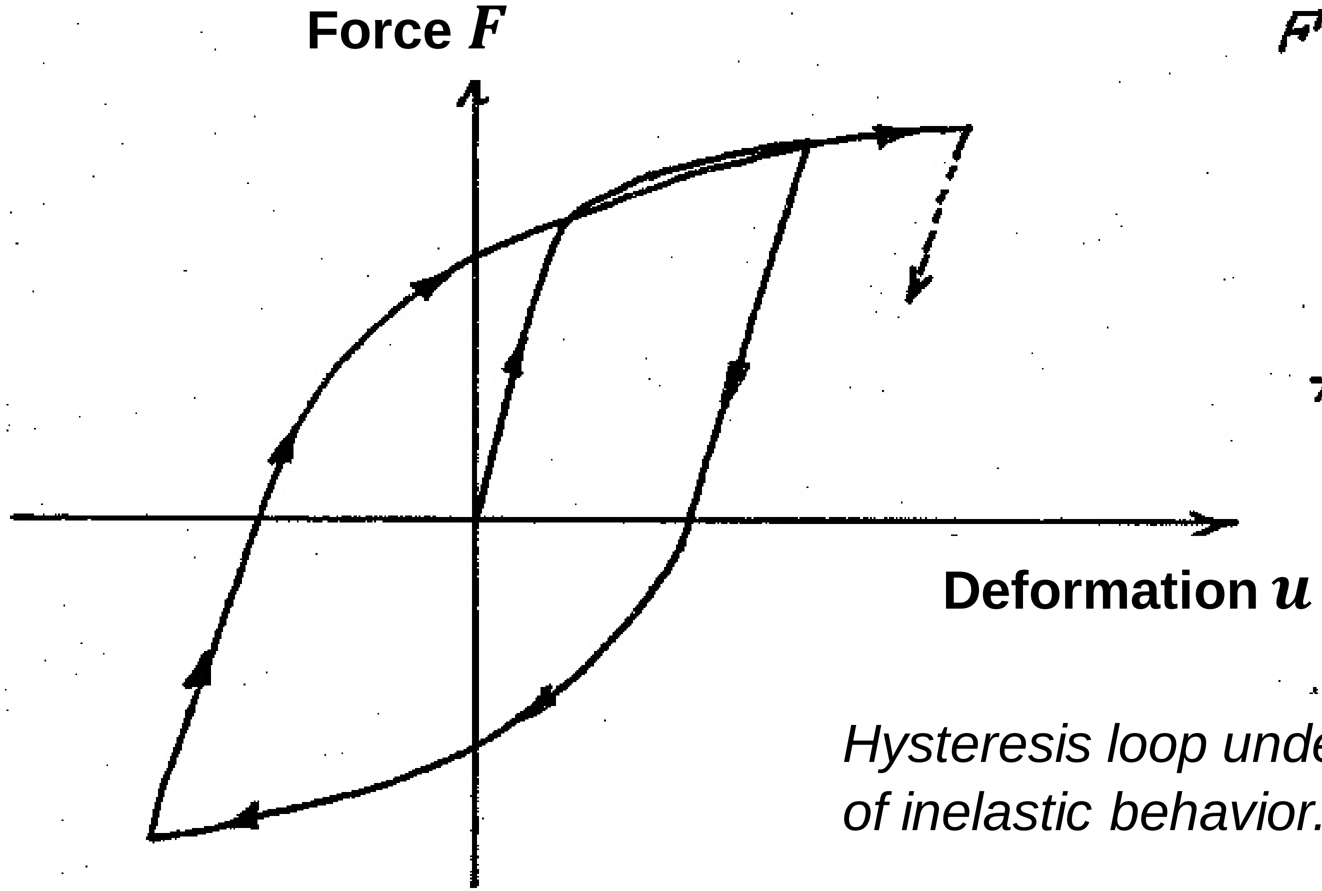
FORCE-DEFORMATION RELATIONS

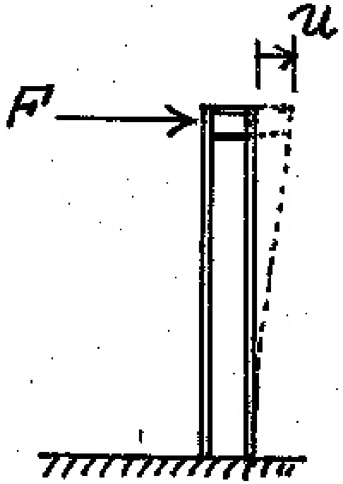
During an earthquake, structures undergo oscillatory motion with reversal of deformation.

The experimental results from cyclic loading conditions indicate that the cyclic force-deformation behavior of a structure depends on

- *the structural material (concrete, steel)*
- *the type of structural members (beam, shear member, axial member)*
- *how members are assembled into the structural system.*

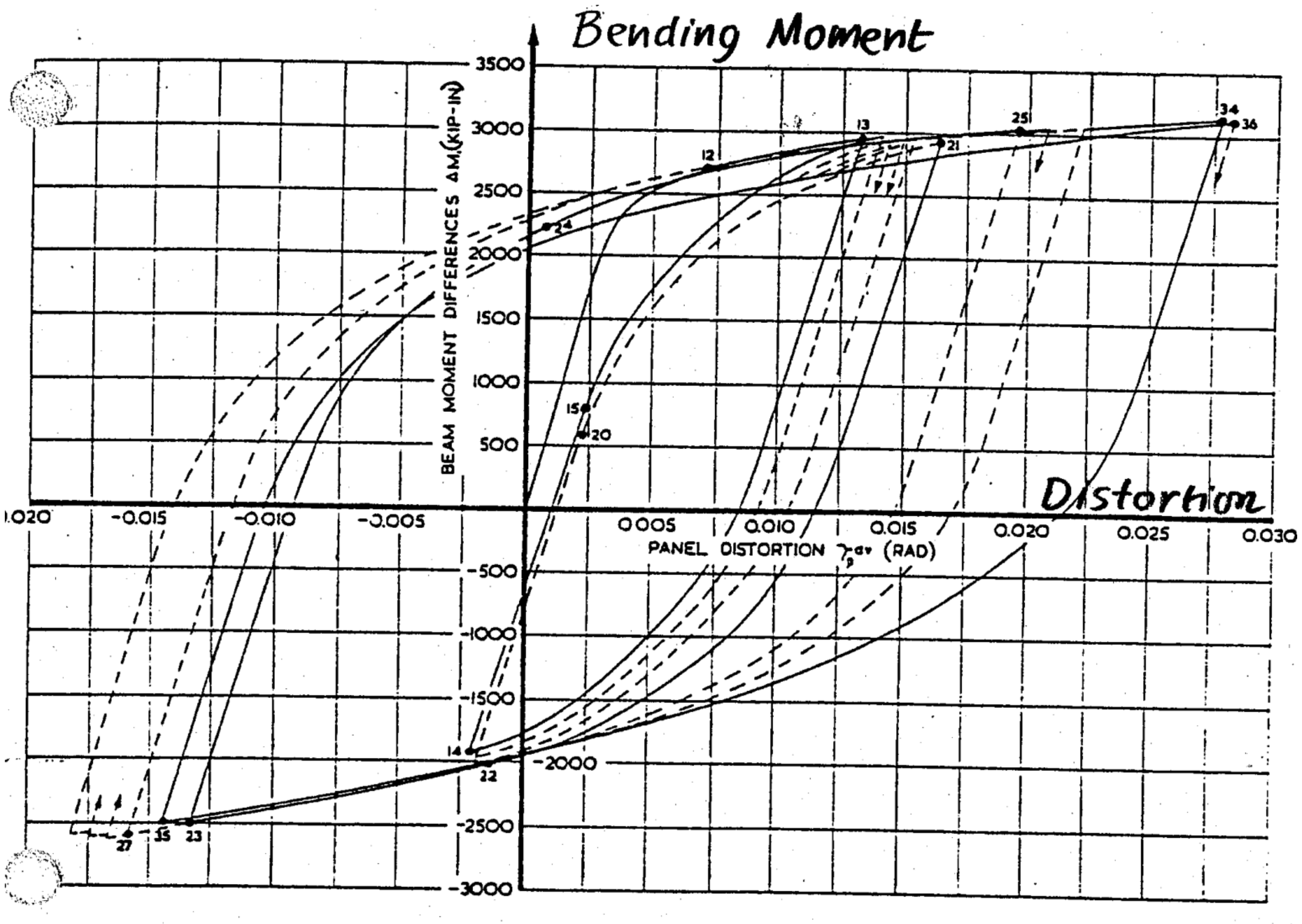
Force-deformation relation of a steel beam





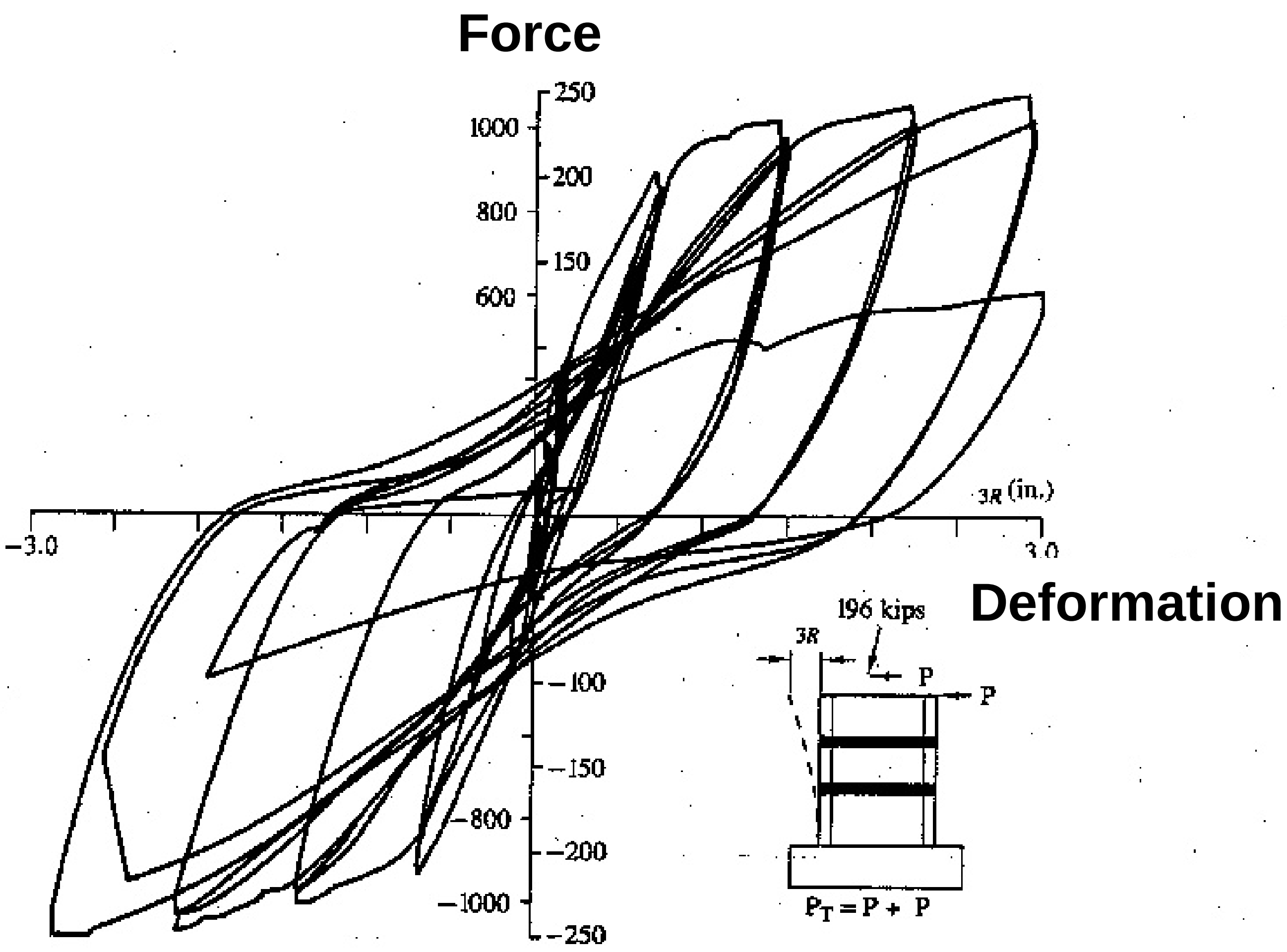
Hysteresis loop under cyclic deformation because of inelastic behavior.

Force-deformation relations of a structural steel component

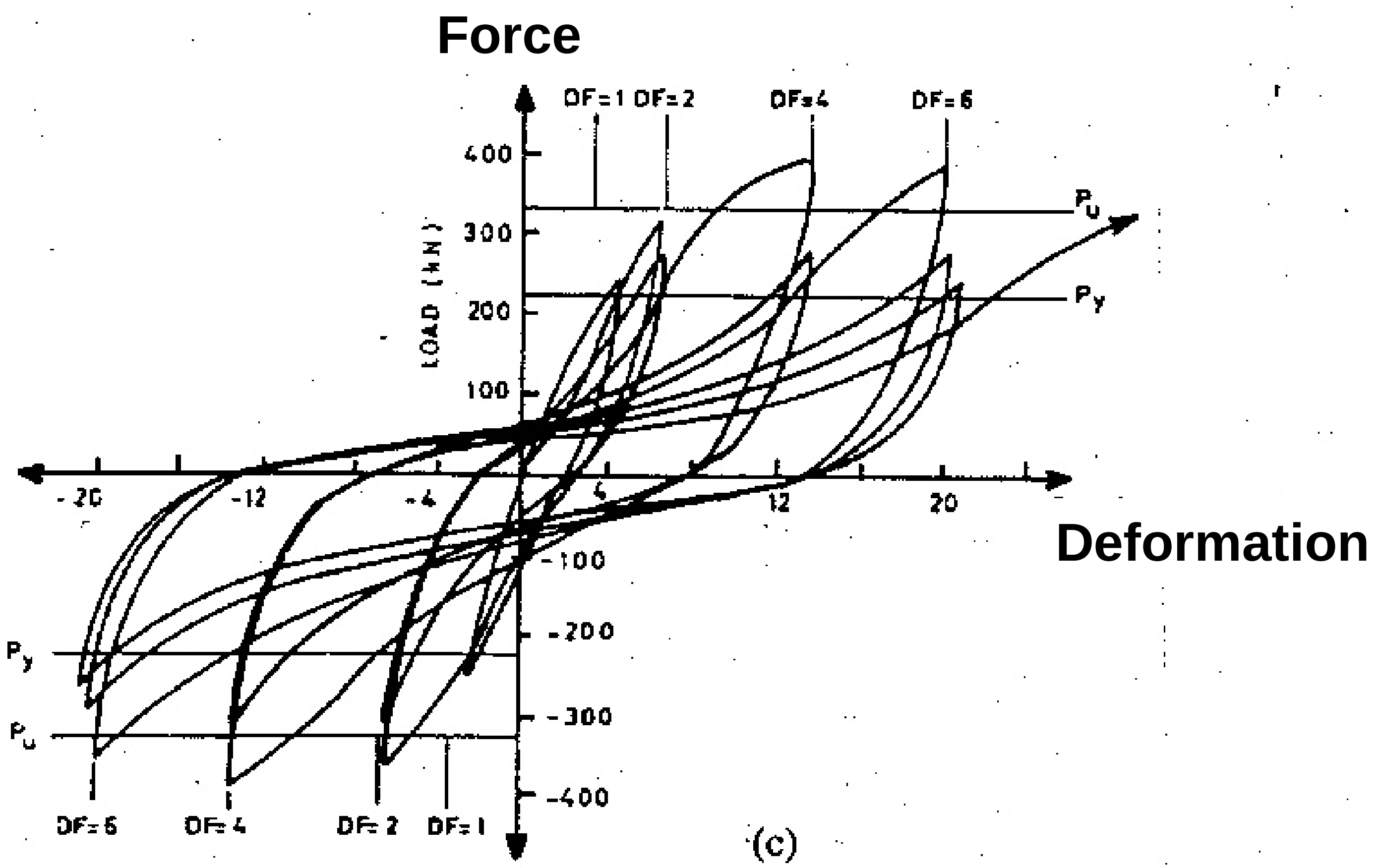


Hysteresis Loop

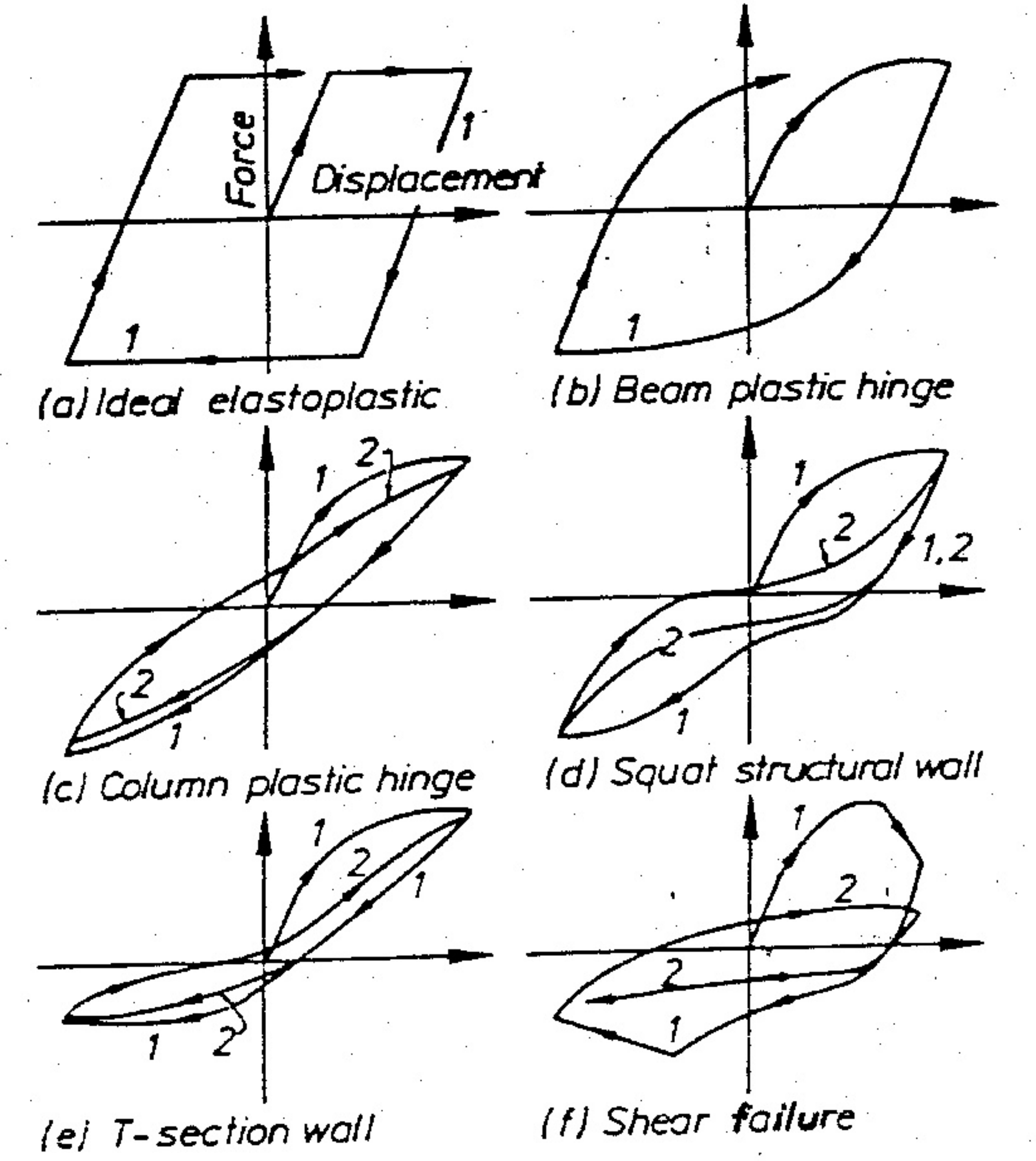
Force-deformation relation of a reinforced concrete structure



Force-deformation relation of a masonry structure

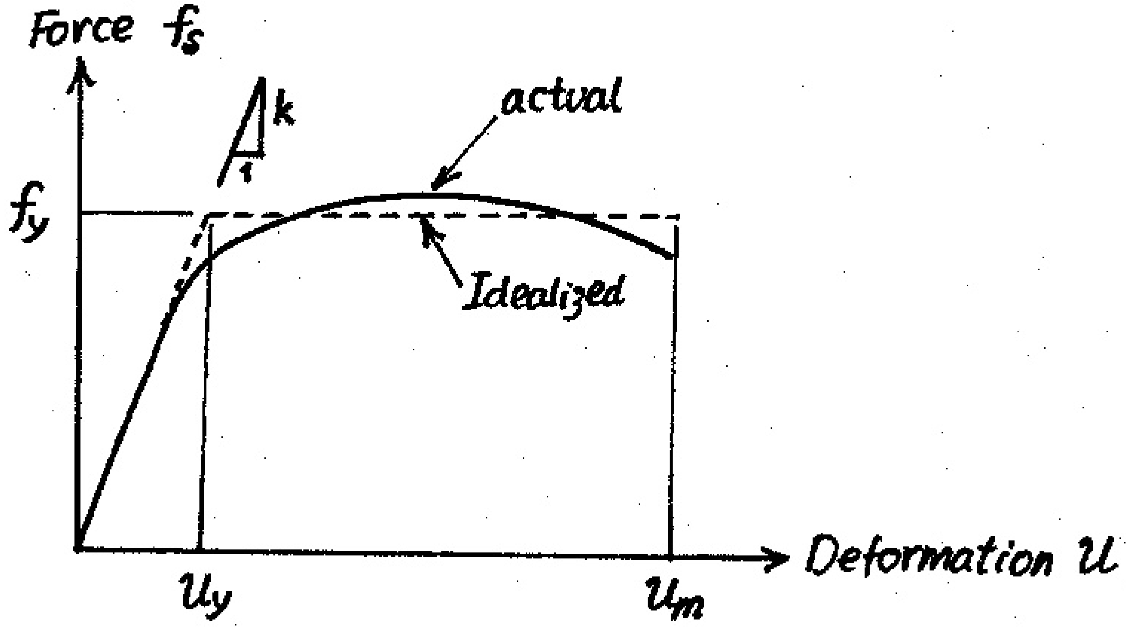


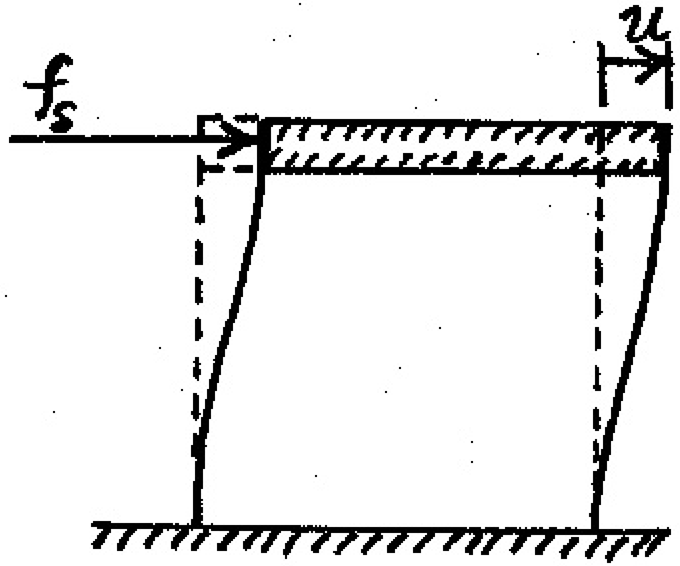
Typical force-deformation hysteresis loop shapes for concrete and masonry structural elements



ELASTOPLASTIC IDEALIZATION

Force-deformation curve during initial loading: Actual and elastoplastic idealization





u_m: *Maximum deformation*

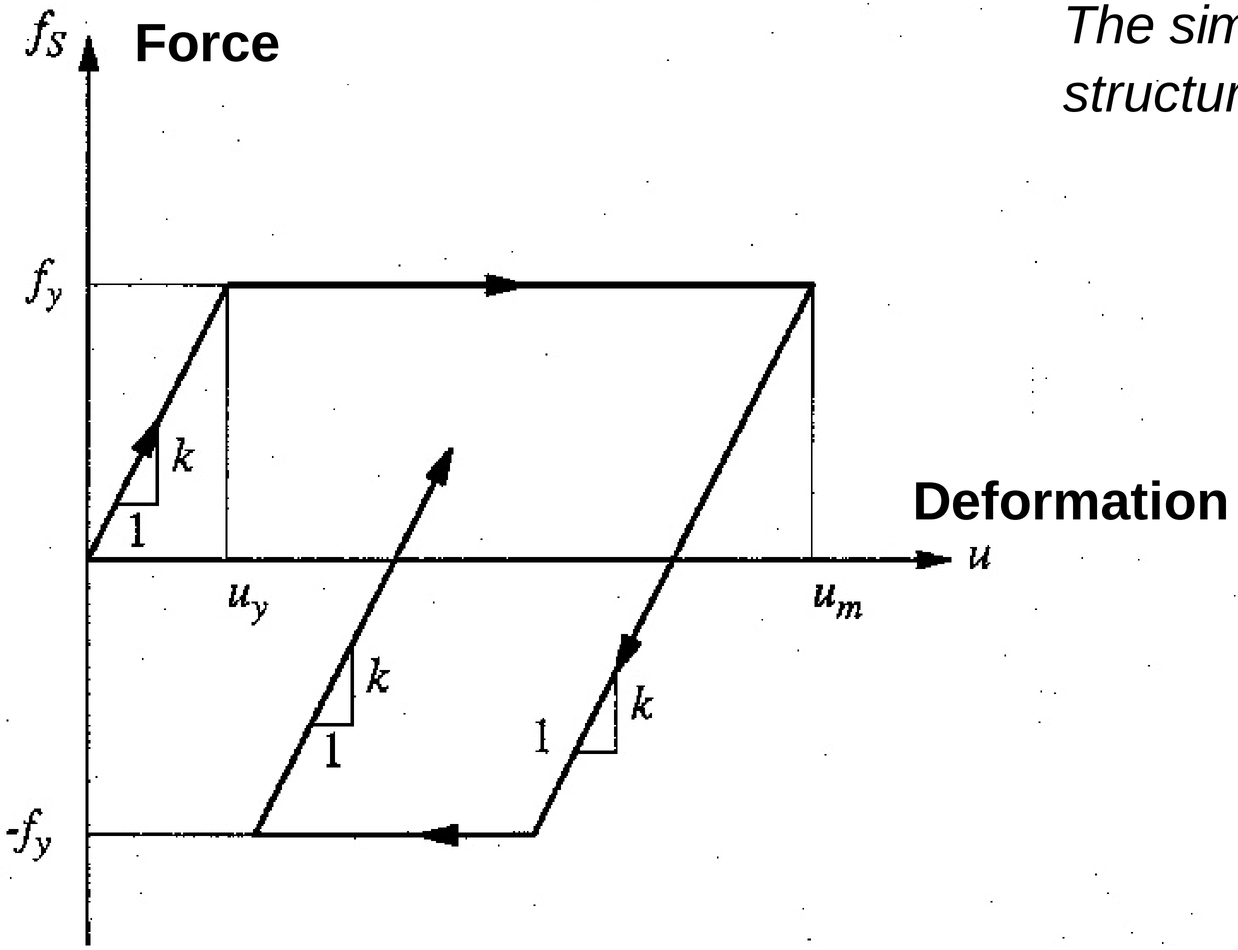
u_y: *Yield deformation*

f_y: *Yield strength*

K: *Stiffness in elastic range*

ELASTOPLASTIC IDEALIZATION

Force-deformation curve of an elastoplastic system for a typical cycle of loading, unloading, and reloading:



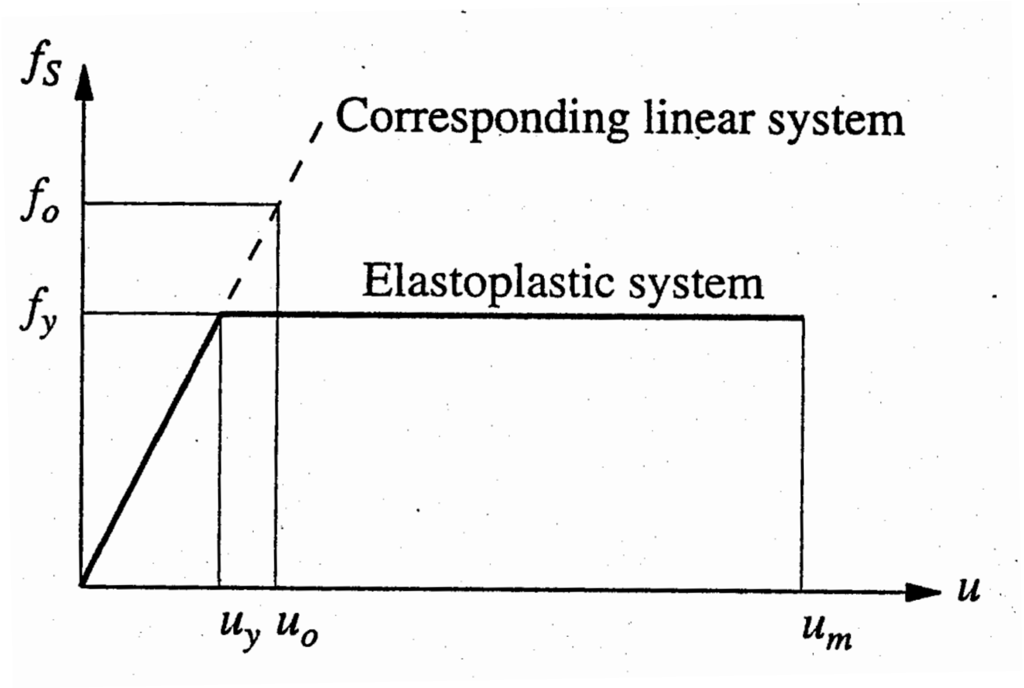
The simplest model of inelastic behavior of simple structure under cyclic loading

$$\boxed{\mu = ductility\ factor = u_m/u_y}$$

The ductility factor is commonly used as ***an index of seismic-induced inelastic damage****.*

ELASTOPLASTIC IDEALIZATION

Elastoplastic system and its corresponding linear system (having the same k in their initial loading phases)



In order to understand the effects of the inelastic force-deformation relation on the earthquake response, it is necessary to evaluate the peak deformation of an elastroplastic system and compare this deformation to the peak deformation of the corresponding linear system.

RESPONSE OF ELASTOPLATIC SYSTEM TO EARTHQUAKE GROUND MOTION

Equation of motion: $$m\frac{d^2u}{dt^2} + c\frac{du}{dt} + f_S(u) = -m\frac{d^2u_g}{dt^2}$$

where $f_S(u)$ is **a nonlinear function of $\boldsymbol{u}$** (as shown earlier).

The response $u(t)$ to an earthquake ground motion $\frac{d^2u_g}{dt^2}$ can be computed from the above equation by a **step-by-step direct integration**.

The nonlinear resisting force f_S can also be obtained from **the elastoplastic force-deformation relation.**

For a low-level ground motion, the system may behave like a linear elastic system with

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \left(T = {}^{1}/_{f}\right),$$

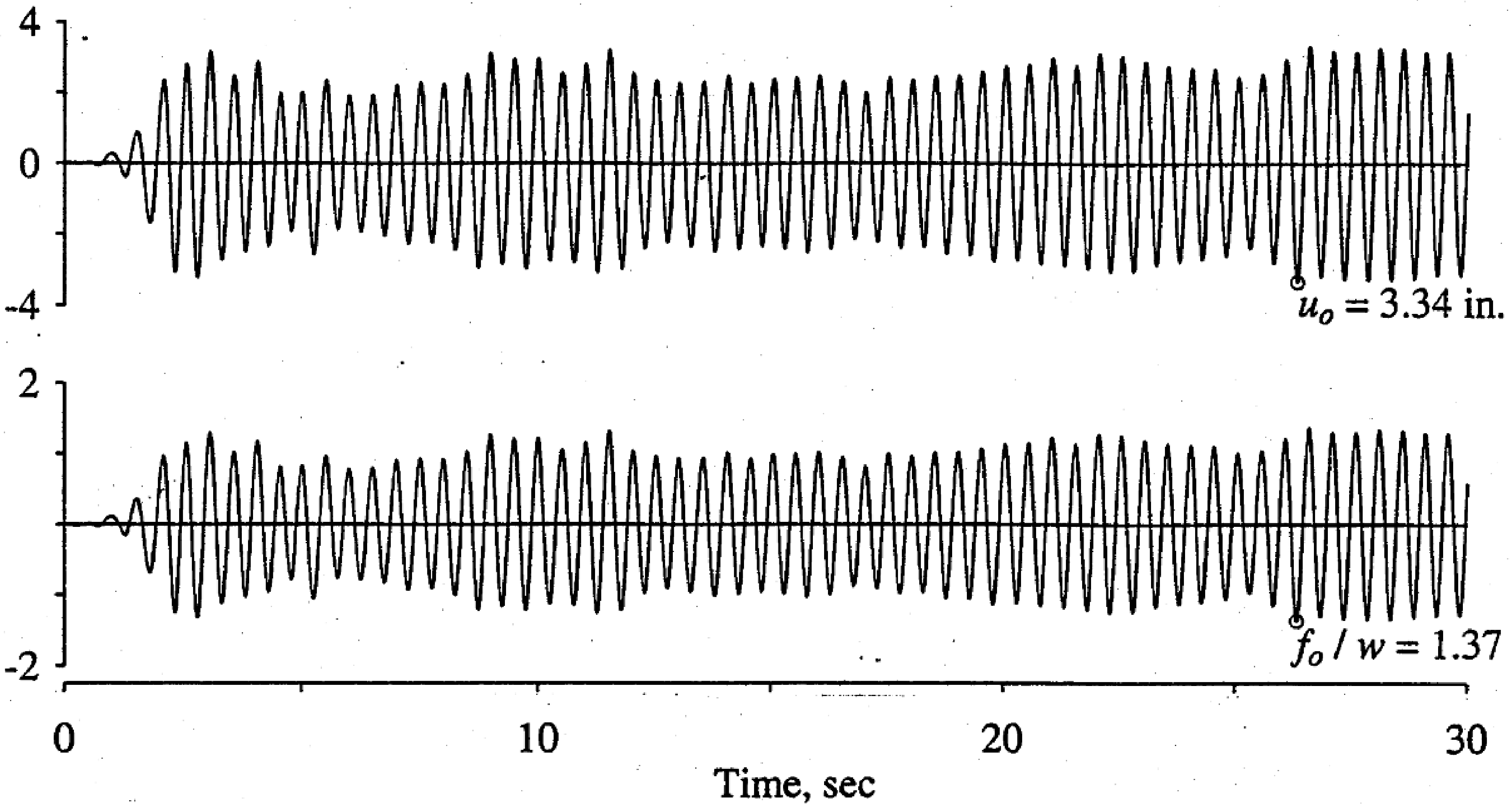
where k= initial stiffness in the elastic range.

EFFECTS OF INELASTICITY ON EARTHQUAKE RESPONSES

Response of a linearly elastic system with T=0.5 sec and $\xi = 0$ to El Centro ground motion:

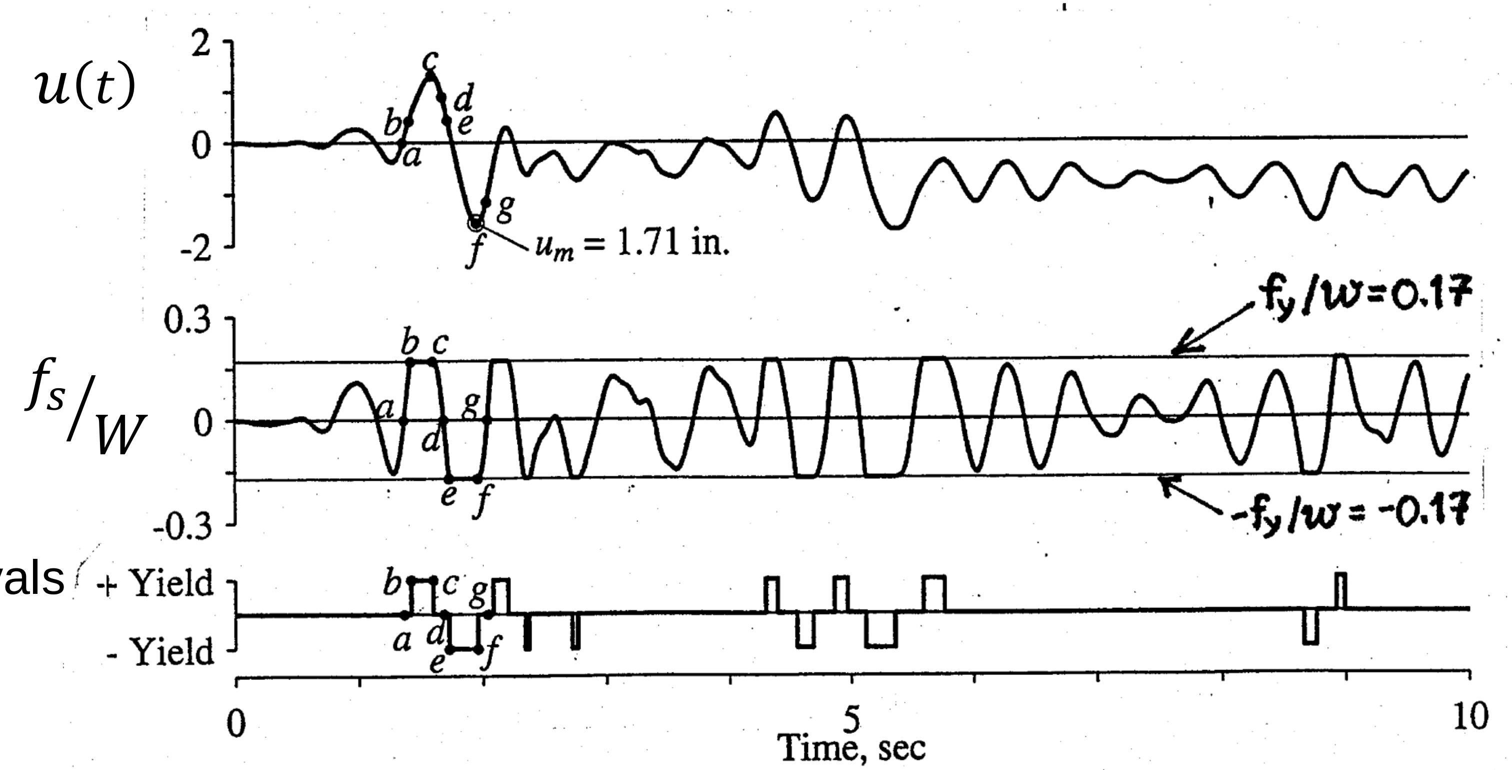
Deformation $u(t)$

Force $f_s/_W$



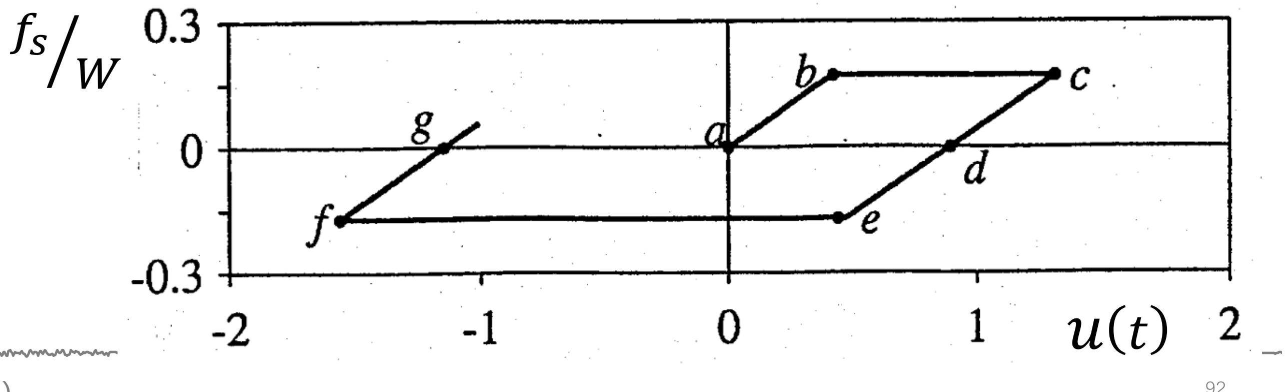
EFFECTS OF INELASTICITY ON EARTHQUAKE RESPONSES

Response of an elastoplastic system having the same mass and initial stiffness as the linear system ($T = 0.5\ and\ \xi = 0$) to El Centro ground motion.



Time intervals of yielding

The yield strength f_y of the system is set to $0.125\ f_0$, that is $f_y = 0.125\ f_0 = 0.125 \times 1.37\ W = 0.17W$



EFFECTS OF INELASTICITY ON EARTHQUAKE RESPONSES

Deformation response and yielding of four elastoplastic systems due to El Centro ground motion;

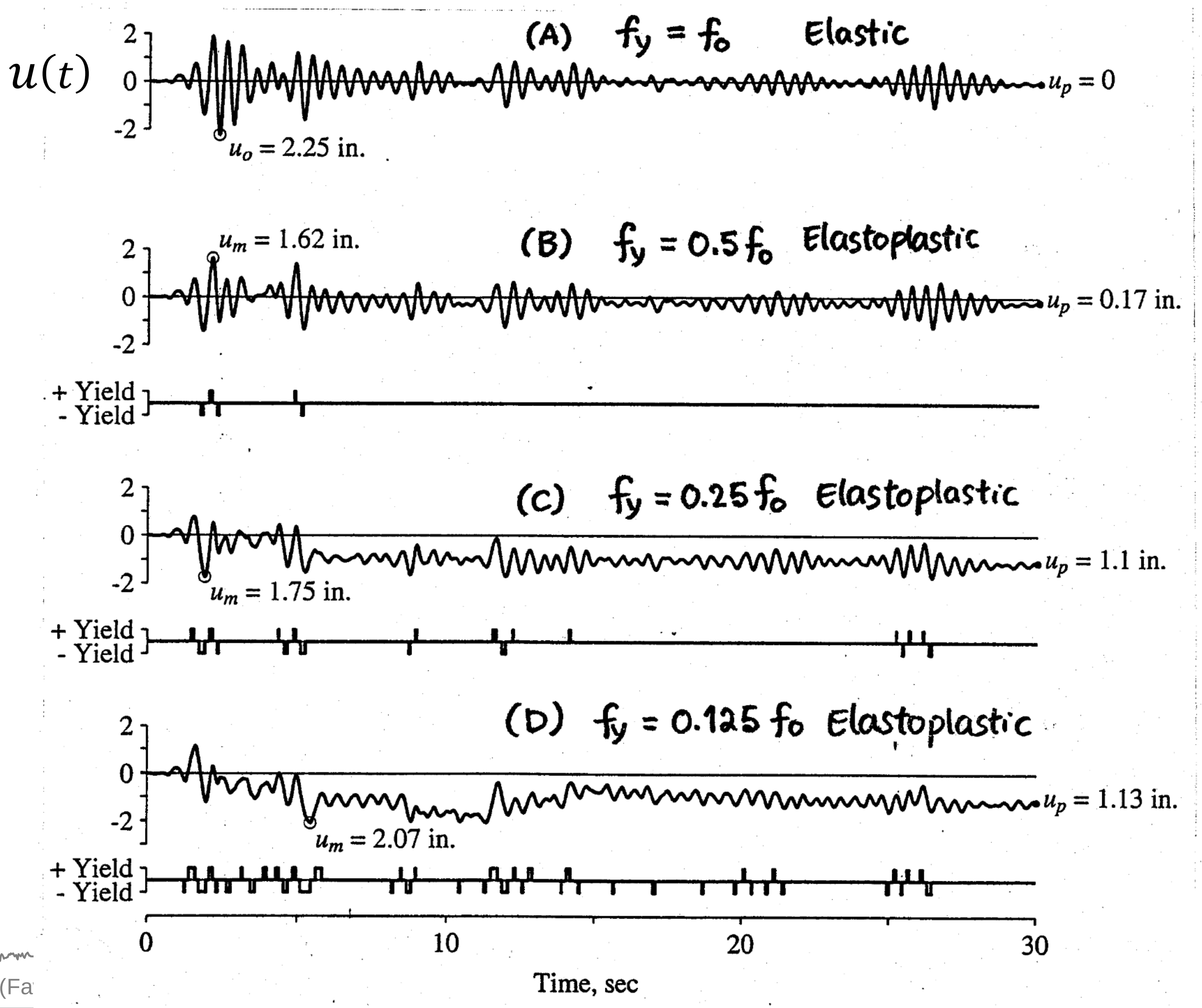
$T = 0.5\,sec, \xi = 0.05$

$and\ f_y = f_0, 0.5 f_0, 0.25\, f_0\ and\ 0.125\, f_0$

All four systems have identical properties in their linearly elastic range, but they differ in their yield strength.

Systems with lower yield strength yield more frequently and for longer intervals.

With more yielding, the permanent deformation u_p of the structure after the ground stops shaking tends to increase.



DUCTILITY FACTOR

Ductility factor for the four elastroplastic systems:

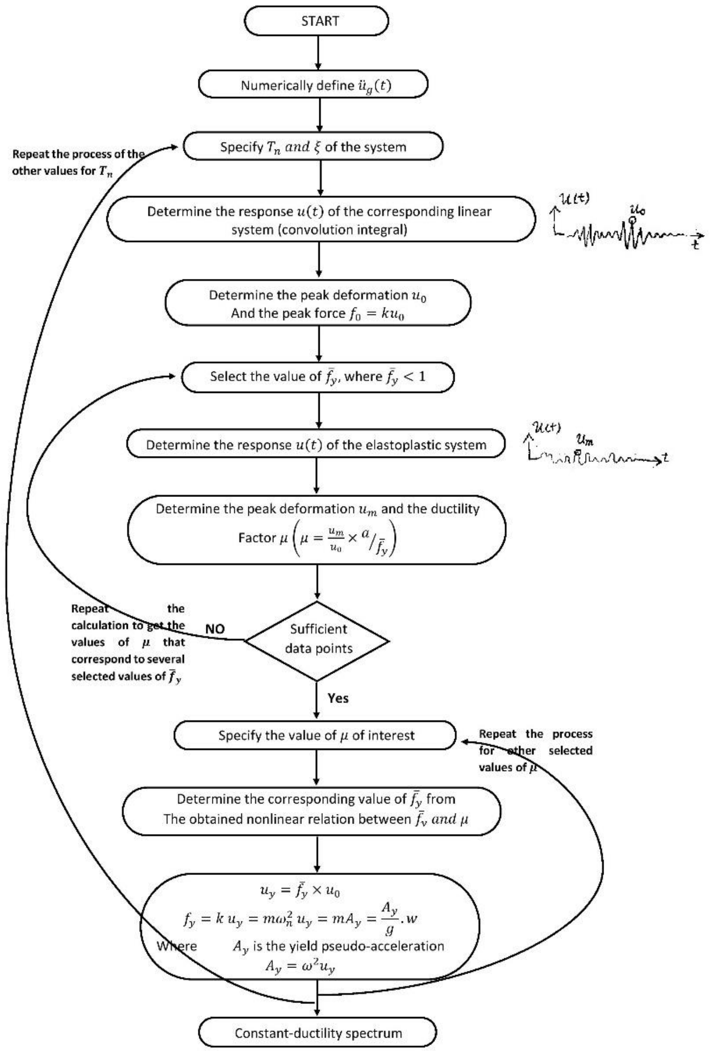
System	Yield strength $\boldsymbol{f_y}$	Yield deform. $\boldsymbol{u_y\ (in)}$	Max. Deform. $\boldsymbol{u_m (in)}$	Ductility factor $\boldsymbol{\mu}$
A	f_0	*2.25*	*2.25*	*1.00*
B	$0.5f_0$	*1.125*	*1.62*	*1.44*
C	$0.25f_0$	*0.562*	*1.75*	*3.11*
D	$0.125f_0$	*0.281*	*2.07*	*7.36*

- *For each system, the computer ductility factor μ is the "**ductility demand**" imposed on elastoplastic system by the ground motion.*
- *The system should be designed such that its "**ductility capacity**" (i.e. the ability to deform beyond the elastic limit) **exceeds the "ductility demand".***

Decreasing in Yield Strength → Increasing in "Ductility Demand".

CONSTANT-DUCTILITY SPECTRUM

The procedure to construct the response spectrum for elastoplastic systems corresponding to specified levels of ductility factor is shown by a flow chart below.



Please see next slides for enlarged flow chart

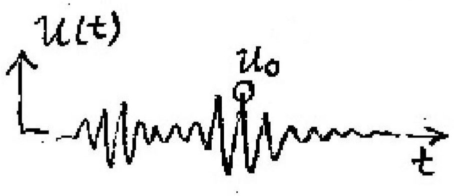
START

Numerically define $\ddot{u}_g(t)$

Specify T_n and ξ of the system

Repeat the process of the other values for T_n

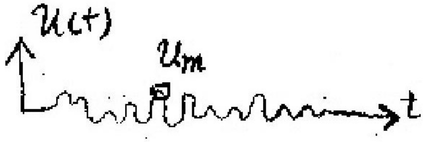
Determine the response $u(t)$ of the corresponding linear system (convolution integral)

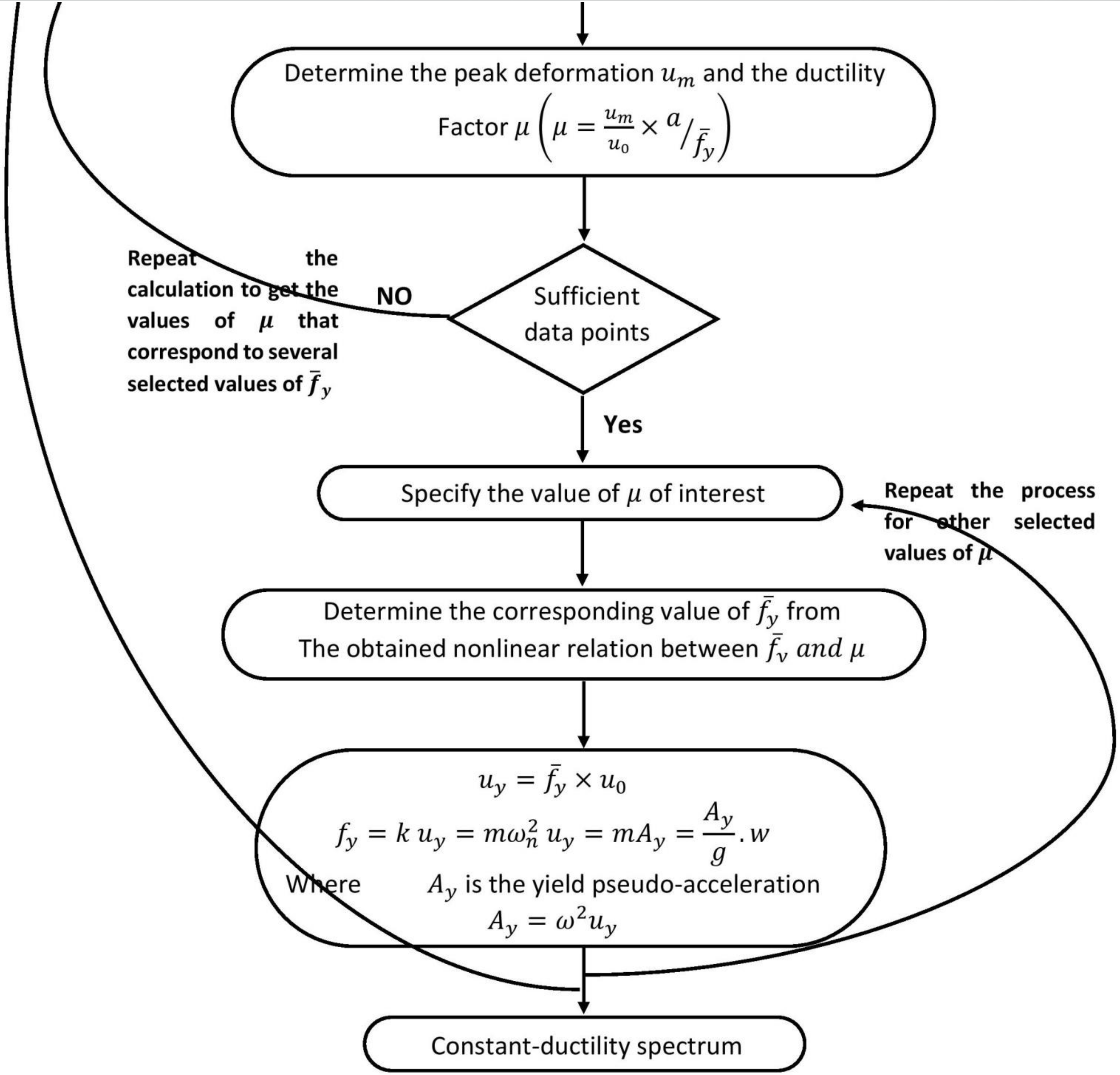


Determine the peak deformation u_0
And the peak force $f_0 = ku_0$

Select the value of $\bar{f}_y$, where $\bar{f}_y < 1$

Determine the response $u(t)$ of the elastoplastic system



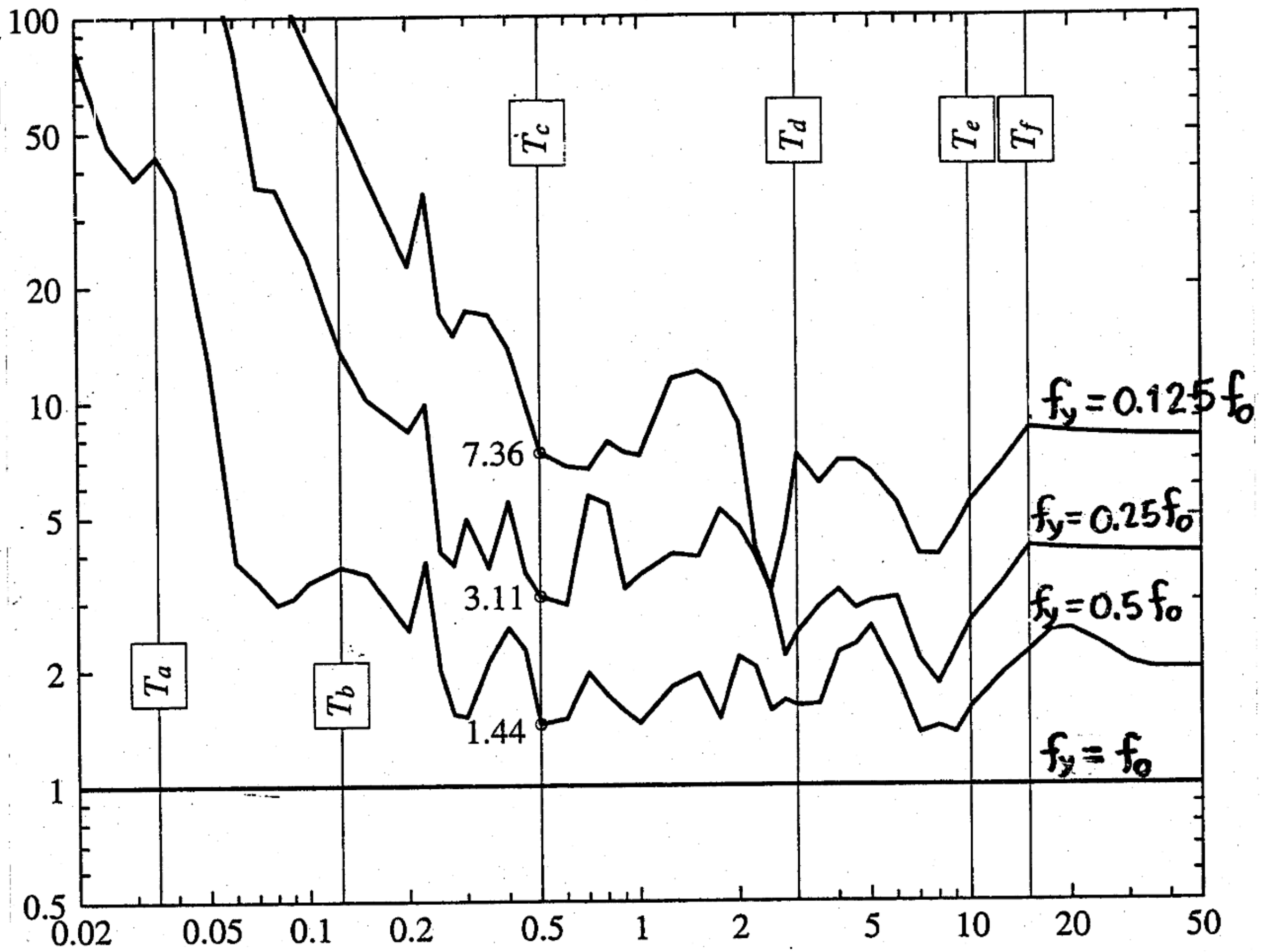


Determine the peak deformation u_m and the ductility Factor μ $\left(\mu = \frac{u_m}{u_0} \times {}^{a}/_{\bar{f}_y}\right)$
Repeat the calculation to get the values of μ that correspond to several selected values of $\bar{f}_y$
NO
Sufficient data points
Yes
Specify the value of μ of interest
Repeat the process for other selected values of μ
Determine the corresponding value of $\bar{f}_y$ from The obtained nonlinear relation between $\bar{f}_v$ and μ
$u_y = \bar{f}_y \times u_0$
$f_y = k\,u_y = m\omega_n^2\,u_y = mA_y = \frac{A_y}{g}.w$
Where A_y is the yield pseudo-acceleration
$A_y = \omega^2 u_y$
Constant-ductility spectrum

DUCTILITY DEMAND Ductility demand for elastroplastic system due to El Centro ground motion:

$\xi = 0.05 \; and \;\; f_y = f_0, 0.5f_0, 0.25f_0 \; and \; 0.125f_0$.

Ductility demand μ



Natural period T, sec

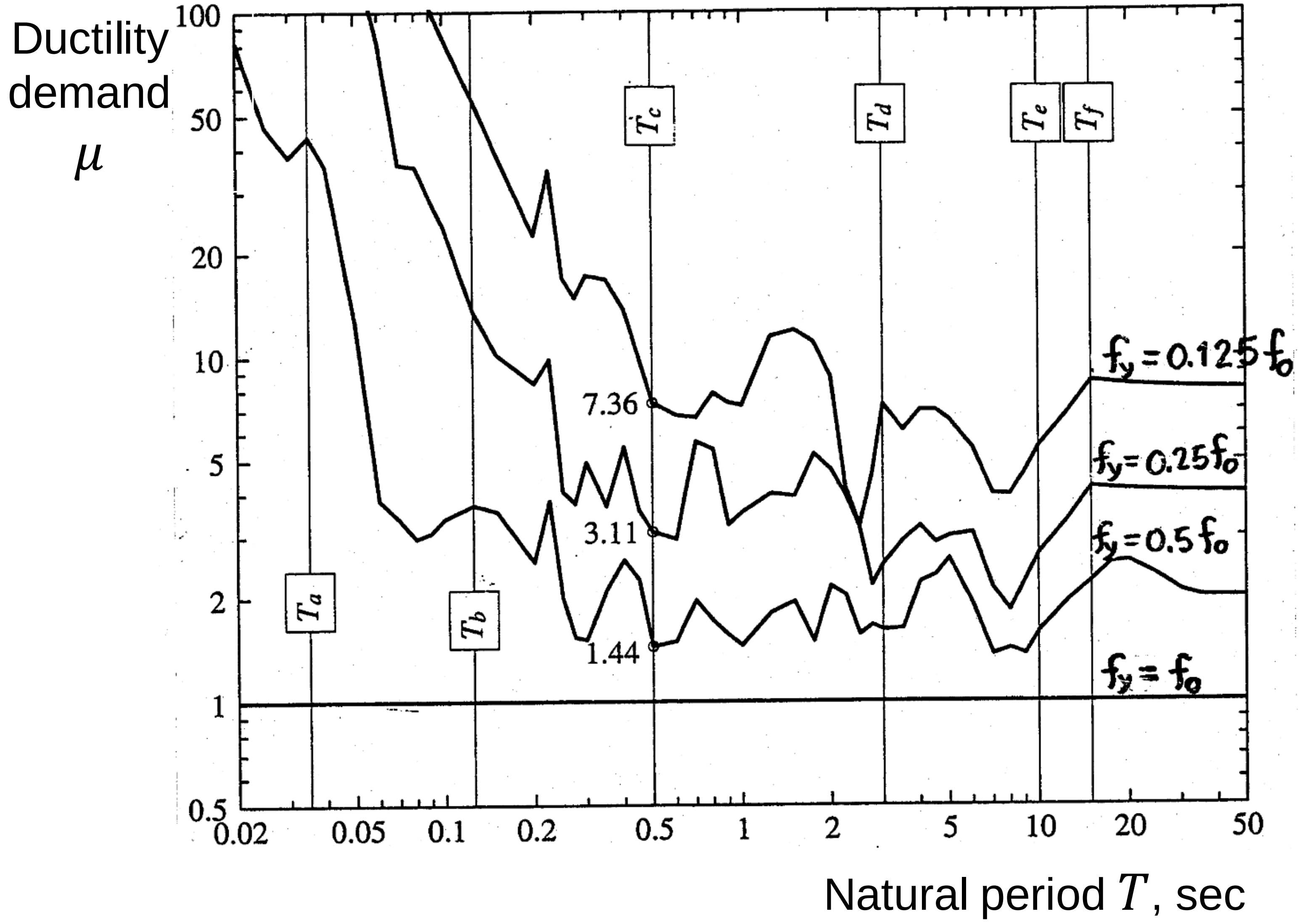
For systems with long natural periods, The ductility demand $\mu \approx f_0/f_y$

For systems with short natural period, The ductility demand μ *can be longer than* f_0/f_y.

It may be more appropriate to design these short-period systems to remain ***elastic:*** *otherwise, the inelastic deformation and ductility demand may be excessive.*

DUCTILITY DEMAND Ductility demand for elastroplastic system due to El Centro ground motion:

$$\xi = 0.05 \; and \;\; f_y = f_0, 0.5f_0, 0.25f_0 \; and \; 0.125f_0.$$



If the acceptable value of ductility factor μ is specified, it is possible to determine the corresponding yield strength f_y by an interpolative procedure.

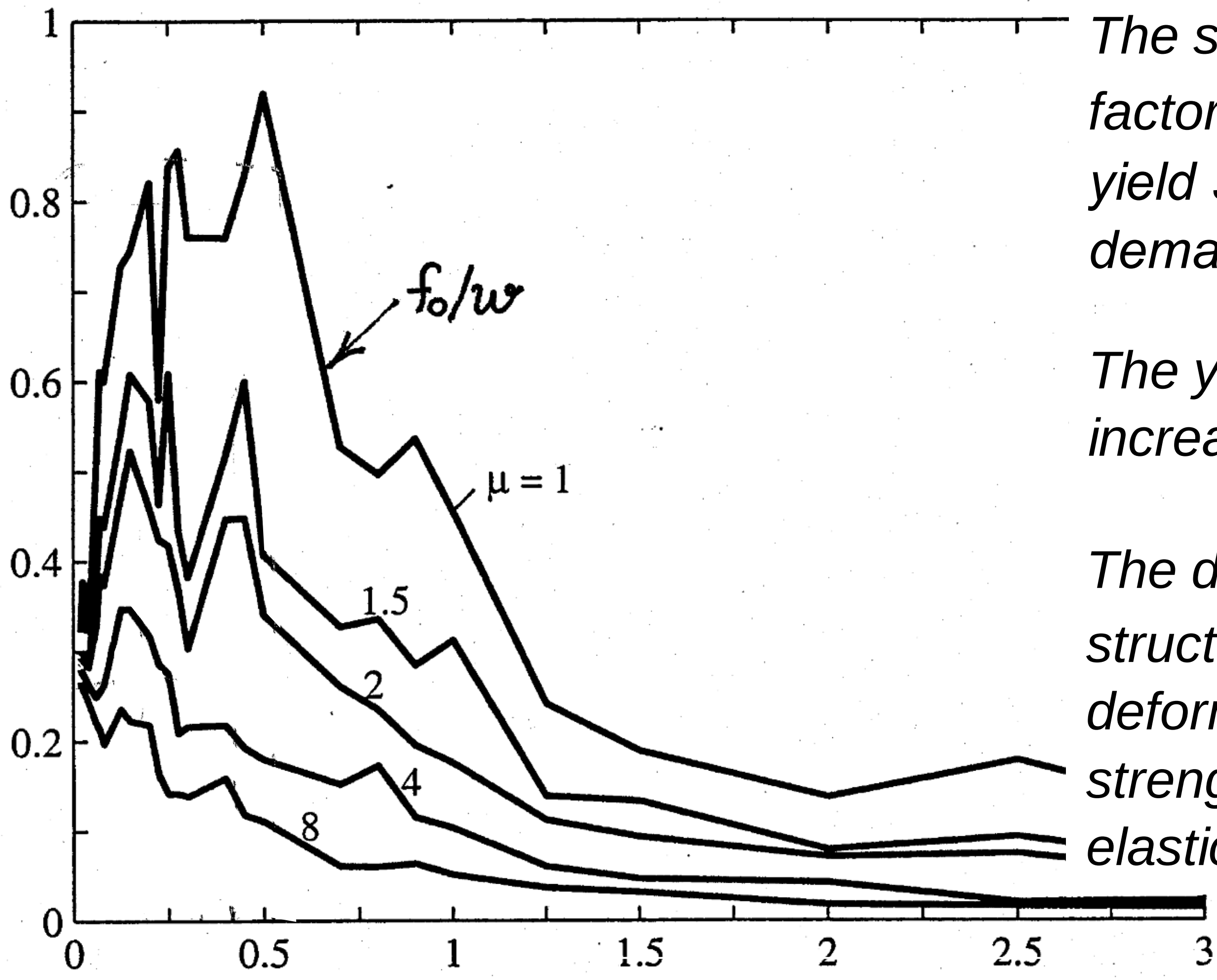
This "f_y" may be considered as the <u>"yield strength demand"</u> for the desired ductility factor μ.

YIELD STRENGTH DEMAND

Yield strength demand for elastroplastic systems ($\xi = 0.05$) for specified ductility $\mu = 1, 1.5, 2, 4, and\ 8$; El Centro ground motion.

Yield strength demand

$f_y/_W$



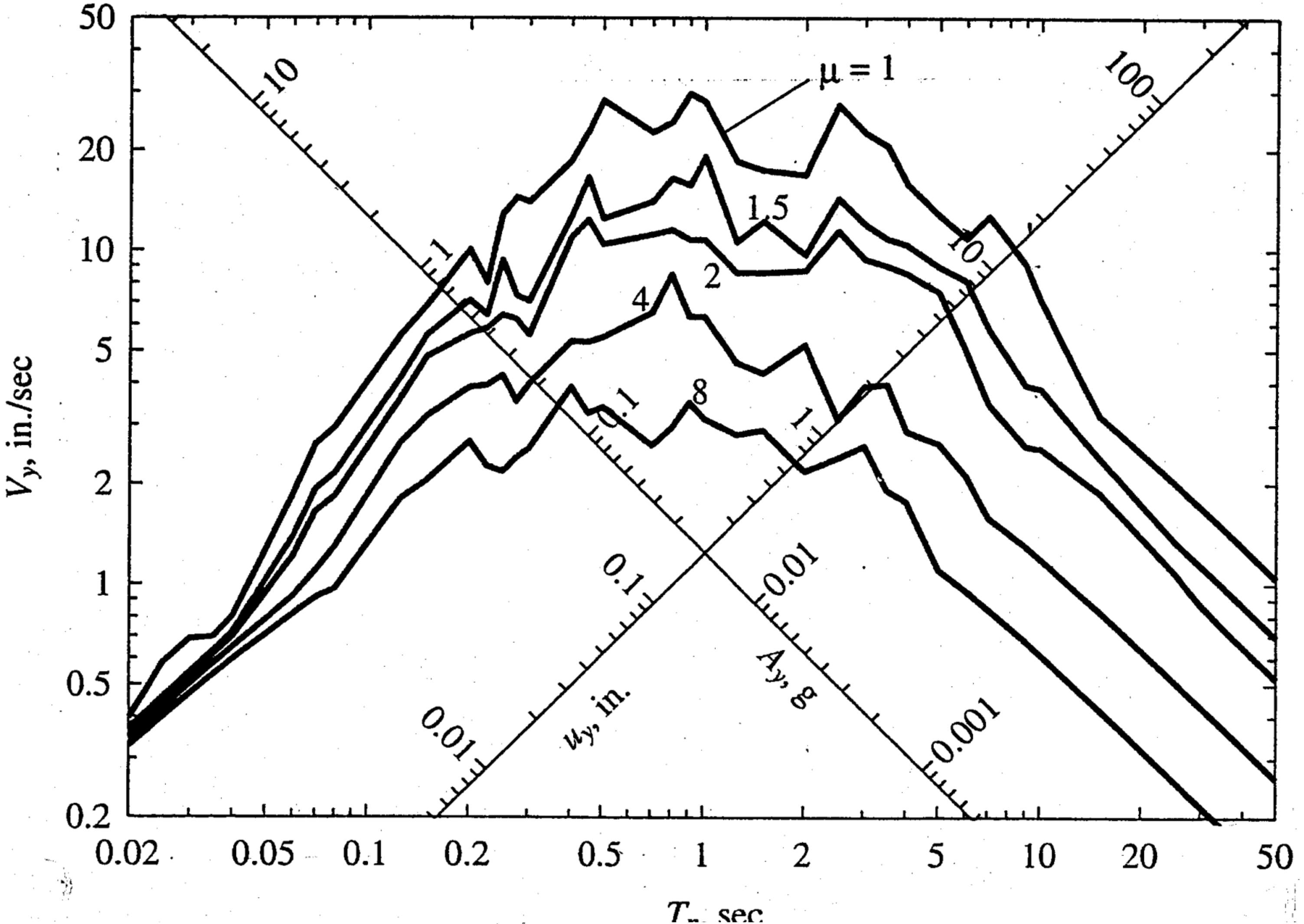
Natural period T, sec

The structure having the specified ductility factor μ should be designed such that its yield strength exceeds the "yield strength demand".

The yield strength demand is reduced with increasing values of the ductility factor.

The design yield strength f_y for a simple structure permitted to undergo inelastic deformation can be much lower than the strength required for the structure to remain elastic.

Combined $D_y - V_y - A_y$ plot of the above constant ductility response spectrum



Note:

$D_y = u_y$

the yield deformation

$V_y = \omega_n u_y$

the yield pseudo-velocity

$A_y = \omega_n^2 u_y$

the yield pseudo-acceleration

DESIGN OPTIONS

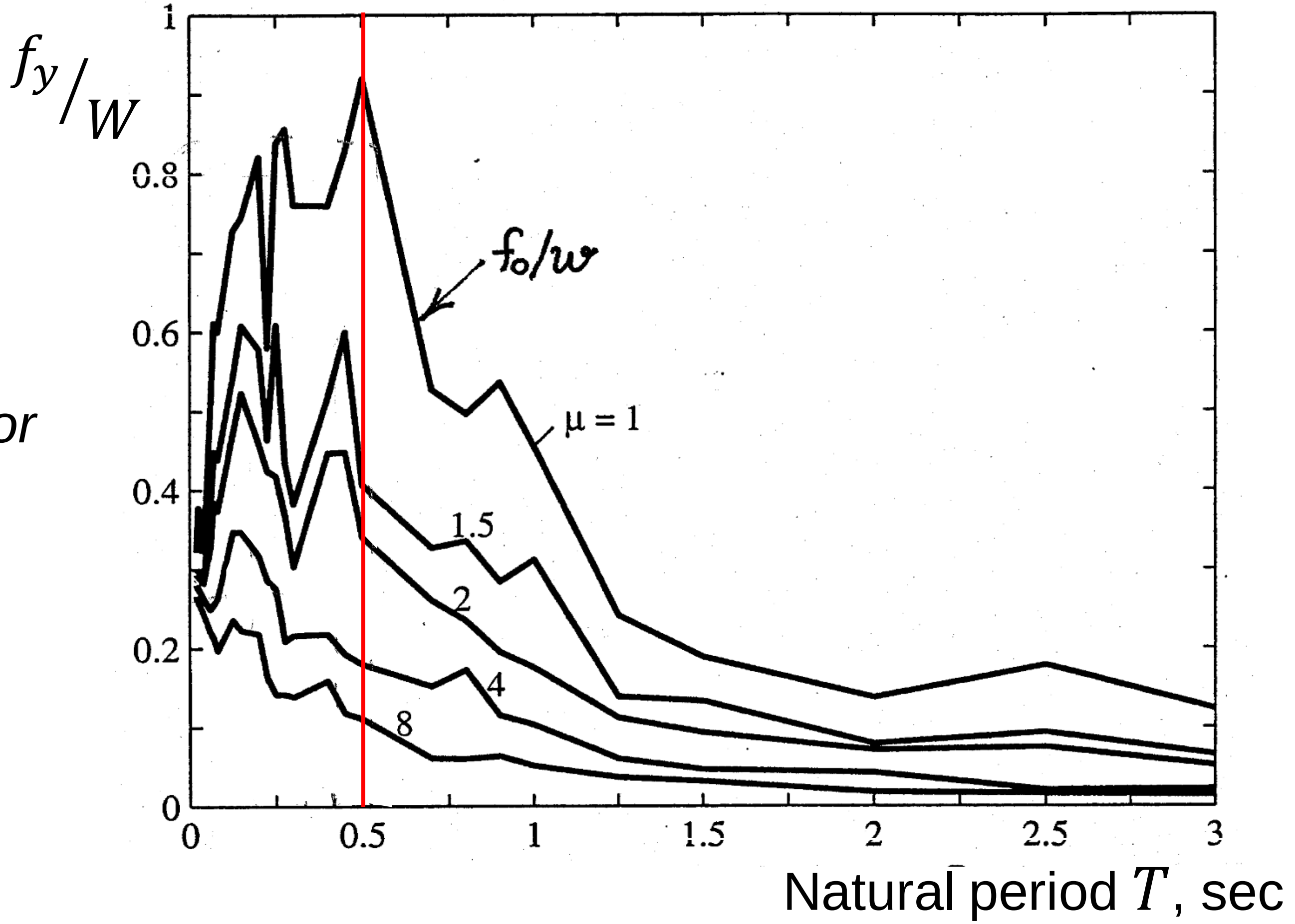
A structure may be designed for earthquake resistance by making it strong, by making it ductile, or by designing it for **economic combinations of both properties.**

Consider a simple structure with $T = 0.5\,\sec\, and\ \xi = 0.05$

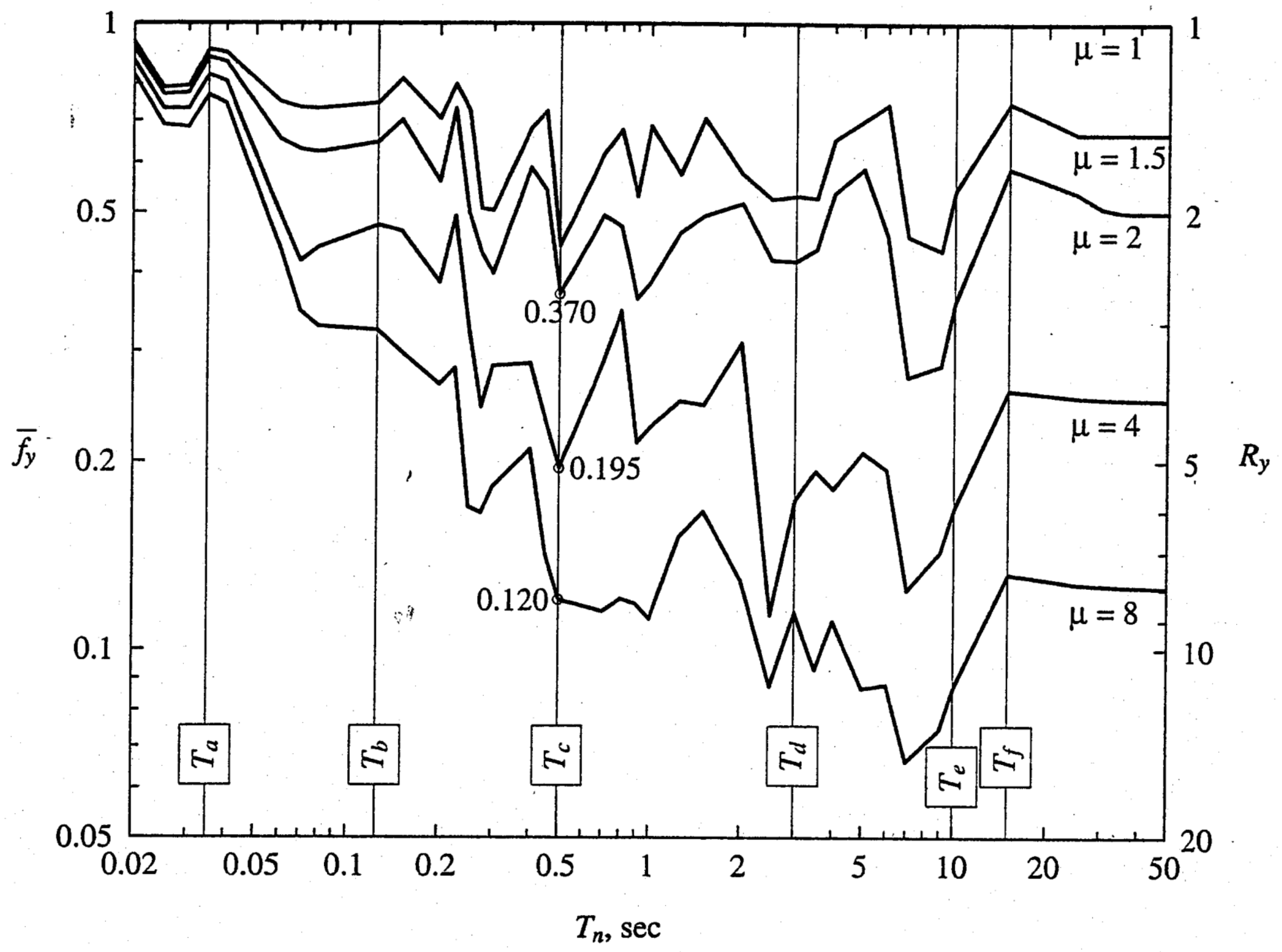
If the structure is designed for strength $f_0 = 0.919\,W$ or larger, it will remain within its linearly elastic range during the El Centro ground motion; therefore, it needs not to be ductile.

On the other hand, if it can develop a ductility factor of 8, it needs to be designed only 12% of the strength f_0 (that is only $0.11W$!)

Alternatively, it may be designed for strength equal to 37% of f_0 and a ductility capacity of 2.



Normalized strength $\bar{f}_y$ of elastoplastic systems as a function of natural vibration period T_n for $\mu = 1, 1.5,\ 2,\ 4$ and 8; $\xi = 5\%$; El Centro ground motion.



DESIGN OPTIONS

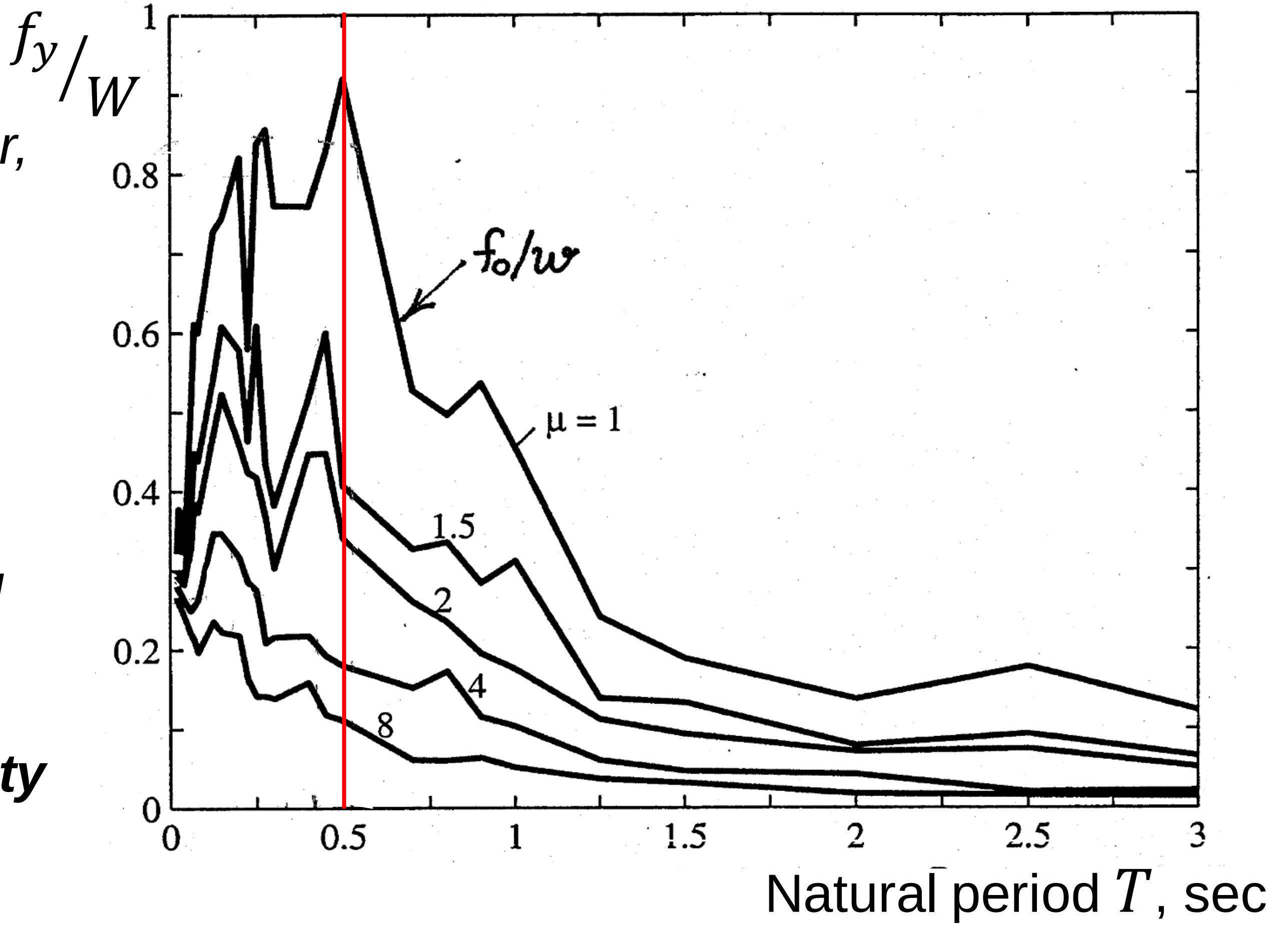
A structure may be designed for earthquake resistance by making it strong, by making it ductile, or by designing it for **economic combinations of both properties.**

Consider a simple structure with $T = 0.5\,\sec\, and\ \xi = 0.05$

For some types of materials and structural member, ductility is difficult to achieve. In such cases, the structure should be designed to have a high yield strength and low ductility.

For others, providing ductility is much easier than providing lateral strength. So, the structure in this case should be designed for low yield strength and high ductility.

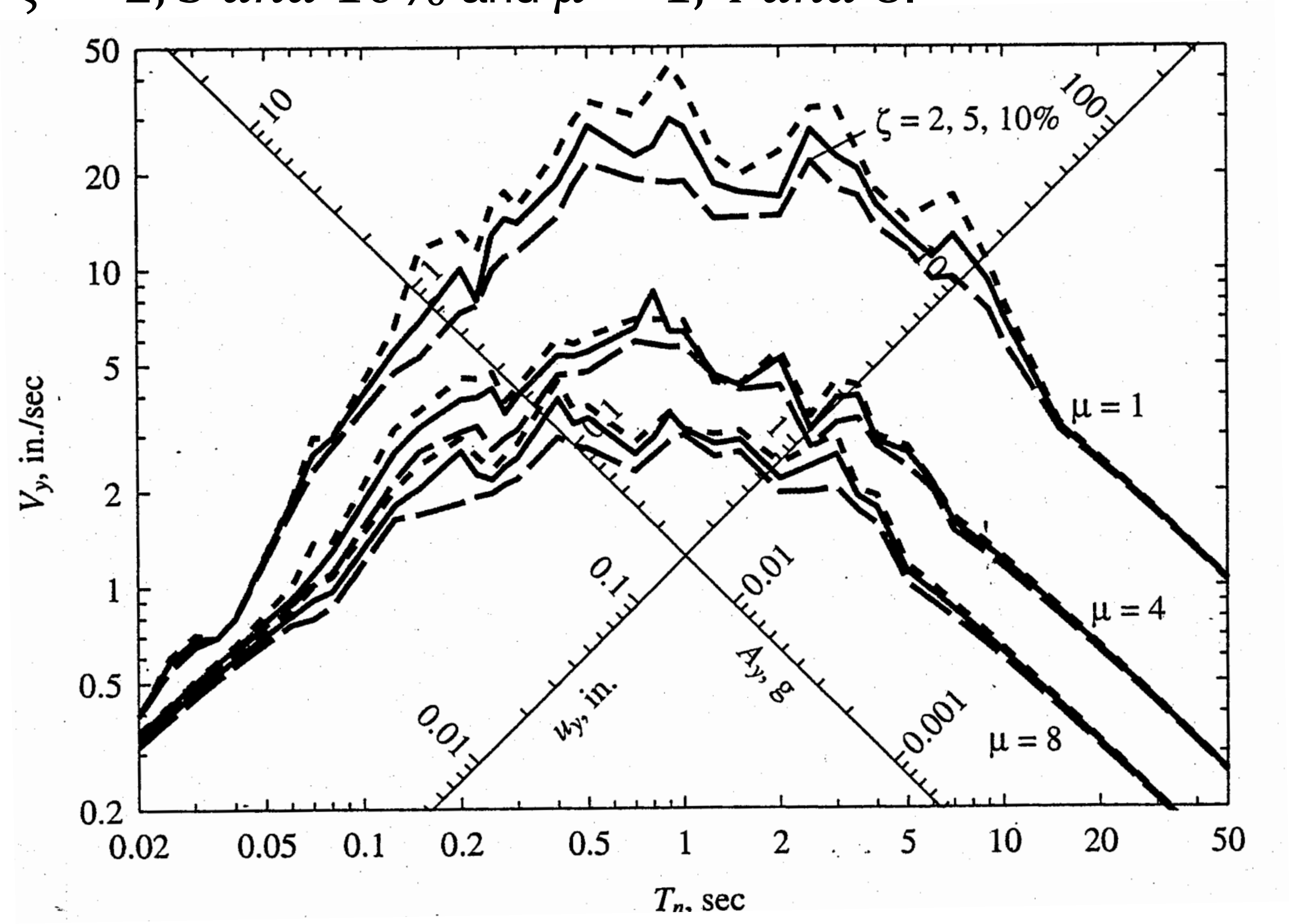
Economic combinations of strength and ductility properties.



RELATIVE EFFECTS OF YIELDING AND DAMPING

Response spectra for elastoplastic systems and EL Centro ground motion;

$\xi = 2, 5\ and\ 10\%$ and $\mu = 1, 4\ and\ 8$.



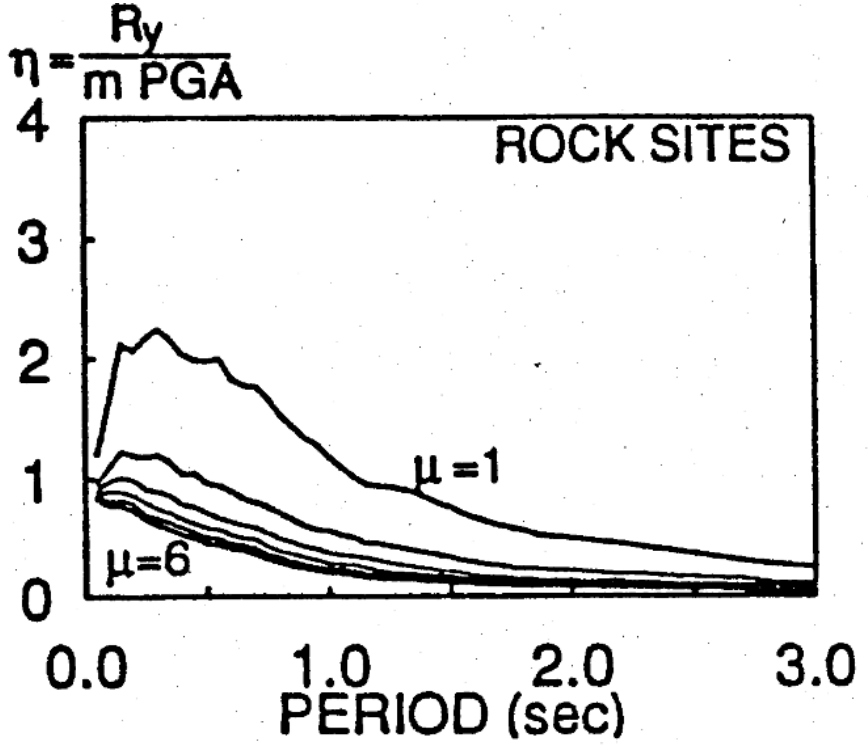
- Both yielding and damping reduce the pseudo-acceleration A_y and hence the peak value of the lateral force for which the system should be designed.

- But the damping is, in general, not as effective as yielding

INELASTIC DESIGN RESPONSE SPECTRA

Obtained from a comprehensive statistical study of yield strength demands on simple structure when subjected to many different ground motions recorded in various earthquakes.

Mean Normalized Strength Demand Spectra for $\mu = 1, 2, 3, 4, 5, 6$



(Miranda, ASCE J.Struc. Eng., 119, No5, May, 1993)

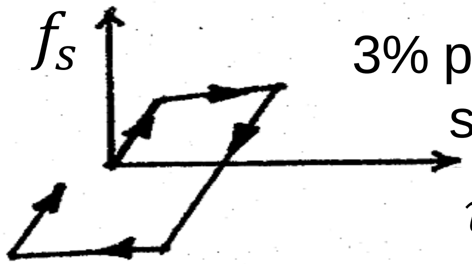
$R_y = the\ system's\ yield\ strength = f_y$

$m = the\ mass\ of\ the\ system$

$PGA = Peak\ ground\ accelaration$

The computed results are based on:

- *124 ground motions*
- *systems with bilinear hysteretic behavior.*

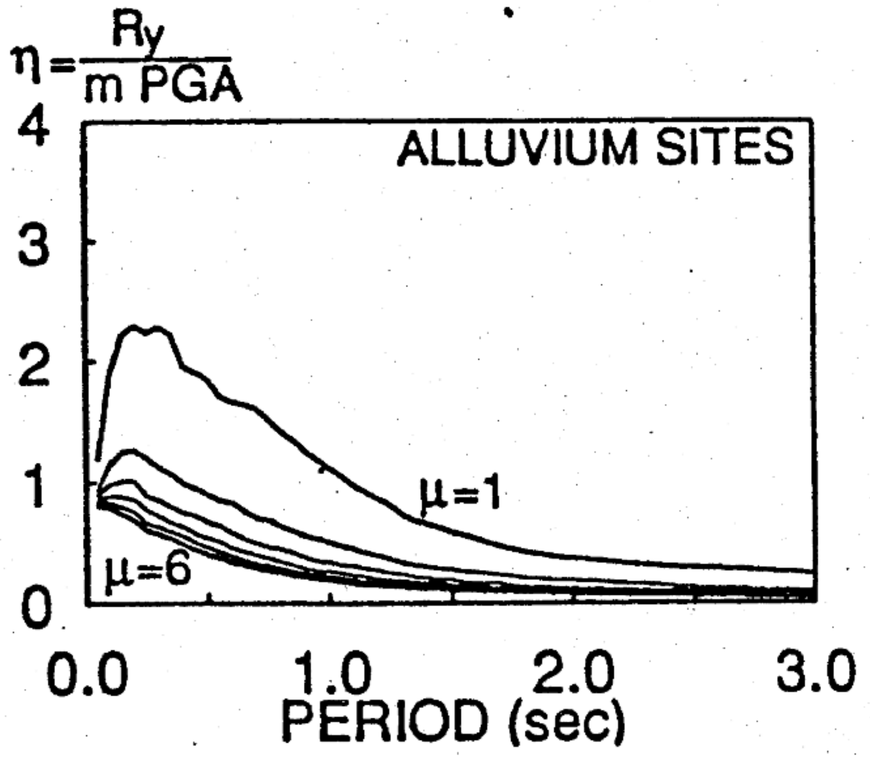


3% post-yielding stiffness

INELASTIC DESIGN RESPONSE SPECTRA

Obtained from a comprehensive statistical study of yield strength demands on simple structure when subjected to many different ground motions recorded in various earthquakes.

Mean Normalized Strength Demand Spectra for $\boldsymbol{\mu = 1, 2, 3, 4, 5, 6}$



$R_y = the\ system's\ yield\ strength = f_y$

$m\ =\ the\ mass\ of\ the\ system$

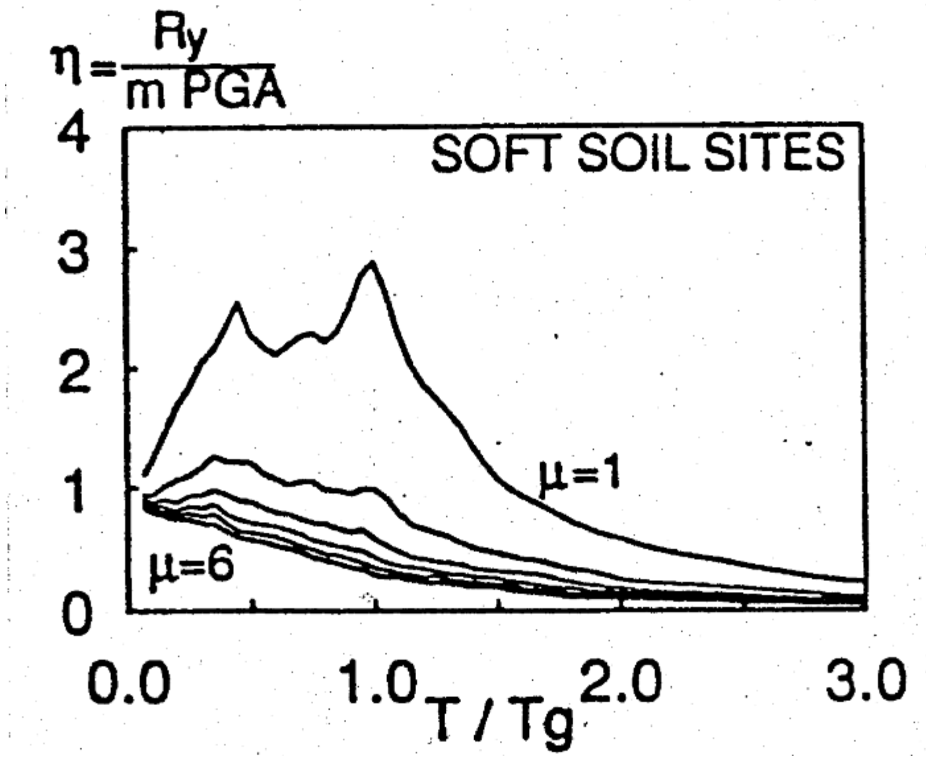
$PGA = Peak\ ground\ accelaration$

(Miranda, ASCE J.Struc. Eng., 119, No5, May, 1993)

INELASTIC DESIGN RESPONSE SPECTRA

Obtained from a comprehensive statistical study of yield strength demands on simple structure when subjected to many different ground motions recorded in various earthquakes.

Mean Normalized Strength Demand Spectra for $\boldsymbol{\mu = 1, 2, 3, 4, 5, 6}$



$R_y = the\ system's\ yield\ strength = f_y$

$m\ =\ the\ mass\ of\ the\ system$

$PGA = Peak\ ground\ accelaration$

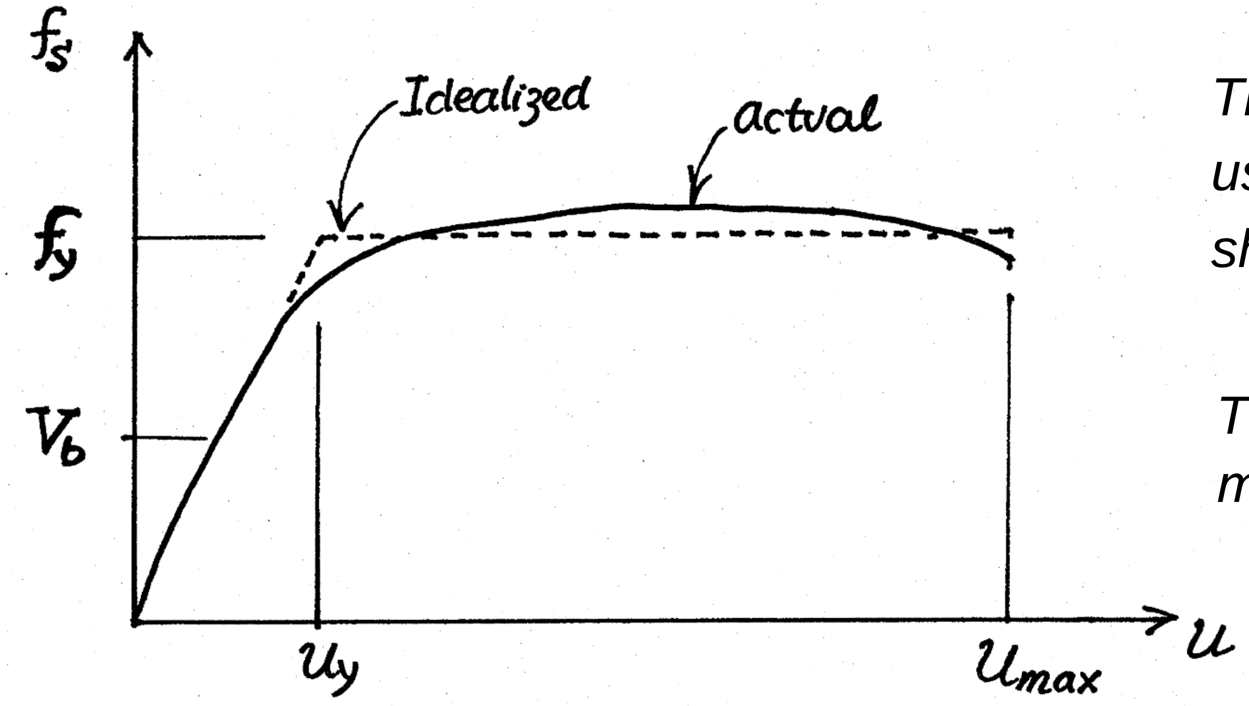
$T_g = the\ predominant\ period\ of\ the\ ground\ motion$

(Miranda, ASCE J.Struc. Eng., 119, No5, May, 1993)

OVERSTRENGTH

Suppose that a structure is designed to resist a code-prescribed design shear V_b by "Allowable Stress Design" method.

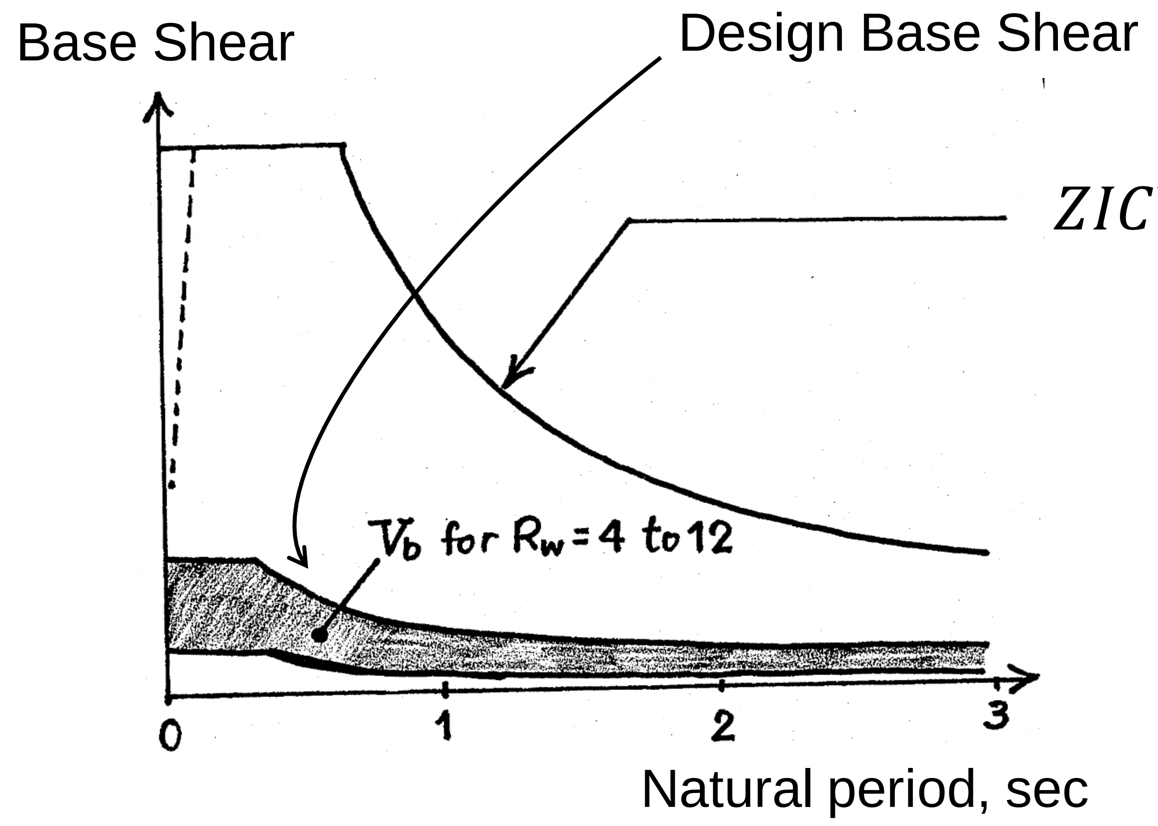
The behavior of the structure under a monotonically increasing load would be:



The yield strength of the structural system f_y is usually considerably higher than the design base shear V_b, (the effect of "overstrength")

Therefore, the design base shear V_b can be set much lower than the "yield strength demand".

UNIFORM BUILDING CODE (1994)



$$V_b = \frac{ZIC}{R_w}\, W$$

$ZICW =$ Elastic Strength Demand (f_0)

$R_w =$ Reduction Factor $= R_\Omega\, R_y$

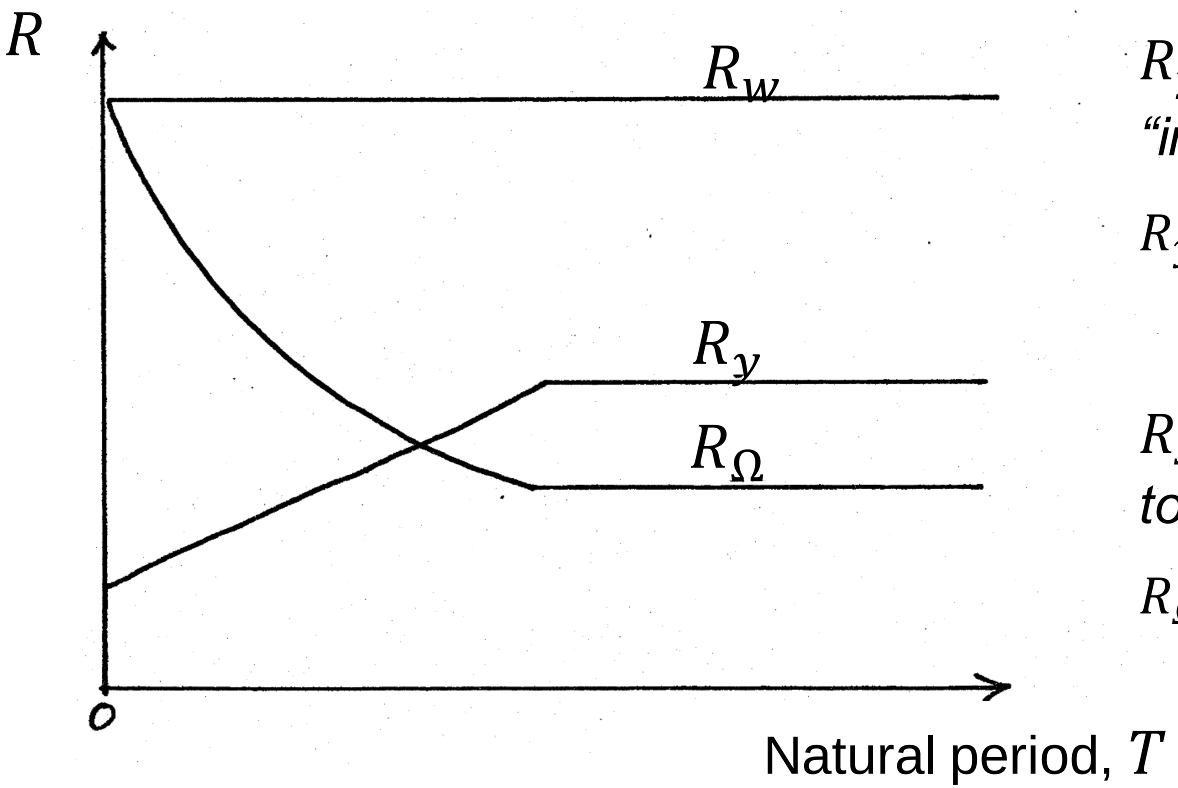
R_y = factor for reducing "elastic strength demand" (f_0) to "inelastic yield strength demand" (f_y)

R_y is associated with the ductility capacity of the system.

R_Ω = factor for reducing "yield strength demand" (f_y) to "design base shear" (V_b).

R_Ω is associated with the overstrength of the system.

REDUCTION FACTOR



R_y = factor for reducing "elastic strength demand" (f_0) to "inelastic yield strength demand" (f_y)

R_y is associated with the ductility capacity of the system.

R_Ω = factor for reducing "yield strength demand" (f_y) to "design base shear" (V_b).

R_Ω is associated with the overstrength of the system.

REDUCTION FACTOR R_w

1994 UNIFORM BUILDING CODE

TABLE 16N-STRUCTURAL SYSTEMS

BASIC STRUCTURAL SYSTEM[1]	LATERAL-FORCE-RESISTING SYSTEM—DESCRIPTION	R_w	H × 304.8 for mm
1. Bearing wall system	1. Light-framed walls with shear panels		
	a. Wood structural panel walls for structures three stories or less	8	65
	b. All other light-framed walls	6	65
	2. Shear walls		
	a. Concrete	6	160
	b. Masonry	6	160
	3. Light steel-framed bearing walls with tension-only bracing	4	65
	4. Braced frames where bracing carries gravity loads		
	a. Steel	6	160
	b. Concrete[4]	4	—
	c. Heavy timber	4	65

Note: $\boldsymbol{H}$ *is height limit applicable to seismic zones 3 and 4*

REDUCTION FACTOR R_w

TABLE 16N-STRUCTURAL SYSTEMS

		R_w	H
2. Building frame system	1. Steel eccentrically braced frame (EBF)	10	240
	2. Light-framed walls with shear panels		
	a. Wood structural panel walls for structures three stories or less	9	65
	b. All other light-framed walls	7	65
	3. Shear walls		
	a. Concrete	8	240
	b. Masonry	8	160
	4. Ordinary braced frames		
	a. Steel	8	160
	b. Concrete[4]	8	—
	c. Heavy timber	8	65
	5. Special concentrically braced frames		
	a. Steel	9	240
3. Moment-resisting frame system	1. Special moment-resisting frames (SMRF)		
	a. Steel	12	N.L.
	b. Concrete	12	N.L.
	2. Masonry moment-resisting wall frame	9	160
	3. Concrete intermediate moment-resisting frames (IMRF)[5]	8	—
	4. Ordinary moment-resisting frames (OMRF)		
	a. Steel[6]	6	160
	b. Concrete[7]	5	—

$Note: \boldsymbol{N.L.} = No\ limit$

REDUCTION FACTOR R_w

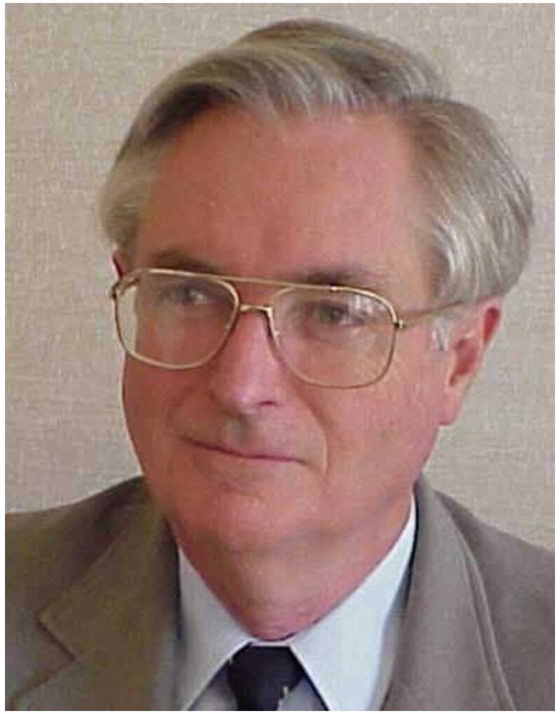
TABLE 16N-STRUCTURAL SYSTEMS

		R_w	H
4. Dual systems	1. Shear walls		
	a. Concrete with SMRF	12	N.L.
	b. Concrete with steel OMRF	6	160
	c. Concrete with concrete IMRF[5]	9	160
	d. Masonry with SMRF	8	160
	e. Masonry with steel OMRF	6	160
	f. Masonry with concrete IMRF[4]	7	—
	2. Steel EBF		
	a. With steel SMRF	12	N.L.
	b. With steel OMRF	6	160
	3. Ordinary braced frames		
	a. Steel with steel SMRF	10	N.L.
	b. Steel with steel OMRF	6	160
	c. Concrete with concrete SMRF[4]	9	—
	d. Concrete with concrete IMRF[4]	6	—
	4. Special concentrically braced frames		
	a. Steel with steel SMRF	11	N.L.
	b. Steel with steel OMRF	6	160
5. Undefined systems	See Sections 1627.8.3 and 1627.9.2	—	—

The Code-based Response Spectrum Analysis (RSA) Procedure

The Code-based Response Spectrum Analysis (RSA) Procedure

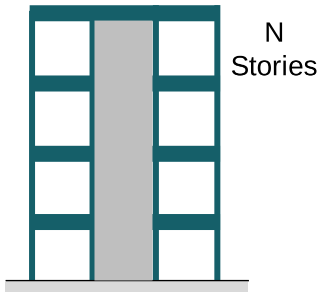
Ray W. Clough (1920 - 2016)



Edward L. Wilson (Born 1931)

The Standard RSA Procedure (ASCE 7-10, IBS 2012, EC 8)

Linear Elastic Model



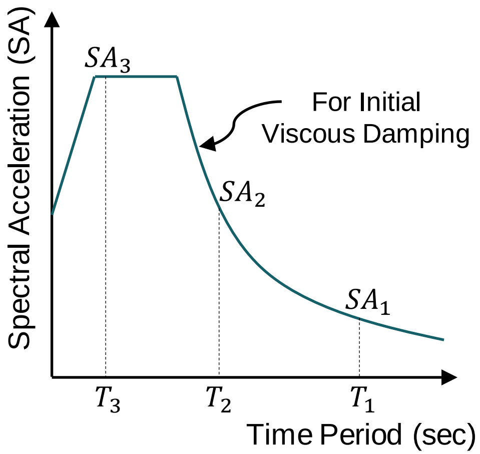
Eigen-value Analysis

$$[\mathbb{K} - \omega^2 \mathbb{M}]\Phi = \mathbb{O}$$

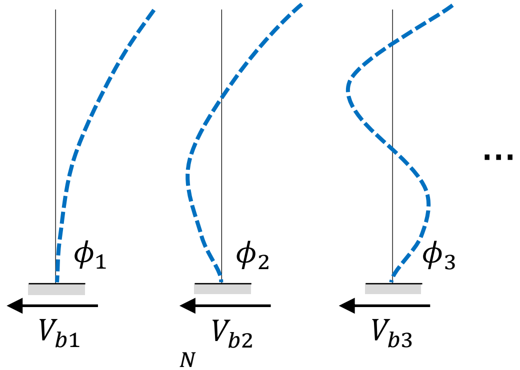
Determine Modal Properties

$$T_i,\ \phi_i,\ \Gamma_i$$

Determine Spectral Acceleration for each Significant Mode



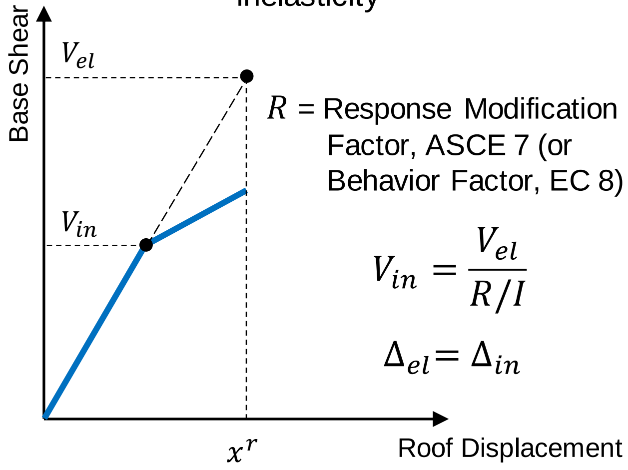
Determine Elastic Base Shears



$$V_{bn} = \sum_{i=1}^{N} \Gamma_n \,.\, m_i \,.\, \phi_{i,n} \,.\, SA_n$$

$$V_{el} = \sqrt{(V_{b1})^2 + (V_{b2})^2 + (V_{b3})^2 + \ \ldots}$$

Reduce Elastic Base Shear to account for inelasticity



R = Response Modification Factor, ASCE 7 (or Behavior Factor, EC 8)

$$V_{in} = \frac{V_{el}}{R/I}$$

$$\Delta_{el} = \Delta_{in}$$

$$V_{Design} = \frac{\sqrt{(V_{b1})^2 + (V_{b2})^2 + (V_{b3})^2 + \ \ldots}}{R/I}$$

$$M_{Design} = \frac{\sqrt{(M_{b1})^2 + (M_{b2})^2 + (M_{b3})^2 + \ \ldots}}{R}$$

$$\Delta = \frac{C_d\sqrt{(\Delta_{el1})^2 + (\Delta_{el2})^2 + (\Delta_{el3})^2 + \ \ldots}}{R/I}$$

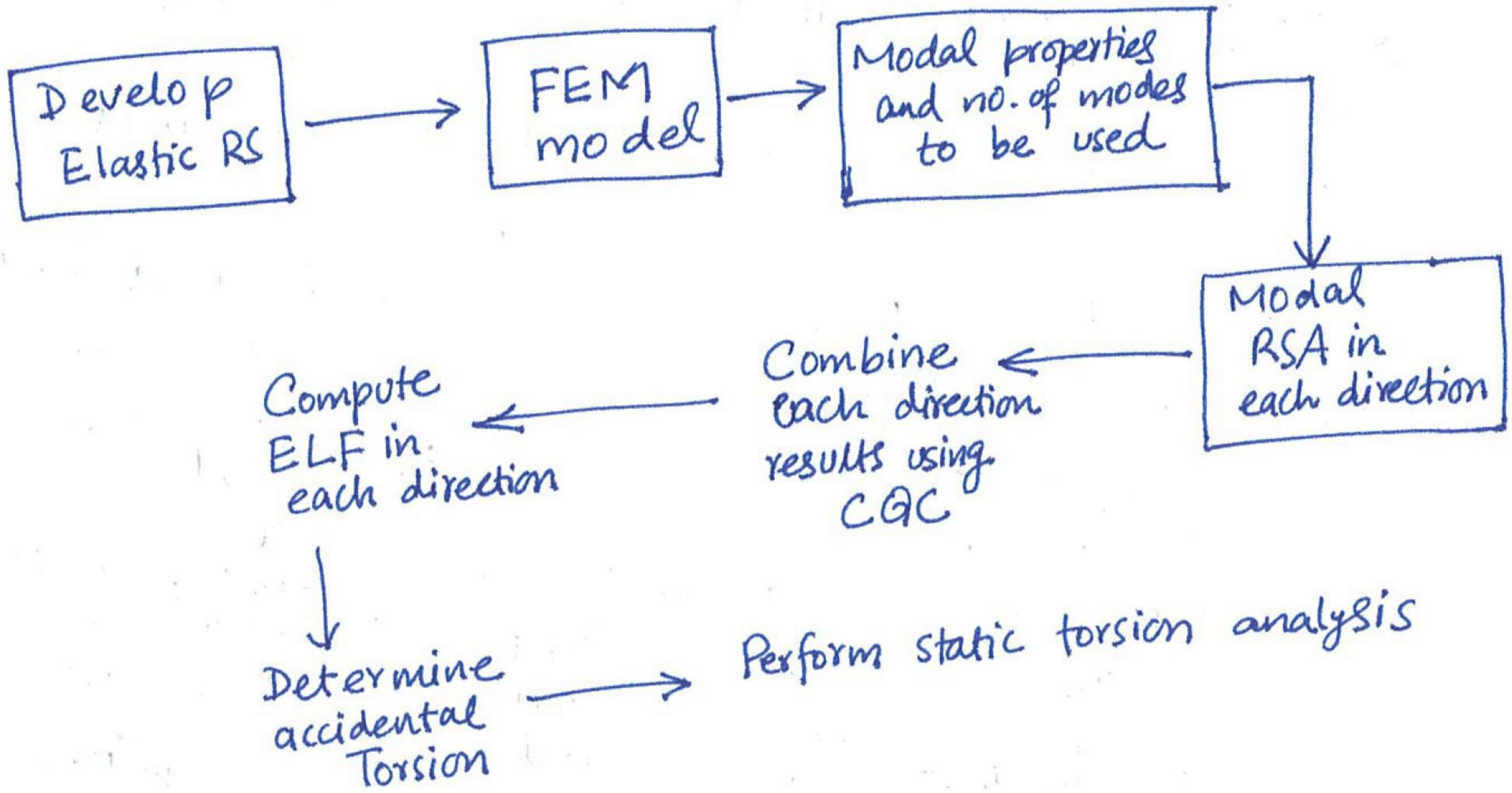
For members not desired to yield during a design earthquake $V_{Design} = \Omega \dfrac{V_{el}}{R}$

Ω = Structural Over-strength Factor

Maximum Displacement during a design earthquake $x^r_{max} = \dfrac{C_d x^r}{I}$

C_d = Displacement Amplification Factor

The Standard RSA Procedure (ASCE 7-10, IBS 2012, EC 8)



Modal Combination Rules

- ABSSUM Rule
  - Add the absolute maximum value from each mode. Not so popular and not used in practice

$$r_o \leq \sum_{n=1}^{N} \left| r_{n0} \right|$$

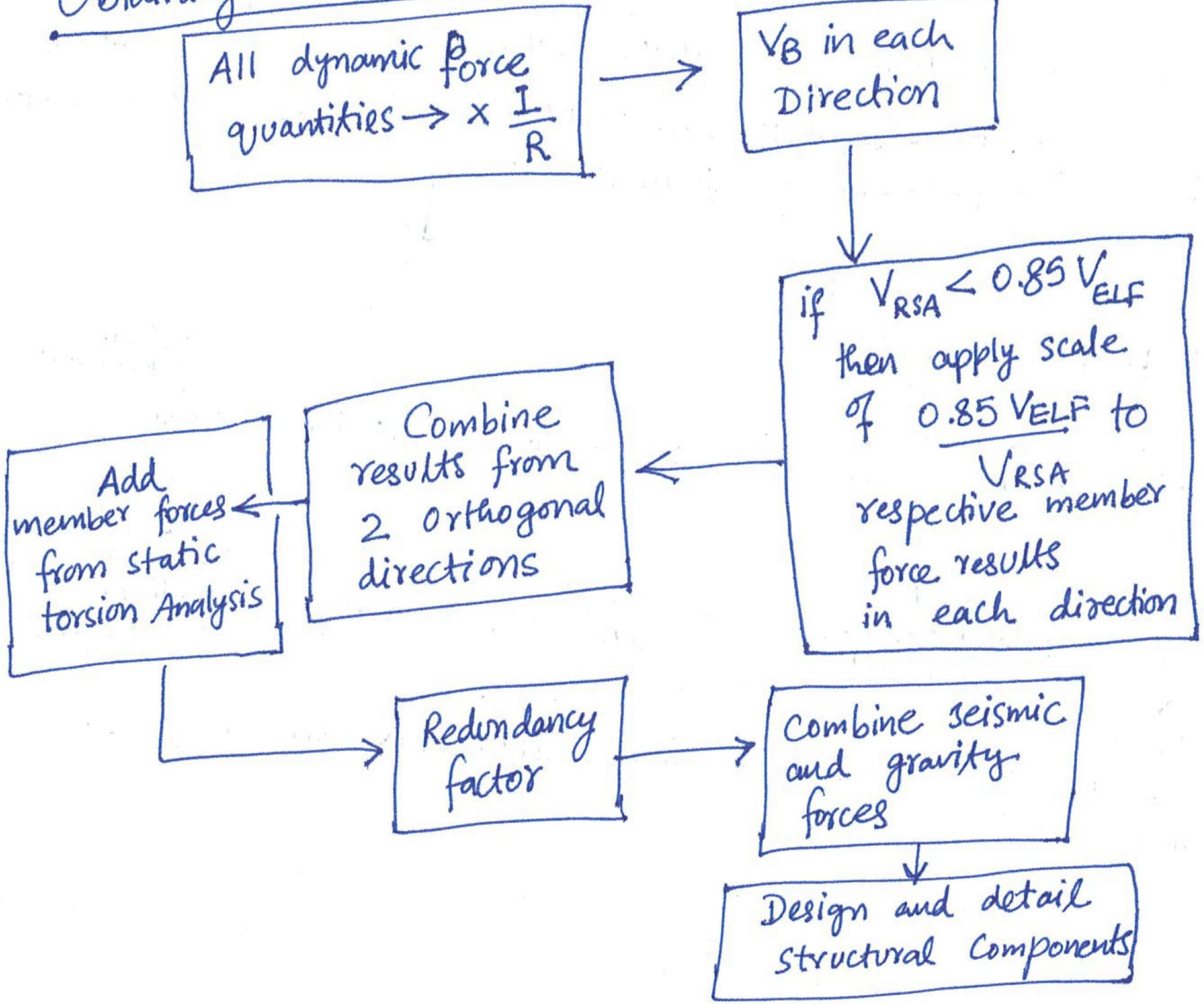
- SRSS
  - Square Root of Sum of Squares of the peak response from each mode. Suitable for well separated natural frequencies.

$$r_o \cong \sqrt{\sum_{n=1}^{N} r_{n0}^2}$$

- CQC
  - Complete Quadric Combination is applicable to large range of structural response and gives better results than SRSS.

$$r_o \cong \sqrt{\sum_{i=1}^{N} \sum_{n=1}^{N} \rho_{in} r_{i0} r_{n0}}$$

Obtaining Member Design Forces :-



The Input – Output Summary

- Input needed for Response Spectrum Analysis
  - Mass and stiffness distribution
  - A Specified Response Spectrum Curve
  - The Response Input Direction
  - The Response Scaling Factors
  - The number of modes to be included
- Output From Response Spectrum Analysis
  - Unsigned displacements, stress resultants and stresses etc.

The RSA Procedure in UBC 97

1631.5 Response Spectrum Analysis.

1631.5.1 Response spectrum representation and interpretation of results. The ground motion representation shall be in accordance with Section 1631.2. The corresponding response parameters, including forces, moments and displacements, shall be denoted as Elastic Response Parameters. Elastic Response Parameters may be reduced in accordance with Section 1631.5.4.

1631.5.2 Number of modes. The requirement of Section 1631.4.1 that all significant modes be included may be satisfied by demonstrating that for the modes considered, at least 90 percent of the participating mass of the structure is included in the calculation of response for each principal horizontal direction.

1631.5.3 Combining modes. The peak member forces, displacements, story forces, story shears and base reactions for each mode shall be combined by recognized methods. When three-dimensional models are used for analysis, modal interaction effects shall be considered when combining modal maxima.

1631.5.4 Reduction of Elastic Response Parameters for design. Elastic Response Parameters may be reduced for purposes of design in accordance with the following items, with the limitation that in no case shall the Elastic Response Parameters be reduced such that the corresponding design base shear is less than the Elastic Response Base Shear divided by the value of R.

1. For all regular structures where the ground motion representation complies with Section 1631.2, Item 1, Elastic Response Parameters may be reduced such that the corresponding design base shear is not less than 90 percent of the base shear determined in accordance with Section 1630.2.

2. For all regular structures where the ground motion representation complies with Section 1631.2, Item 2, Elastic Response Parameters may be reduced such that the corresponding design base shear is not less than 80 percent of the base shear determined in accordance with Section 1630.2.

3. For all irregular structures, regardless of the ground motion representation, Elastic Response Parameters may be reduced such that the corresponding design base shear is not less than 100 percent of the base shear determined in accordance with Section 1630.2.

The corresponding reduced design seismic forces shall be used for design in accordance with Section 1612.

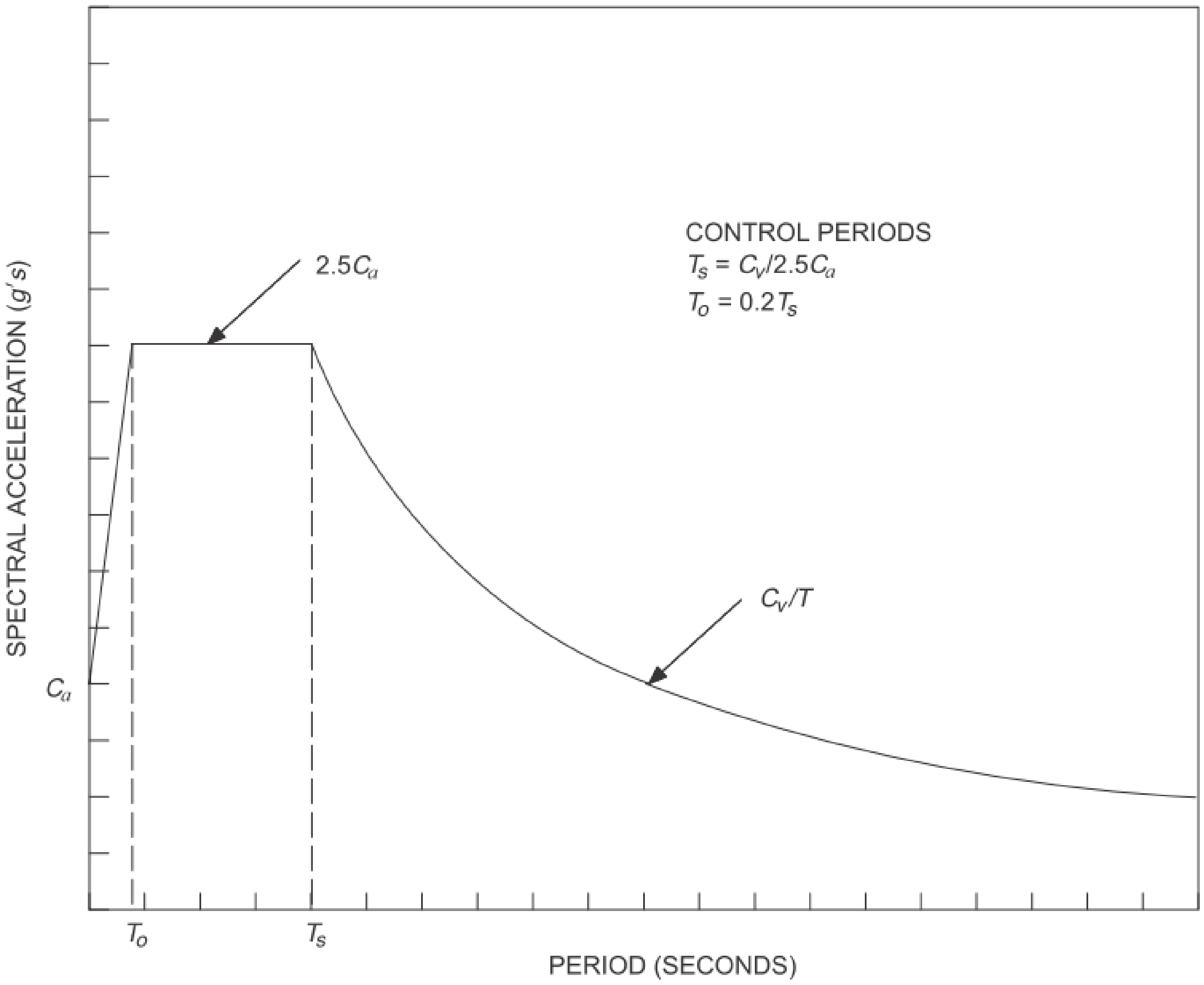


FIGURE 16-3—DESIGN RESPONSE SPECTRA

Dynamic Analysis Procedure in IBC 2000

SECTION 1618
DYNAMIC ANALYSIS PROCEDURE FOR THE SEISMIC DESIGN OF BUILDINGS

1618.1 Dynamic anaylsis procedures. The following dynamic analysis procedures performed in accordance with the requirements of this section may be used in lieu of equivalent lateral force procedure of Section 1617.4:

1. Modal Response Spectra Analysis.
2. Linear Time-history Analysis.
3. Nonlinear Time-history Analysis.

1618.1.1 Modeling. A mathematical model of the building shall be constructed that represents the spatial distribution of mass and stiffness throughout the structure. For regular structures with independent orthogonal seismic-force-resisting systems, independent two-dimensional models may be constructed to represent each system. For irregular structures without independent orthogonal systems, a three-dimensional model incorporating a minimum of three dynamic degrees of freedom consisting of translation in two orthogonal plan directions and torsional rotation about the vertical axis shall be included at each level of the building. Where the diaphragms are not rigid compared to the vertical elements of the lateral-force-resisting system, the model shall include representation of the diaphragm's flexibility and such additional dynamic degrees of freedom as are required to account for the participation of the diaphragm in the structure's dynamic response. In addition, the model shall comply with the following:

1. Stiffness properties of concrete and masonry elements shall include the effects of cracked sections.
2. For steel moment frame systems, the contribution of panel zone deformations to overall story drift shall be included.

1615.1.4 General procedure response spectrum. The general design response spectrum curve shall be developed as indicated in Figure 1615.1.4 and as follows:

1. For periods less than or equal to T_0, the design spectral response acceleration, S_a, shall be given by Equation 16-20.
2. For periods greater than or equal to T_0 and less than or equal to the T_S, the design spectral response acceleration, S_a, shall be taken equal to S_{DS}.
3. For periods greater than T_S, the design spectral response acceleration, S_a, shall be given by Equation 16-21.

$$S_a = 0.6\frac{S_{DS}}{T_O}T + 0.4S_{DS} \qquad \textbf{(Equation 16-20)}$$

$$S_a = \frac{S_{D1}}{T} \qquad \textbf{(Equation 16-21)}$$

The RSA Procedure in IBC 2000

Design Spectral Values

- Adjust Maximum Considered Earthquake (MCE) values of Ss and $S1$ for local site effects

$$S_{MS} = F_a \times S_s$$
$$S_{M1} = F_v \times S_1$$

- Calculate the spectral design values

$$S_{DS} = 2/3 \times S_{MS}$$
$$S_{D1} = 2/3 \times S_{M1}$$

where:

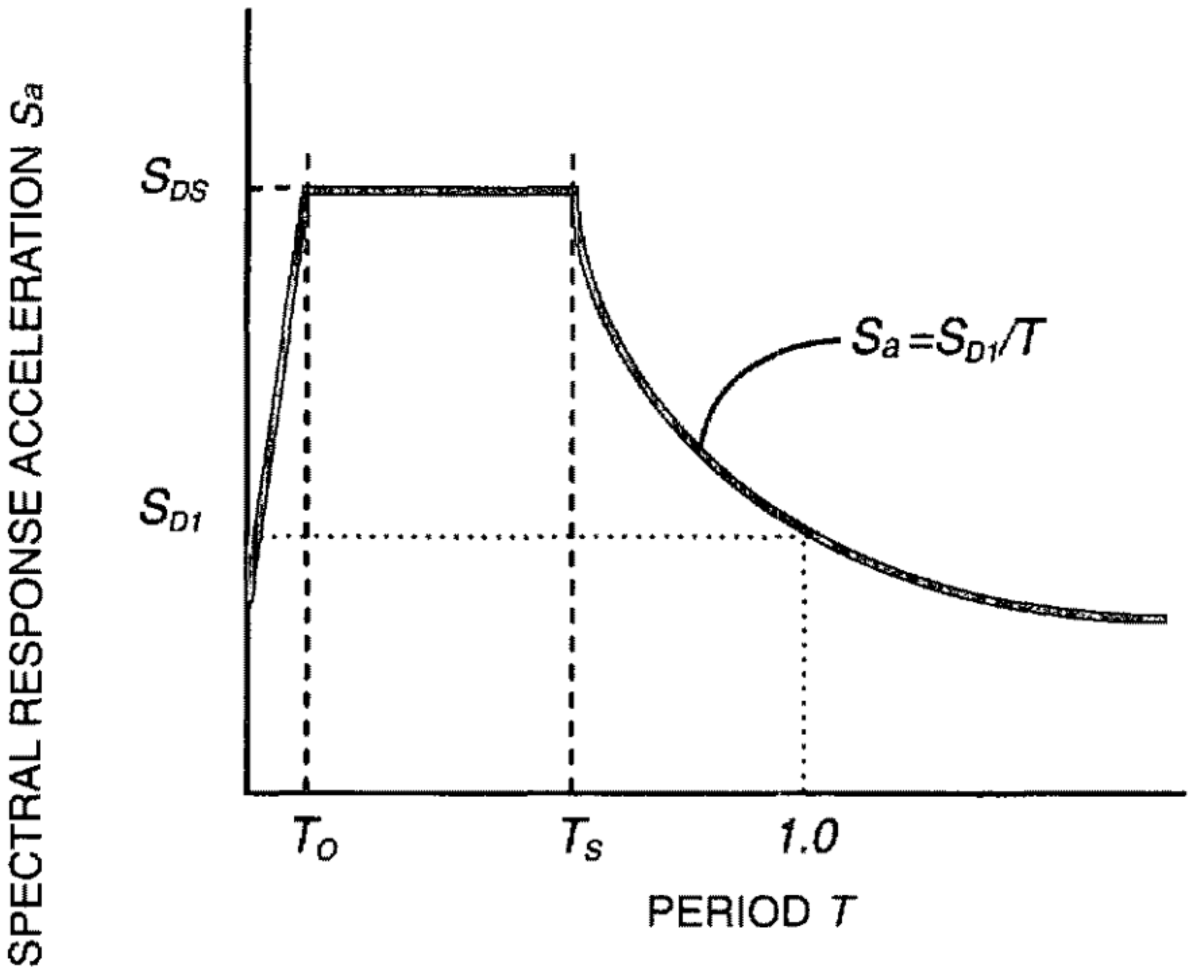
S_{DS} = The design spectral response acceleration at short periods as determined in Section 1615.1.3.

S_{D1} = The design spectral response acceleration at 1 second period as determined in Section 1615.1.3.

T = Fundamental period (in seconds) of the structure (Section 1617.4.2).

$T_O = 0.2\ S_{D1}/S_{DS}$

$T_S = S_{D1}/S_{DS}$


SPECTRAL RESPONSE ACCELERATION Sa
SDS
SD1
Sa =SD1/T
TO
TS
1.0
PERIOD T

1618.2 Modes. An analysis shall be conducted to determine the natural modes of vibration for the building including the period of each mode, the modal shape vector, ϕ, the mass participation factor, and the modal mass. The analysis shall include a sufficient number of modes to obtain a combined modal mass participation of at least 90 percent of the actual building mass in each of two orthogonal directions.

1618.3 Modal properties. The required periods, mode shapes, and participation factors of the building shall be calculated by established methods of structural analysis for the fixed base condition using the masses and elastic stiffnesses of the seismic-force-resisting system.

1618.4 Modal base shear. The portion of the base shear contributed by the m^{th} mode, V_m, shall be determined from the following equations:

$$V_m = C_{sm}\overline{W}_m \qquad \textbf{(Equation 16-51)}$$

$$\overline{W}_m = \frac{\left(\sum_{i=1}^{n} w_i \phi_{im}\right)^2}{\sum_{i=1}^{n} w_i \phi_{im}^2} \qquad \textbf{(Equation 16-52)}$$

where:

C_{sm} = The modal seismic response coefficient determined in Equation 16-53.

$\overline{W}_m$ = The effective modal gravity load.

w_i = The portion of the total gravity load, W, of the building at Level i, where W = the total dead load and other loads listed below:

1. In areas used for storage, a minimum of 25 percent of the reduced floor live load (floor live load in public garages and open parking structures need not be included).
2. Where an allowance for partition load is included in the floor load design, the actual partition weight or a minimum weight of 10 pounds per square foot (0.479 kN/m^2) of floor area, whichever is greater.
3. Total operating weight of permanent equipment.
4. Twenty percent of flat roof snow load where the flat roof snow load exceeds 30 pounds per square foot (1.44 kN/m^2).

ϕ_{im} = The displacement amplitude at the i^{th} level of the building when vibrating in its m^{th} mode.

The modal seismic response coefficient, C_{sm}, shall be determined by the following equation:

$$C_{sm} = \frac{S_{am}}{\left(\frac{R}{I_E}\right)} \qquad \textbf{(Equation 16-53)}$$

where:

I_E = The occupancy importance factor determined in accordance with Section 1616.2.

S_{am} = The modal design spectral response acceleration at period T_m determined from either the general design response spectrum of Section 1615.1 or a site-specific response spectrum per Section 1615.2.

R = The response modification factor determined from Table 1617.6.

1618.5 Modal forces, deflections and drifts. The modal force, F_{xm}, at each level shall be determined by the following equations:

$$F_{XM} = C_{vxm} V_m \qquad \text{(Equation 16-55)}$$

$$C_{vxm} = \frac{w_x \phi_{xm}}{\sum_{i=1}^{n} w_i \phi_{im}} \qquad \text{(Equation 16-56)}$$

where:

C_{vxm} = The vertical distribution factor in the m^{th} mode.

V_m = The total design lateral force or shear at the base in the m^{th} mode.

w_i, w_x = The portion of the total gravity load of the building, W, located or assigned to Level i or x.

ϕ_{im} = The displacement amplitude at the i^{th} level of the building when vibrating in its m^{th} mode.

ϕ_{xm} = The displacement amplitude at the x^{th} level of the building when vibrating in its m^{th} mode.

The modal deflection at each level, δ_{xm}, shall be determined by the following equations:

$$\delta_{xm} = \frac{C_d \delta_{xem}}{I_E} \qquad \text{(Equation 16-57)}$$

$$\delta_{xem} = \left(\frac{g}{4\pi^2}\right)\left(\frac{T_m^2 F_{xm}}{W_x}\right) \qquad \text{(Equation 16-58)}$$

where:

C_d = The deflection amplification factor determined from Table 1617.6.

F_{xm} = The portion of the seismic base shear in the m^{th} mode, induced at Level x.

g = The acceleration due to gravity (ft/s² or m/s²).

I_E = The occupancy importance factor determined in accordance with Section 1616.2.

T_m = The modal period of vibration, in seconds, of the m^{th} mode of the building.

w_x = The portion of the total gravity load of the building, W, located or assigned to Level x.

δ_{xem} = The deflection of Level x in the m^{th} mode at the center of the mass at Level x determined by an elastic analysis.

1618.7 Design values. The design value for the modal base shear, V_t; each of the story shear, moment and drift quantities; and the deflection at each level shall be determined by combining their modal values as obtained from Sections 1618.5 and 1618.6. The combination shall be carried out by taking the square root of the sum of the squares of each of the modal values or by the complete quadratic combination (CQC) technique.

The base shear, V, using the equivalent lateral force procedure in Section 1617.4 shall be calculated using a fundamental period of the building, T, in seconds, of 1.2 times the coefficient for upper limit on the calculated period, C_u, taken from Table 1617.4.2, times the approximate fundamental period of the building, T_a, calculated in accordance with Section 1617.4.2.1. Where the thus calculated base shear, V, is greater than the modal base shear, V_t, the design story shears, moments, drifts and floor deflections shall be multiplied by C_m, the modification factor:

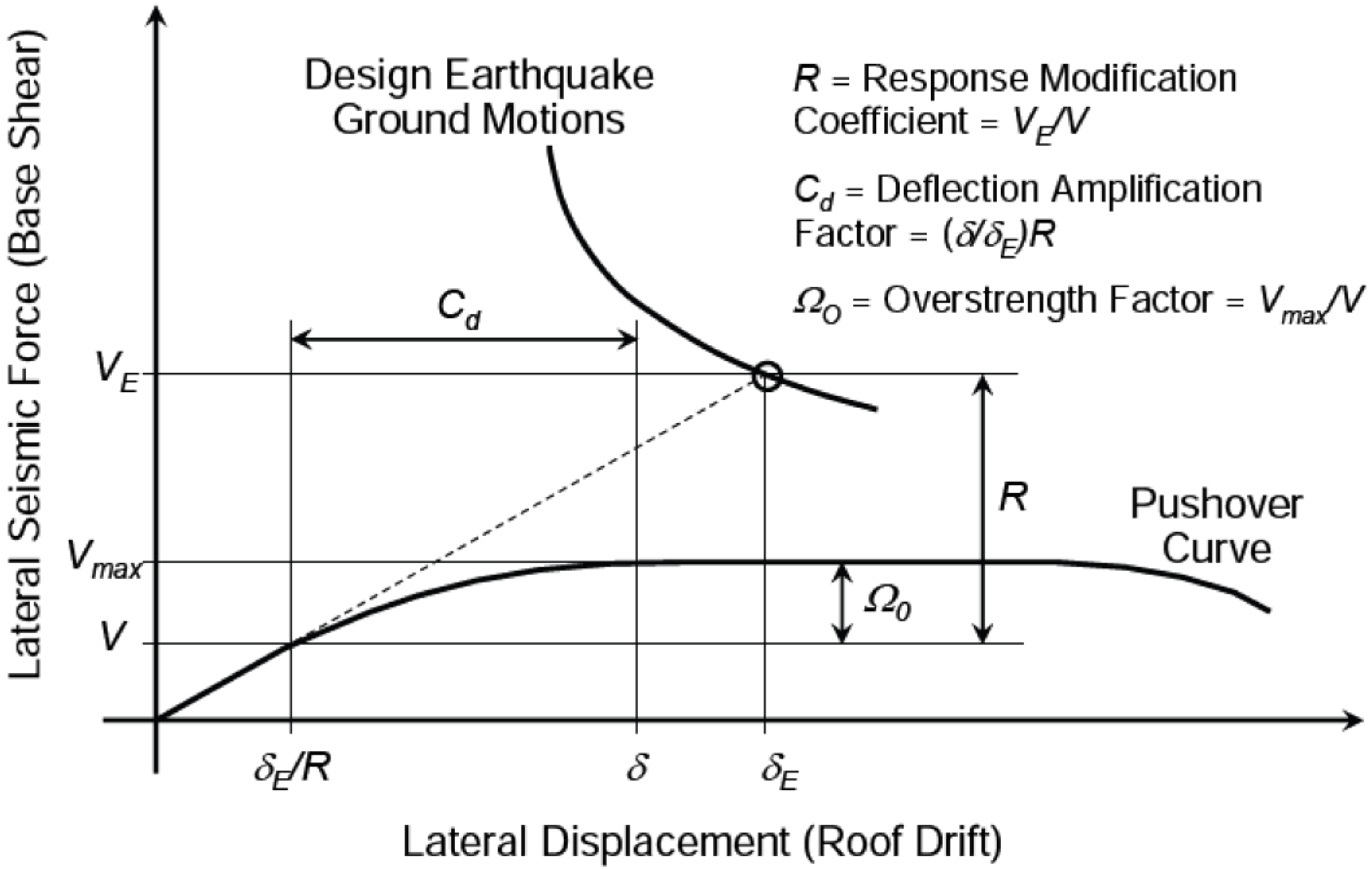
$$C_m = \frac{V}{V_t} \qquad \textbf{(Equation 16-59)}$$

where

V = The equivalent lateral force procedure base shear, calculated in accordance with this section and Section 1617.4.

V_t = The modal base shear, calculated in accordance with this section.

The modal base shear, V_t, need not exceed the base shear calculated from the equivalent lateral force procedure in Section 1617.4. However, for buildings with a value of the design spectral response acceleration at 1 second period, S_{D1}, of 0.2 or greater, as determined in Section 1615.1.3, with a period T, as determined in Section 1617.4.2, of 0.7 second or greater, and located on Site Class E or F sites (Section 1615.1.1), the design base shear shall not be less than that determined using the equivalent lateral force procedure in Section 1617.4.


Design Earthquake
Ground Motions
R = Response Modification
Coefficient = V_E/V
C_d = Deflection Amplification
Factor = $(\delta/\delta_E)R$
Ω_O = Overstrength Factor = V_{max}/V
C_d
V_E
R
Pushover
Curve
V_{max}
Ω_0
V
δ_E/R
δ
δ_E
Lateral Seismic Force (Base Shear)
Lateral Displacement (Roof Drift)

Routine Design Office Practice

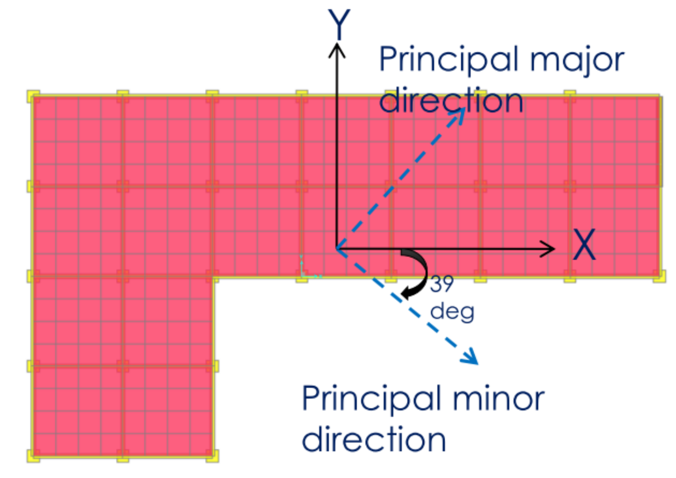
- Run equivalent static analysis according to code
- Find principal directions from modal base shear
- Run response spectrum analysis along principal directions with scale factor of 9.81 (m/s^2)
- Find the scale factor for scaling elastic response spectrum results to equivalent static base shear
- Run response spectrum analysis with calculated scale factor in principal directions

Scaling the Results

- Reduce the elastic response for design purpose, but design base shear is not less than elastic base shear divided by R.
- Design base shear shall not be less than 85% of static lateral force base shear according to ASCE 7-05.
- In ASCE 7-16, base shear shall be scaled to 100% of static lateral force base shear.

Principal Directions

- Lack of definitions of the principal directions in code
- The direction of the base reaction of the mode shape associated with the fundamental frequency of the system is used to define the principal direction.
    - Run modal analysis
    - Extract base shear of mode 1
    - Find angle between X and Y components of base shear
    - Another direction is 90 degrees apart



Fx = -1,041 kN
Fy = 846 kN
Angle = tan-1 (Fy/Fx)
= -39 deg.

Courtesy: Mr. Thanung Htut Aung (AIT Solutions)

Directional and Orthogonal Effects

- Seismic forces act in both principal directions of the building simultaneously.
- But seismic effects in two directions are unlikely to reach their maxima simultaneously.
- 100% of seismic forces in one principal direction combined with 30% of seismic forces in the orthogonal direction.

Accidental Torsion

- Arise from several factors
  - Rotational components of ground motions
  - Effects of non-structural elements
  - Actual distribution of dead and live loads
  - Uncertainties in defining building's material properties for dynamic analysis
- Generally 5% of eccentricity from center of mass is considered

- Structures assigned to Seismic Design Category C, D, E, or F, where Type 1a or 1b torsional irregularity exists, accidental torsional moment needs to be amplified.

$$A_x = \left(\frac{\delta_{max}}{1.2\delta_{avg}}\right)^2 \qquad (12.8\text{-}14)$$

where

δ_{max} = the maximum displacement at Level x (in. or mm) computed assuming $A_x = 1$

δ_{avg} = the average of the displacements at the extreme points of the structure at Level x computed assuming $A_x = 1$ (in. or mm)

EXCEPTION: The accidental torsional moment need not be amplified for structures of light-frame construction.

The torsional amplification factor (A_x) is not required to exceed 3.0. The more severe loading for each element shall be considered for design.

Table 12.3-1 Horizontal Structural Irregularities

Type	Description	Reference Section	Seismic Design Category Application
1a.	**Torsional Irregularity:** Torsional irregularity is defined to exist where the maximum story drift, computed including accidental torsion with $A_x = 1.0$, at one end of the structure transverse to an axis is more than 1.2 times the average of the story drifts at the two ends of the structure. Torsional irregularity requirements in the reference sections apply only to structures in which the diaphragms are rigid or semirigid.	12.3.3.4 12.7.3 12.8.4.3 12.12.1 Table 12.6-1 16.3.4	D, E, and F B, C, D, E, and F C, D, E, and F C, D, E, and F D, E, and F B, C, D, E, and F
1b.	**Extreme Torsional Irregularity:** Extreme torsional irregularity is defined to exist where the maximum story drift, computed including accidental torsion with $A_x = 1.0$, at one end of the structure transverse to an axis is more than 1.4 times the average of the story drifts at the two ends of the structure. Extreme torsional irregularity requirements in the reference sections apply only to structures in which the diaphragms are rigid or semirigid.	12.3.3.1 12.3.3.4 12.3.4.2 12.7.3 12.8.4.3 12.12.1 Table 12.6-1 16.3.4	E and F D D B, C, and D C and D C and D D B, C, and D
2.	**Reentrant Corner Irregularity:** Reentrant corner irregularity is defined to exist where both plan projections of the structure beyond a reentrant corner are greater than 15% of the plan dimension of the structure in the given direction.	12.3.3.4 Table 12.6-1	D, E, and F D, E, and F
3.	**Diaphragm Discontinuity Irregularity:** Diaphragm discontinuity irregularity is defined to exist where there is a diaphragm with an abrupt discontinuity or variation in stiffness, including one that has a cutout or open area greater than 50% of the gross enclosed diaphragm area, or a change in effective diaphragm stiffness of more than 50% from one story to the next.	12.3.3.4 Table 12.6-1	D, E, and F D, E, and F
4.	**Out-of-Plane Offset Irregularity:** Out-of-plane offset irregularity is defined to exist where there is a discontinuity in a lateral force-resistance path, such as an out-of-plane offset of at least one of the vertical elements.	12.3.3.3 12.3.3.4 12.7.3 Table 12.6-1 16.3.4	B, C, D, E, and F D, E, and F B, C, D, E, and F D, E, and F B, C, D, E, and F
5.	**Nonparallel System Irregularity:** Nonparallel system irregularity is defined to exist where vertical lateral force-resisting elements are not parallel to the major orthogonal axes of the seismic force-resisting system.	12.5.3 12.7.3 Table 12.6-1 16.3.4	C, D, E, and F B, C, D, E, and F D, E, and F B, C, D, E, and F

Seismic Design as per ASCE 7-10

All members should be designed for these effects.

Seismic Load Effect (E)
(for load Combinations)

$$E = E_h \pm E_v$$

$E_h = \rho Q_E$

- ρ → redundancy factor (1 or 1.3)
- Q_E → effects from $\frac{V_e}{R}$

$E_v = 0.2 S_{DS} D$ → effect of dead load

So combinations for strength design,

$$(1.2 + 0.2 S_{DS}) D + \rho Q_E + L + 0.2S$$

$$(0.9 - 0.2 S_{DS}) D + \rho Q_E + 1.6H$$

Seismic load Effect Including Ω_o

$$E_m = E_{mh} \pm E_v$$

E_{mh} → effect of seismic forces including Ω

$= \Omega_o Q_E$

Q_E → effects from $\frac{V_e}{R}$

$$(1.2 + 0.2 S_{DS}) D + \Omega_o Q_E + L + 0.2S$$

$$(0.9 - 0.2 S_{DS}) D + \Omega_o Q + 1.6H$$

The RSA Procedure (ASCE 7-10)

→ Seismic Design Catagory (SDC) → assigned to a structure based on its risk category, and the severity of design EQ motio.

→ Nominal strength → without any reduction factor.

Design strength = Nominal strength × ϕ

Table 1.5–1 → Risk Catagory of Buildings

I, II, III, IV

Table 1.5-2 → Importance Factors of Risk category of buildings. $I_e = 1$ (for I and II)

$= 1.25$ for III

$= 1.5$ for IV.

SDC → based on S_{DS} or S_{D1} and Risk Category →

A, B, C, D, E, F

E: $S_1 > 0.75g$ for I, II, III

F: $S_1 > 0.75g$ for IV

Mapped Acceleration Parametes S_s, S_1 → Site class A, B, C, D, E → Site Coefficients F_a, F_v → Risk-targeted MCE SA para-meters S_{MS}, S_{M1} → Design SA parametes S_{DS}, S_{D1} → Design Response Spectrum → MCE Response Spectrum = 1.5×DBE

The RSA Procedure (ASCE 7-10)

→ For SDC D, E, F → PGA shall be determined based on (a) A site-specific study taking into account soil amplification effects or (b) $PGA_M = F_{PGA}\,PGA$

PGA_M → adjusted for site class effects

F_{PGA} → site coefficient Table 11.8-1

→ R, Ω_o and C_d (Table 12.2-1)

R → base shear

Ω_o → element design forces

C_d → design story drift

→ Combining factored loads using strength design.

Basic combinations

1) $1.4D$
2) $1.2D + 1.6L + 0.5(L_r, S \text{ or } R)$
3) $1.2D + 1.6(L_r, S \text{ or } R) + (L \text{ or } 0.5W)$
4) $1.2D + 1.0W + L + 0.5(L_r, S \text{ or } R)$
5) $1.2D + 1.0E + L + 0.2S$
6) $0.9D + 1.0W$
7) $0.9D + 1.0E$

The RSA Procedure (ASCE 7-10)

→ Redundancy → Sometimes synonym of "Alternative loading path". The ability of structure to redistribute among its members/connections the loads which can no longer be carried by some other damaged portions. Non-redundant structures → fail immediately under local damage.

A "ρ" redundancy factor is assigned to seismic force resisting system.

$\rho = 1$ (SDC B, C and .....)

$\rho = 1.3$ (SDC D, E, F except ...)

The RSA Procedure (ASCE 7-10)

→ The "E" in load combination 5 is

$$E = E_h + E_v$$

For load combination 7, $E = E_h - E_v$

$E_h = \rho Q_E$ → from V (application of horizontal forces simultaneously in 2 directions). at right angles.

$E_V = 0.2 S_{DS} D$ └ dead load

↓

(some exceptions)

→ So basic combinations for strength design

5) $(1.2 + 0.2 S_{DS}) D + \rho Q_E + L + 0.2S$

6) $(0.9 \cancel{+} 0.2 S_{DS}) D + \rho Q_E + 1.6 H$ └ lateral earth pressure

→ Where specifically required, conditions requiring overstrength factor →

For Load Combination 5) $E_m = E_{mh} + E_v$

6) $E_m = E_{mh} - E_v$

$E_{mh} = \Omega_o Q_E$

↳ need not exceed the max force that can develop in the element as determined by a rational, plastic mechanism analysis or NL analysis.

so

$(1.2 + 0.2 S_{DS}) D + \Omega_o Q_E + L + 0.2S$

$(1.2 - 0.2 S_{DS}) D + \Omega_o Q_E + 1.6 H$

→ Direction of loading :-

SDC B → permitted to be applied independently in each of the two orthogonal directions (Interaction effects neglected).

The RSA Procedure (ASCE 7-10)

SDC C → Minimum as SDC B.
If irregularity type 5 — Orthogonal combination procedure
— Simultaneous application of orthogonal ground motions.

SDC D, E, F → Minimum as SDC
+ .....

→ Table 12.6-1 for "Permitted Analytical Procedures"

Structural Characteristics	ELF	RSA	RHA
⋮	P	P	P
	⋮	⋮	⋮
	NP	P	P

→ Modeling Criteria

- for determining seismic loads → fixed base permitted
- Effective seismic weight = D
  + 0.25 L
  + Partitions
  + Operating
  + Snow
  + Landscaping
- Spatial distribution of mass and stiffness
- Concrete → cracked sections
- Steel frame → the contribution of panel zone deformations to overall story drift shall be included.
- For 3D, a minimum of 3 dynamic DOF (2 Trans, 1 rot) shall be included at each level.

Strength Design Load Combinations (ASCE 7-16)

Gravity Load Combinations

1) $1.4DL$

2) $1.2DL + 1.6LL$

Seismic Load Combinations

1) $1.2DL + 1.0EQX + 0.3EQY + 0.5LL$

2) $1.2DL + 0.3EQX + 1.0EQY + 0.5LL$

3) $0.9DL + 1.0EQX + 0.3EQY$

4) $0.9DL + 0.3EQX + 1.0EQY$

$EQ = \rho E_h \pm E_v$

$E_v = 0.2\, S_{DS}\, DL$

e.g. If $S_{DS} = 0.7$ and $\rho = 1.3$,

$E_v = 0.2 \times 0.7 \times DL = 0.14\, DL$

1) $(1.2 + 0.14)DL + 1.3EQX + 0.39EQY + 0.5LL$

or $1.34DL + 1.3EQX + 0.39EQY + 0.5LL$

2) $1.34DL + 0.39EQX + 1.3EQY + 0.5LL$

3) $(0.9 - 0.14)DL + 1.3EQX + 0.39EQY$

or $0.76DL + 1.3EQX + 0.39EQY$

4) $0.76DL + 0.39EQX + 1.3EQY$

Response Spectrum Analysis Procedure in BCP (2007)

5.31 Dynamic Analysis Procedures

5.31.1 *General*
Dynamic analyses procedures, when used, shall conform to the criteria established in this section. The analysis shall be based on an appropriate ground motion representation and shall be performed using accepted principles of dynamics. Structures that are designed in accordance with this section shall comply with all other applicable requirements of these provisions.

5.31.2 *Ground Motion*
The ground motion representation shall, as a minimum, be one having a 10-percent probability of being exceeded in 50 years, shall not be reduced by the quantity R and may be one of the following:

1. An elastic design response spectrum constructed in accordance with Figure 5.1, using the values of C_a and C_v consistent with the specific site. The design acceleration ordinates shall be multiplied by the acceleration of gravity, 9.815 m/sec^2 (386.4 in/sec^2).
2. A site-specific elastic design response spectrum based on the geologic, tectonic, seismologic and soil characteristics associated with the specific site. The spectrum shall be developed for a damping ratio of 0.05, unless a different value is shown to be consistent with the anticipated structural behavior at the intensity of shaking established for the site.

3. Ground motion time histories developed for the specific site shall be representative of actual earthquake motions. Response spectra from time histories, either individually or in combination, shall approximate the site design spectrum conforming to Section 5.31.2, Item 2.
4. For structures on Soil Profile Type S_F, the following requirements shall apply when required by Section 5.29.8.4, Item 4:

   4.1 The ground motion representation shall be developed in accordance with Items 2 and 3.
   4.2 Possible amplification of building response due to the effects of soil-structure interaction and lengthening of building period caused by inelastic behavior shall be considered.
5. The vertical component of ground motion may be defined by scaling corresponding horizontal accelerations by a factor of two-thirds. Alternative factors may be used when substantiated by site specific data. Where the Near Source Factor, N_a, is greater than 1.0, site-specific vertical response spectra shall be used in lieu of the factor of two-thirds.

5.31.4 *Description of Analysis Procedures.*

5.31.4.1 *Response spectrum analysis.* An elastic dynamic analysis of a structure utilizing the peak dynamic response of all modes having a significant contribution to total structural response. Peak modal responses are calculated using the ordinates of the appropriate response spectrum curve which correspond to the modal periods. Maximum modal contributions are combined in a statistical manner to obtain an approximate total structural response.

5.31.4.2 *Time-history analysis.* An analysis of the dynamic response of a structure at each increment of time when the base is subjected to a specific ground motion time history.

5.31.5 *Response Spectrum Analysis.*

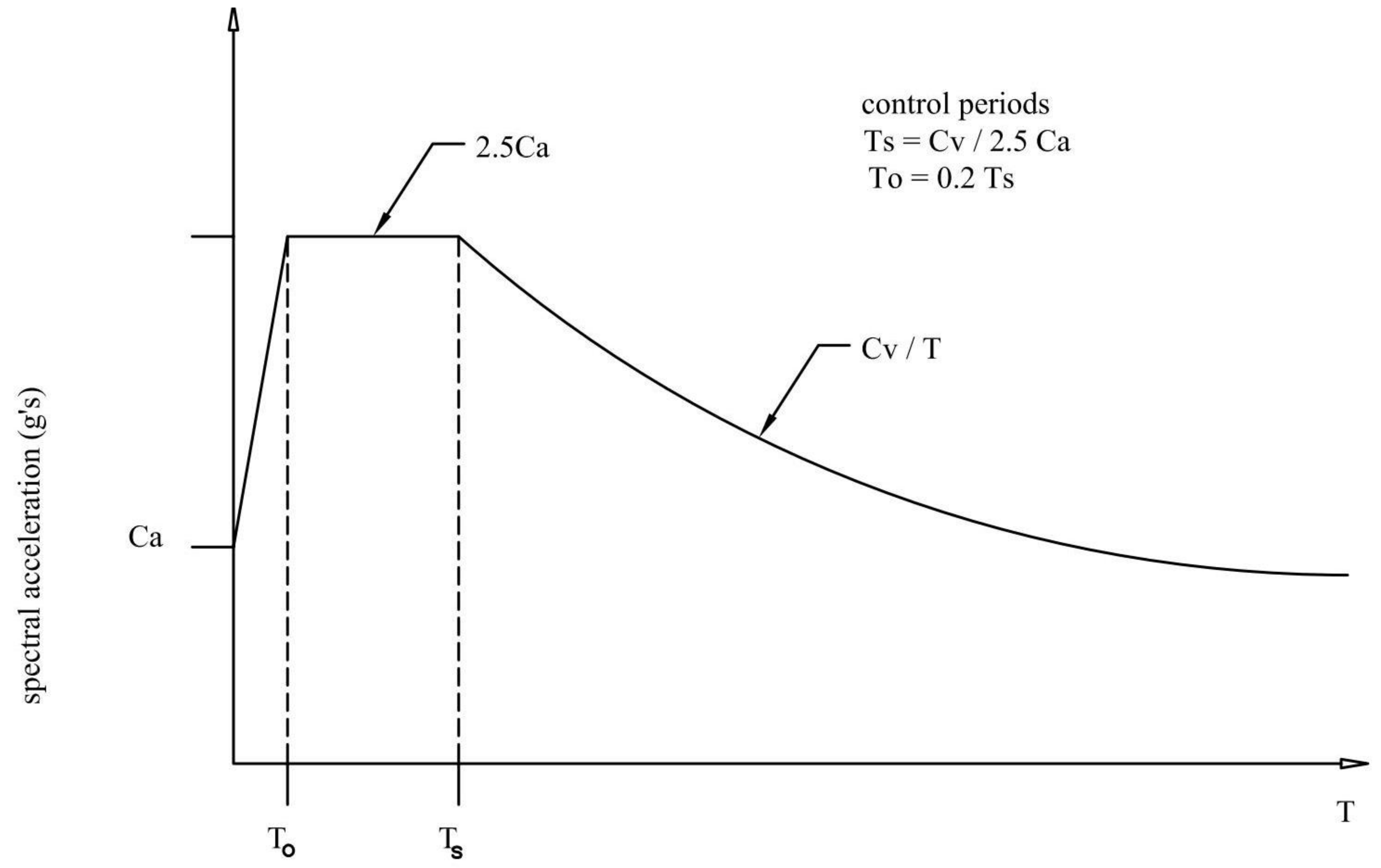
5.31.5.1 *Response spectrum representation and interpretation of results.* The ground motion representation shall be in accordance with Section 5.31.2. The corresponding response parameters, including forces, moments and displacements, shall be denoted as Elastic Response Parameters. Elastic Response Parameters may be reduced in accordance with Section 5.31.5.4.

5.31.5.2 *Number of modes.* The requirement of Section 5.31.4.1 that all significant modes be included may be satisfied by demonstrating that for the modes considered, at least 90 percent of the participating mass of the structure is included in the calculation of response for each principal horizontal direction.

5.31.5.3 *Combining modes.* The peak member forces, displacements, storey forces, storey shears and base reactions for each mode shall be combined by recognized methods. When three-dimensional models are used for analysis, modal interaction effects shall be considered when combining modal maxima.

5.31.5.4 *Reduction of Elastic Response Parameters for design*
Elastic Response Parameters may be reduced for purposes of design in accordance with the following items, with the limitation that in no case shall the Elastic Response Parameters be reduced such that the corresponding design base shear is less than the Elastic Response Base Shear divided by the value of R.

Design Response Spectrum (UBC 1997, p 2-38)



5.31.5.4 *Reduction of Elastic Response Parameters for design*

Elastic Response Parameters may be reduced for purposes of design in accordance with the following items, with the limitation that in no case shall the Elastic Response Parameters be reduced such that the corresponding design base shear is less than the Elastic Response Base Shear divided by the value of *R*.

1. For all regular structures where the ground motion representation complies with Section 5.31.2, Item 1, Elastic Response Parameters may be reduced such that the corresponding design base shear is not less than 90 percent of the base shear determined in accordance with Section 5.30.2.
2. For all regular structures where the ground motion representation complies with Section 5.31.2, Item 2, Elastic Response Parameters may be reduced such that the corresponding design base shear is not less than 80 percent of the base shear determined in accordance with Section 5.30.2.
3. For all irregular structures, regardless of the ground motion representation, Elastic Response Parameters may be reduced such that the corresponding design base shear is not less than 100 percent of the base shear determined in accordance with Section 5.30.2. The corresponding reduced design seismic forces shall be used for design in accordance with Section 5.12.

The Response Spectrum Analysis (RSA) Procedure

- The RSA Procedure in IBC 2003
  - ***Check yourself***
- The RSA Procedure in ASCE 7-05
  - ***Check yourself***
- The RSA Procedure in IBC 2006
  - ***Check yourself***
- The RSA Procedure in IBC 2009
  - ***Check yourself***
- The RSA Procedure in ASCE 7-10
  - ***Check yourself***
- The RSA Procedure in IBC 2012
  - ***Check yourself***
- The RSA Procedure in ASCE 7-16
  - ***Check yourself***

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- The RSA Procedure in EC 2003/2008
  - ***Check yourself***
- The RSA Procedure in AS Codes
  - ***Check yourself***
- The RSA Procedure in BS 8110
  - ***Check yourself***
- The RSA Procedure in CSA Codes
  - ***Check yourself***
- The RSA Procedure in IS Codes
  - ***Check yourself***
- The RSA Procedure in MNBC
  - ***Check yourself***
- The RSA Procedure in NZS
  - ***Check yourself***

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The RSA Procedure as prescribed in ASCE 7-16

Software Demonstration

ETABS v 2016

Thank you