CE 809 - Structural Dynamics

Lecture 7: EQ Response of SDF Systems - Concept of Elastic Response Spectrum

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The Concept of Response Spectrum

The governing equation of motion of an SDF system subjected to a ground motion $\ddot{u}_g(t)$ can be written as follows.

$$m\,\ddot{u}(t) + c\,\dot{u}(t) + k\,u(t) = -m\,\ddot{u}_g(t)$$

$$c = 2\,m\,\xi\,\omega \quad , \quad \omega^2 = \frac{k}{m}$$

$$\ddot{u}(t) + 2\,\xi\,\omega\,\dot{u}(t) + \omega^2 u(t) = -\ddot{u}_g(t)$$

Solution

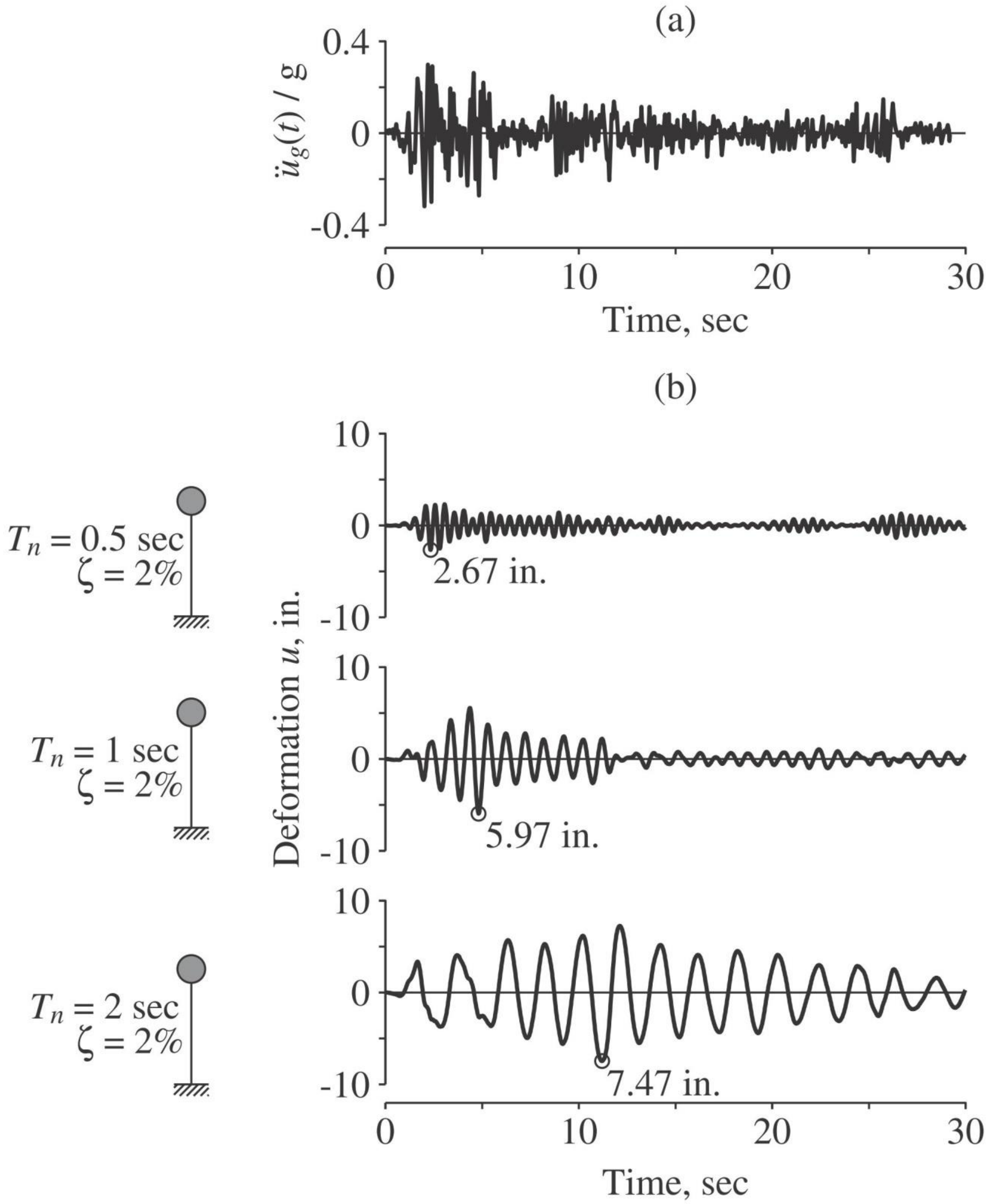
$$u(t, T, \xi)$$
$$\dot{u}(t, T, \xi)$$
$$\ddot{u}(t, T, \xi)$$

$$u_o(T, \xi) = max|u(t, T, \xi)|$$

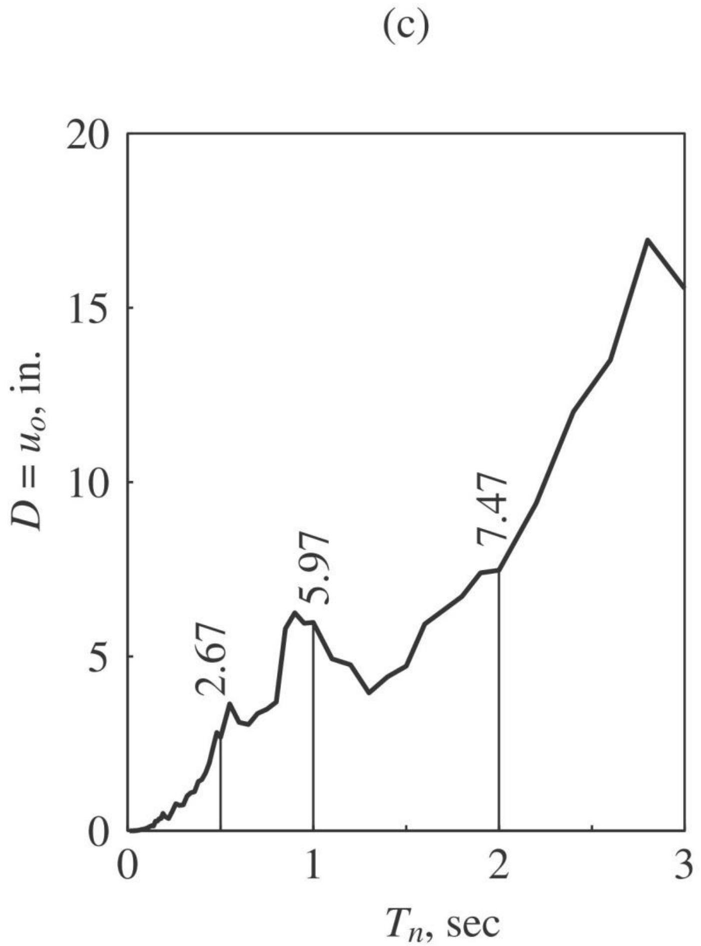
$$\dot{u}_o(T, \xi) = max|\dot{u}(t, T, \xi)|$$

$$\ddot{u}_o(T, \xi) = max|\ddot{u}(t, T, \xi)|$$

The deformation response spectrum is a plot of u_o against T for fixed ξ. A similar plot for $\dot{u}_o$ is the velocity response spectrum, and for $\ddot{u}_o$ is the acceleration response spectrum.



(a) Ground acceleration; (b) deformation response of three SDF systems with ξ = 2% and T = 0.5, 1, and 2 sec; (c) deformation response spectrum for ξ = 2%.



Pseudo-velocity response spectrum

- Consider a quantity V for an SDF system with natural frequency ω related to its peak deformation $D \equiv u_o$ due to earthquake ground motion:

$$V = \omega D = \frac{2\pi}{T} D$$

- The quantity V has units of velocity. This is pseudo-velocity.

- Using the above expression, the displacement response spectrum can be converted to the Pseudo-velocity response spectrum.

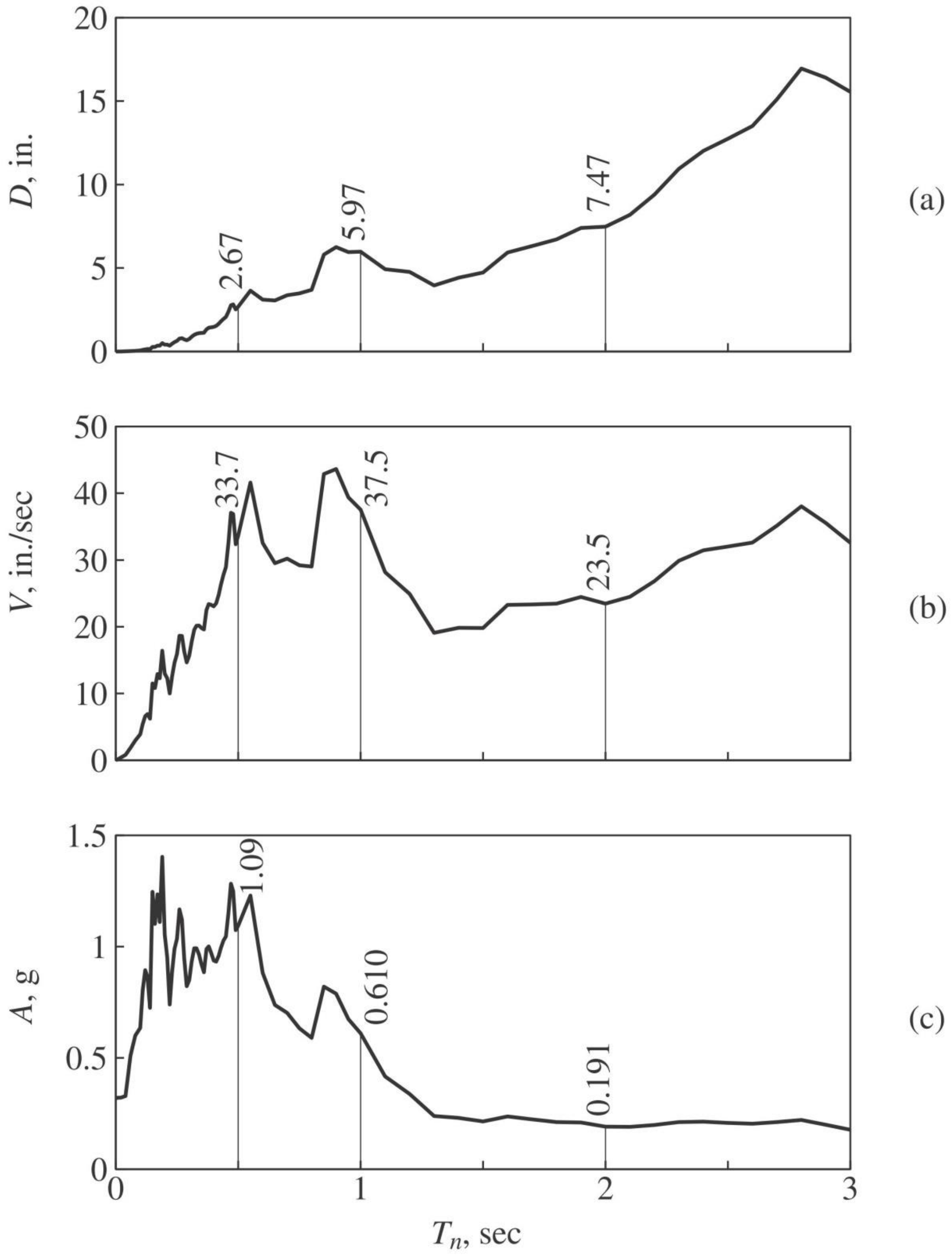
Pseudo-acceleration response spectrum

- Consider a quantity A for an SDF system with natural frequency ω related to its peak deformation $D \equiv u_o$ due to earthquake ground motion:

$$A = \omega^2 D = \left(\frac{2\pi}{T}\right)^2 D$$

- The quantity A has units of acceleration. This is pseudo-acceleration.

- Using the above expression, the displacement response spectrum can be converted to the Pseudo-acceleration response spectrum.



Response spectra ($\xi = 0.02$) for El Centro ground motion: (a) deformation response spectrum; (b) pseudo-velocity response spectrum; (c) pseudo-acceleration response spectrum.

Pseudo-acceleration response spectrum

- The quantity A is related to the peak value of base shear V_{bo} or the peak value of the equivalent static force f_{so}.

$$V_{bo} = f_{so} = mA$$

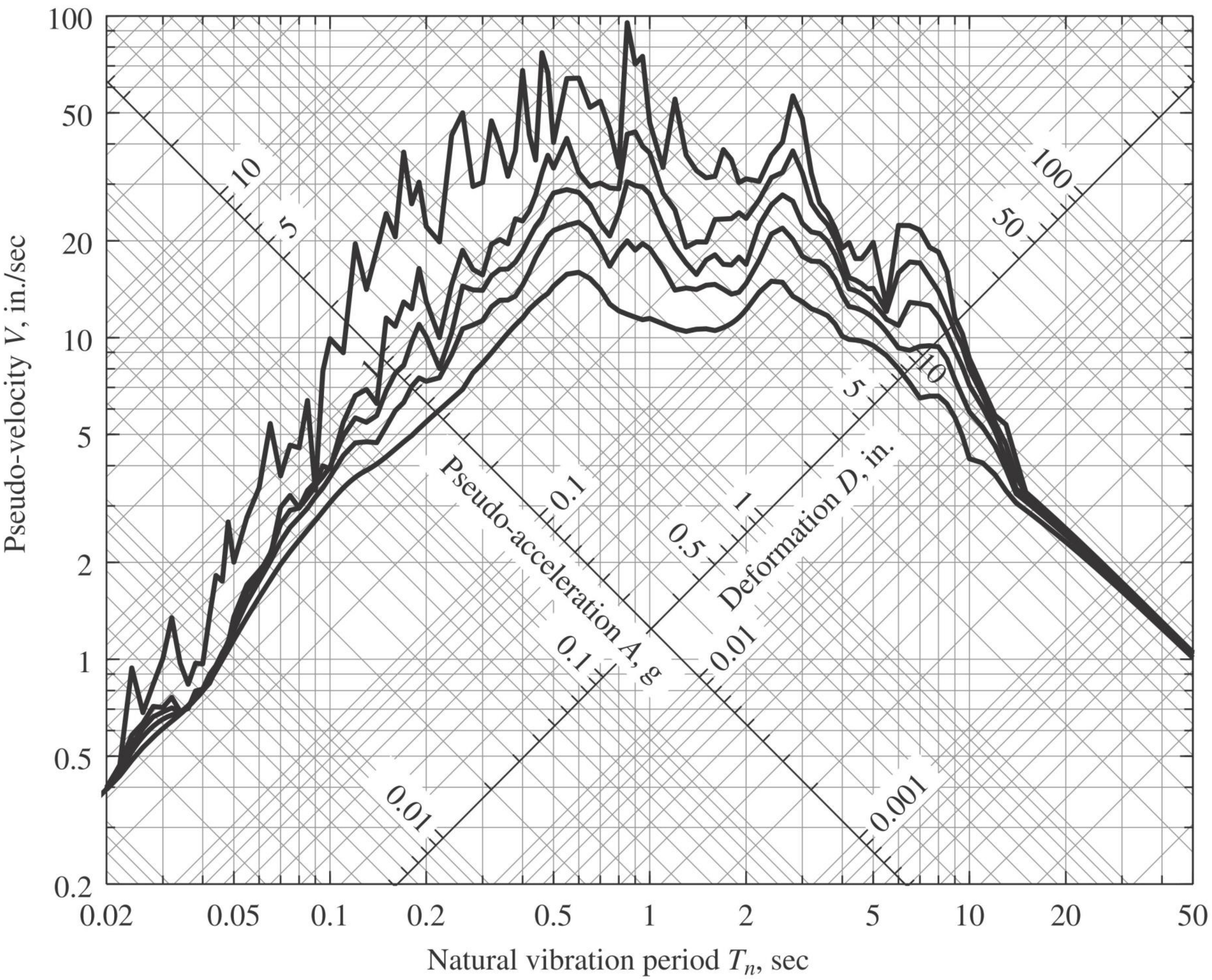
- The peak base shear can be written in the form

$$V_{bo} = \frac{A}{g}w \quad , \quad f_{so} = \frac{A}{g}w$$

Combined D-V-A response spectrum

- The three spectra (deformation, pseudo-velocity, and pseudo-acceleration) are simply different ways of presenting the same information on structural response for a given ground motion.

- Knowing one of the spectra, the other two can be obtained by algebraic operations mentioned earlier.



Combined D–V –A response spectrum for El Centro ground motion

ξ = 0, 2, 5, 10, and 20%.

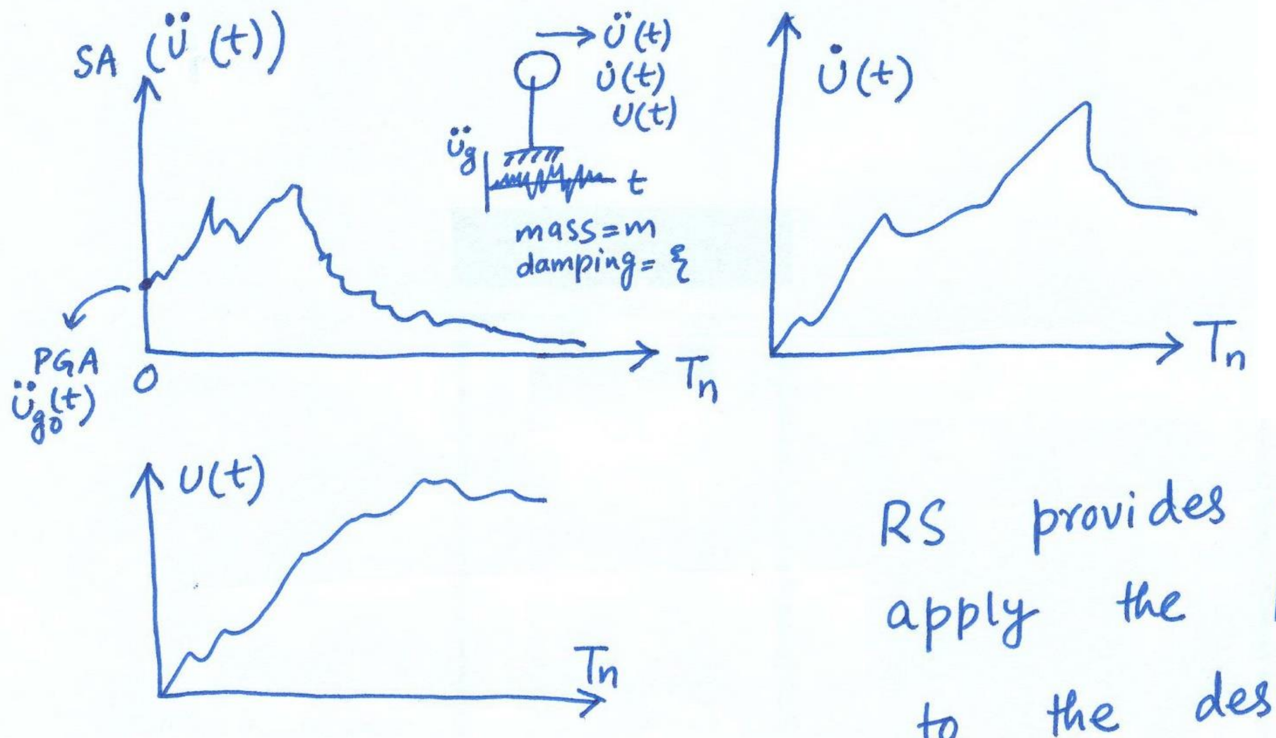
Construction of Response Spectrum

The response spectrum for a given ground motion component $\ddot{u}_g(t)$ can be developed by implementation of the following steps:

1) Numerically define the ground acceleration $\ddot{u}_g(t)$; typically, the ground motion ordinates are defined every 0.02 sec.
2) Select the natural vibration period T and damping ratio ξ of an SDF system.
3) Compute the deformation response $u(t)$ of this SDF system due to the ground motion $\ddot{u}_g(t)$ by any of the numerical methods.
4) Determine u_o, the peak value of $u(t)$.

Construction of Response Spectrum

5) The spectral ordinates are $D = u_o$, $V = (2\pi/T)D$, and $A = (2\pi/T)^2 D$.
6) Repeat steps 2 to 5 for a range of T and ξ values covering all possible systems of engineering interest.
7) Present the results of steps 2 to 6 graphically to produce three separate spectra or a combined spectrum.



RS provides a practical approach to apply the knowledge of structural dynamics to the design of of structures and development of lateral force Requirements in building codes.

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Response spectrum Analysis procedure

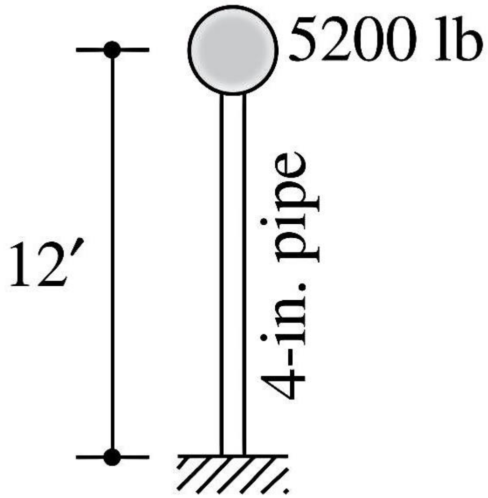
Use of Response Spectrum – SDF Systems

Solved Example

A 12-ft-long vertical cantilever, a 4-in.-nominal-diameter standard steel pipe, supports a 5200-lb weight attached at the tip as shown in Figure.

The properties of the pipe are: outside diameter, $d_o = 4.5\ in.$, inside diameter $d_i = 4.026\ in.$, thickness $t = 0.237\ in.$, and second moment of cross-sectional area, $I = 7.23\ in^4$, elastic modulus $E = 29{,}000\ ksi$, and weight $= 10.79\ lb/foot\ length$.

Determine the peak deformation and bending stress in the cantilever due to the El Centro ground motion. Assume that $\xi = 2\%$.



Solution The lateral stiffness of this SDF system is

$$k = \frac{3EI}{L^3} = \frac{3(29 \times 10^3)7.23}{(12 \times 12)^3} = 0.211 \text{ kip/in.}$$

The total weight of the pipe is $10.79 \times 12 = 129.5$ lb, which may be neglected relative to the lumped weight of 5200 lb. Thus

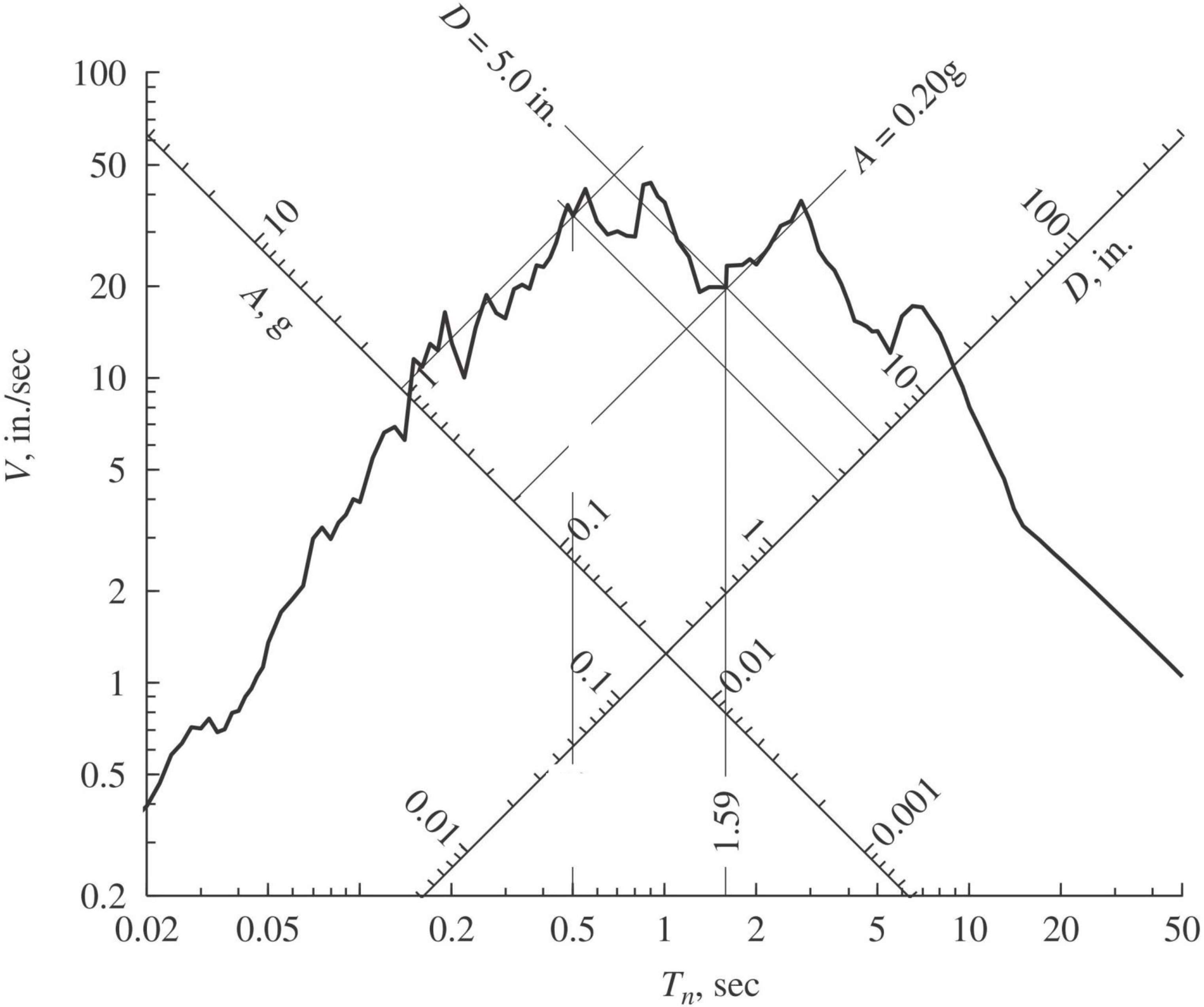
$$m = \frac{w}{g} = \frac{5.20}{386} = 0.01347 \text{ kip-sec}^2/\text{in.}$$

The natural vibration frequency and period of the system are

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.211}{0.01347}} = 3.958 \text{ rad/sec} \qquad T_n = 1.59 \text{ sec}$$

From the response spectrum curve for $\zeta = 2\%$ (Fig. E6.2b), for $T_n = 1.59$ sec, $D = 5.0$ in. and $A = 0.20$g. The peak deformation is

$$u_o = D = 5.0 \text{ in.}$$



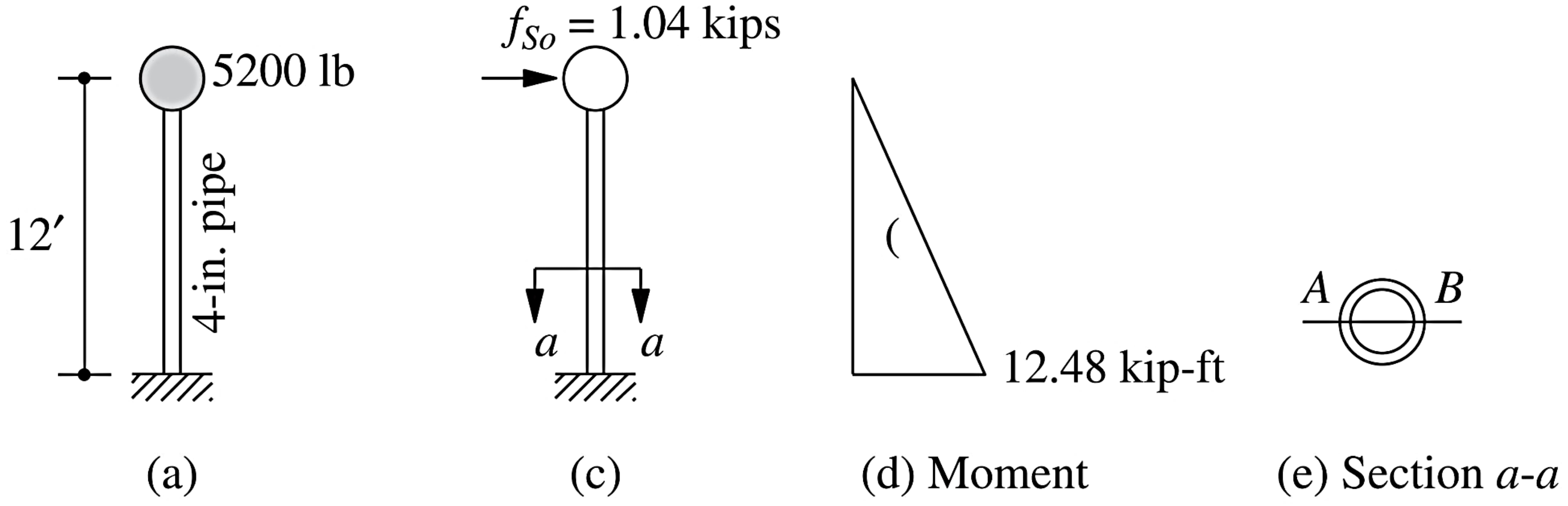
D = 5.0 in.
A = 0.20g
V, in./sec
T_n, sec
A, g
D, in.
100
50
20
10
5
2
1
0.5
0.2
0.02
0.05
0.2
0.5
1
2
5
10
20
50
10
1
0.1
0.01
0.001
100
10
1
0.1
0.01
1.59

The peak value of the equivalent static force is

$$f_{So} = \frac{A}{g} w = 0.20 \times 5.2 = 1.04 \text{ kips}$$

The bending moment diagram is shown in Fig. E6.2d with the maximum moment at the base = 12.48 kip-ft. Points *A* and *B* shown in Fig. E6.2e are the locations of maximum bending stress:

$$\sigma_{\max} = \frac{Mc}{I} = \frac{(12.48 \times 12)(4.5/2)}{7.23} = 46.5 \text{ ksi}$$



Thank you