CE 809 - Structural Dynamics

Lecture 5: Response of SDF Systems to Impulse Loading

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Impulse Loading

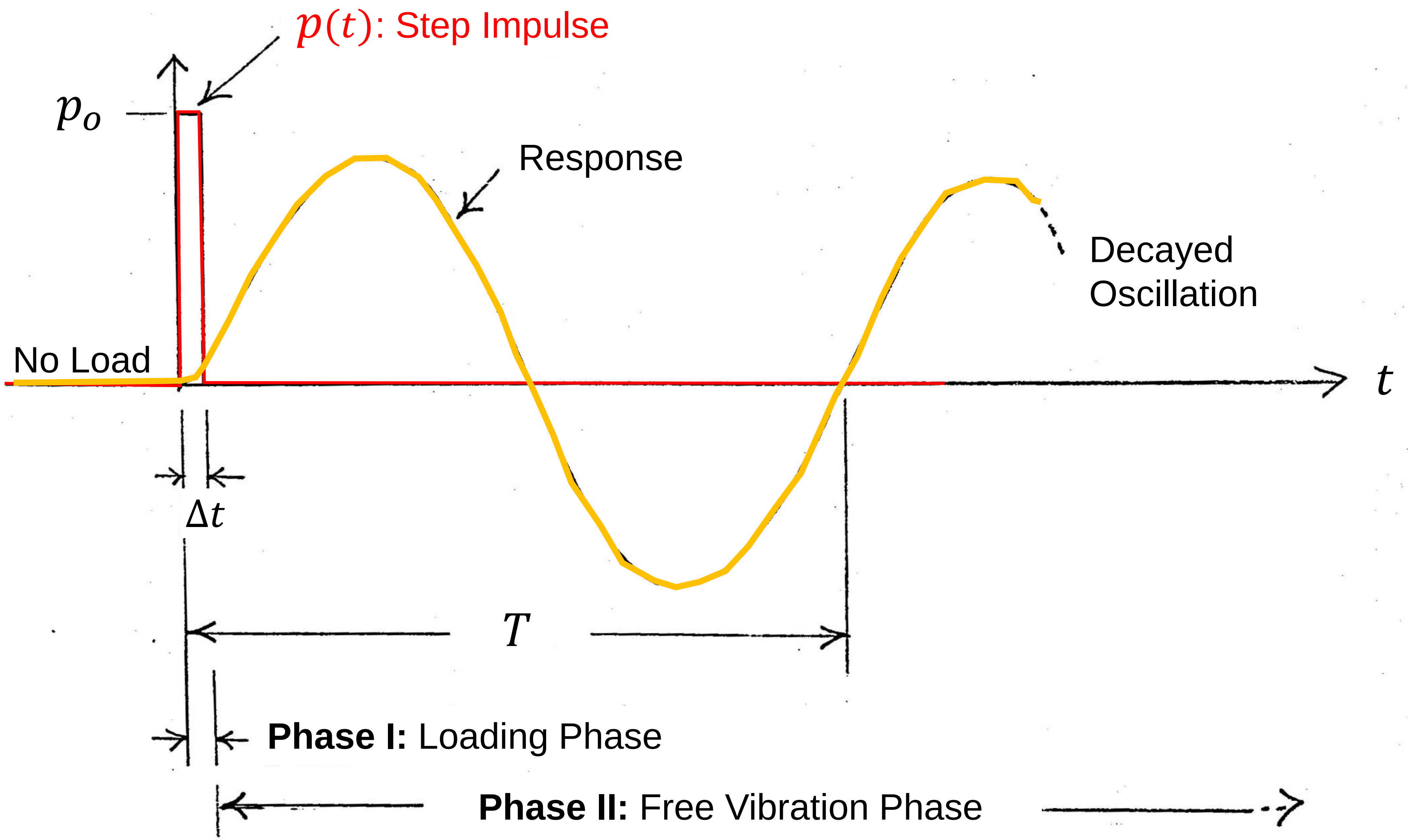
- Impulsive forces, shock loads, short duration loads.

- Impact, blast wave, explosion

- Sudden stop/break of trucks & automobiles & travelling cranes

The study of impulse response is also important for the **analysis of response to arbitrary loadings**.

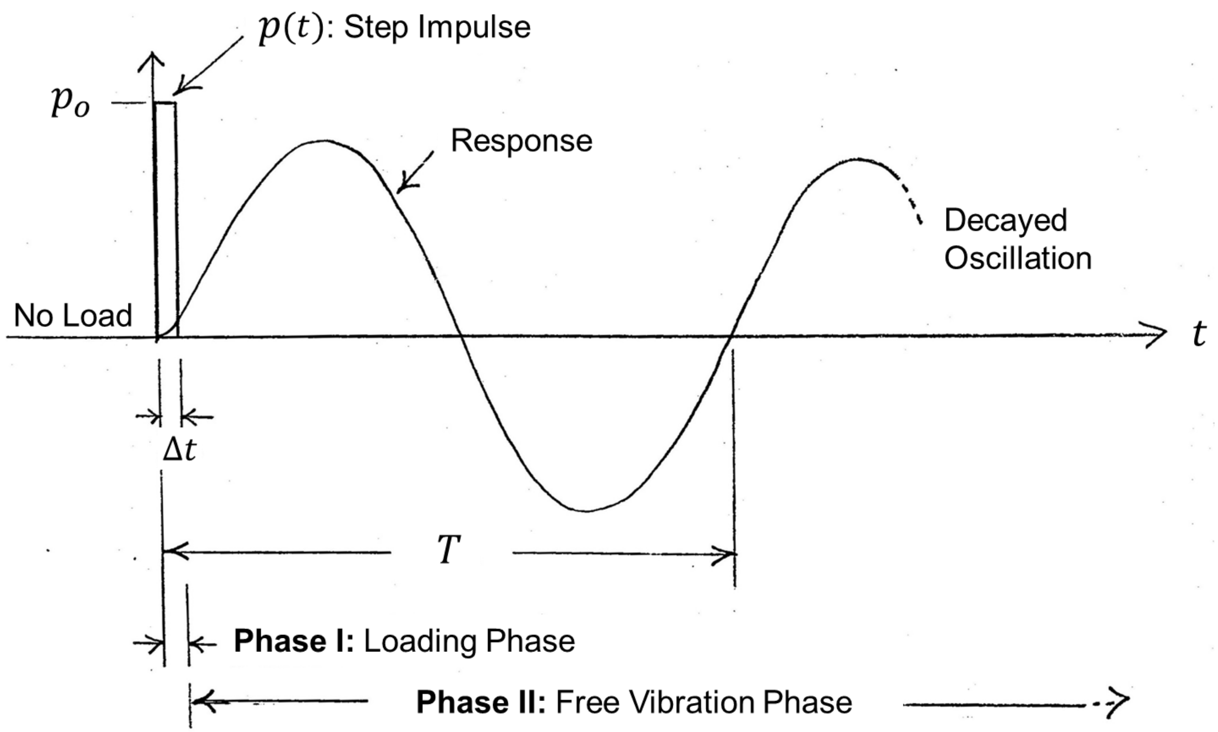
Response to Impulse Loading



Response to Impulse Loading

Impulse Force Magnitude = p_o, start at $t = 0$, Duration = Δt, where $\Delta t / T << 1$

Structure Initial at-rest: $u(0) = 0,\ \dot{u}(0) = 0$

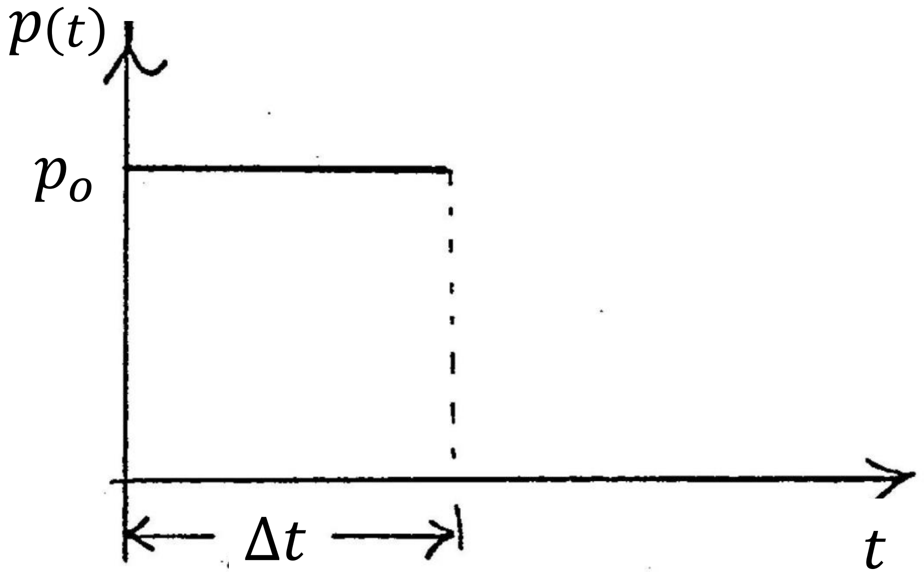


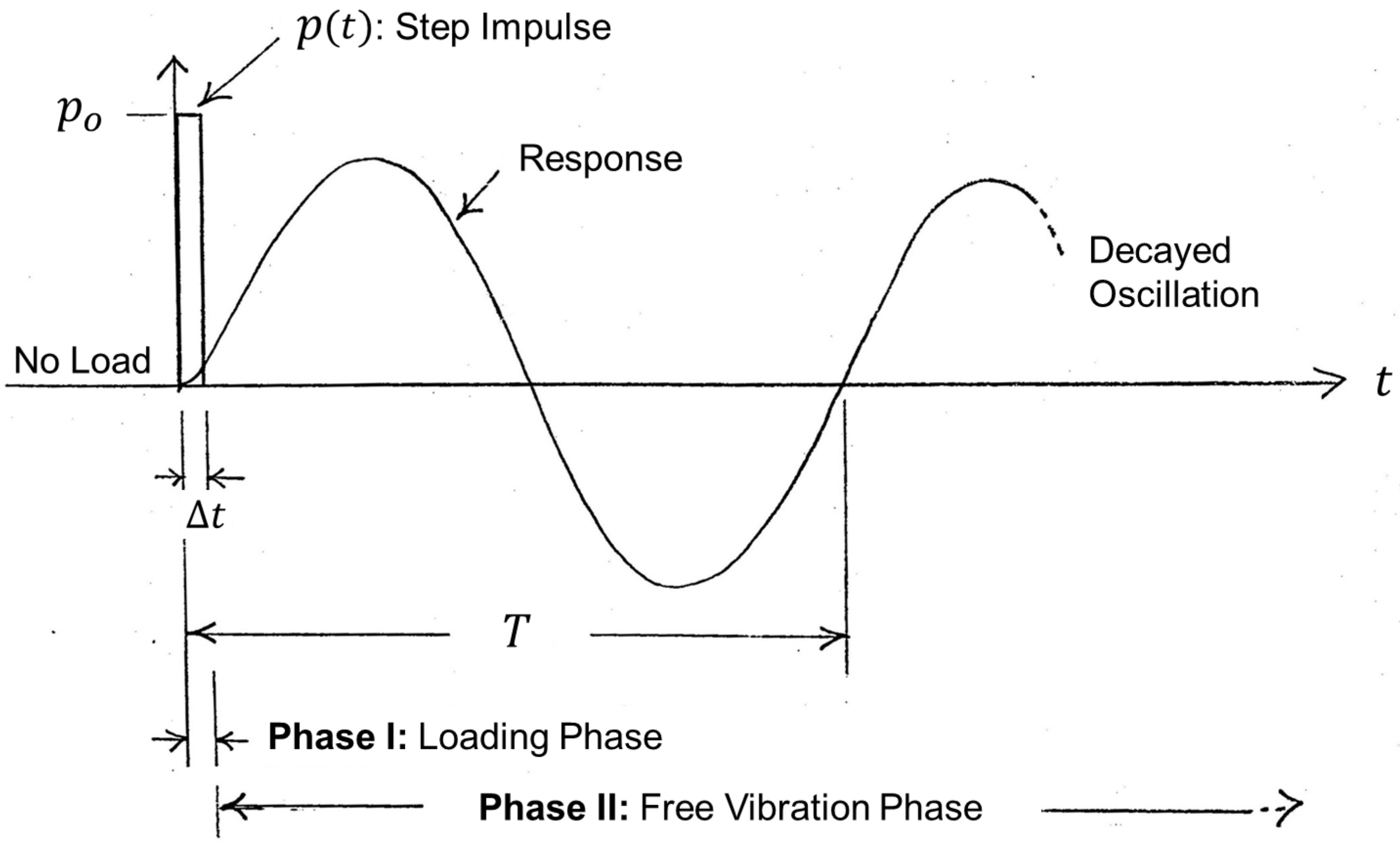
Response to Impulse Loading

Phase 1:

The particular solution to a step loading is simply a static deflection:

$$u_p(t) = \frac{p_o}{k} \quad \text{........... (1)}$$





Response to Impulse Loading

Phase 1:

$$u_p(t) = \frac{p_o}{k}$$

This solution satisfies the equation of motion:

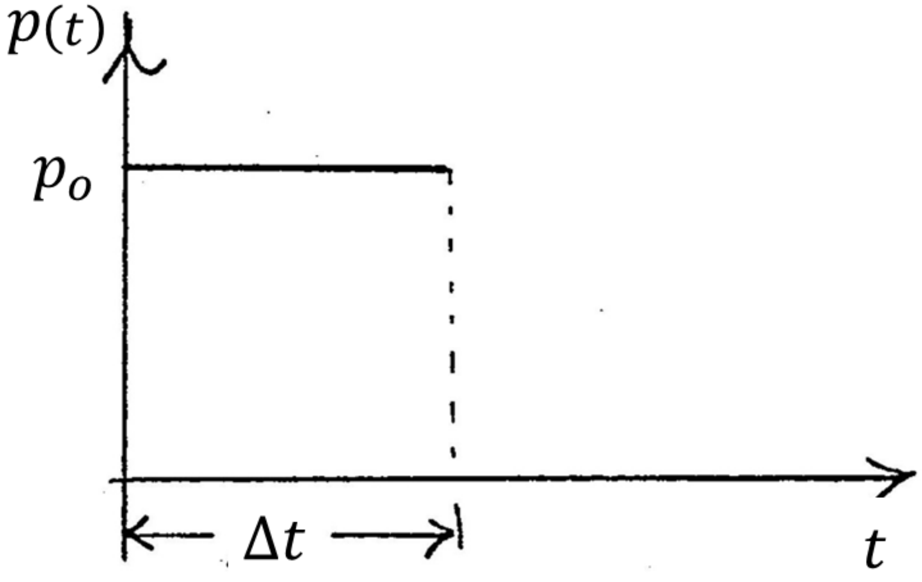
$$\dot{u}_p(t) = 0$$

$$\ddot{u}_p(t) = 0$$

Substitute the above relations into the equation of motion:

$$m\,\ddot{u}_p(t) + c\,\dot{u}_p(t) + k\,u_p(t) = p_0$$

$$0 + 0 + k\frac{p_0}{k} = p_0$$



Response to Impulse Loading

Phase 1:

A general solution:

$$u(t) = u_p(t) + u_h(t)$$

$$u(t) = \frac{p_o}{k} + e^{-\xi \omega t} [A \sin(\omega_D t) + B \cos(\omega_D t)] \qquad \text{………..} \ (2)$$

Where

A and B are determined such that the at-rest initial conditions are satisfied.

Response to Impulse Loading

Phase 1:

$$u(0) = \frac{p_o}{k} + B = 0 \qquad \dot{u}(0) = \omega_D A - \xi\omega B = 0$$

$$\left.\begin{aligned} B &= -\frac{p_o}{k} \\ A &= \frac{\xi\omega B}{\omega_D} \end{aligned}\right\} \quad \text{……….. (3)}$$

So we obtain:

$$u(t) = \frac{p_o}{k} + e^{-\xi\omega t}\left[-\frac{\xi\omega}{\omega_D}\frac{p_o}{k}\sin(\omega_D t) - \frac{p_o}{k}\cos(\omega_D t)\right] \quad \text{……….. (4)}$$

(valid in the range $0 < t < \Delta t$)

Response to Impulse Loading

Phase 1:

$$u(t) = \frac{p_o}{k} + e^{-\xi\,\omega\,t}\left[-\frac{\xi\,\omega}{\omega_D}\frac{p_o}{k}\sin(\omega_D t) - \frac{p_o}{k}\cos(\omega_D t)\right] \qquad \text{..........} (4)$$

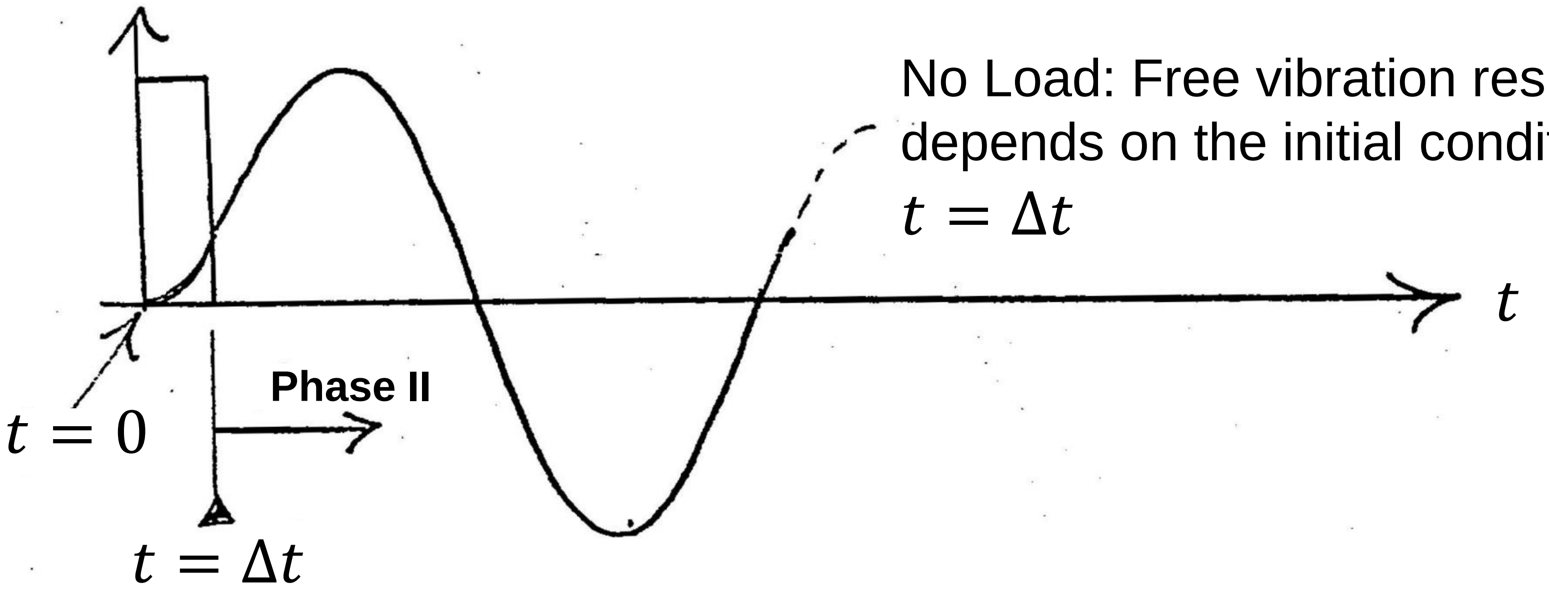
Due to fact that $0 < t < \Delta t$ and $\Delta t/T << 1$ and $\xi << 1$

$$u(t) \approx \frac{p_o}{k} - \frac{p_o}{k}\cos(\omega t) \approx \frac{p_o}{k}(1 - \cos(\omega t)) \qquad \text{..........} (5)$$

Equation (5) shows the approximate response during the loading phase.

Response to Impulse Loading

Phase 2:



No Load: Free vibration response depends on the initial conditions at $t = \Delta t$

Response to Impulse Loading

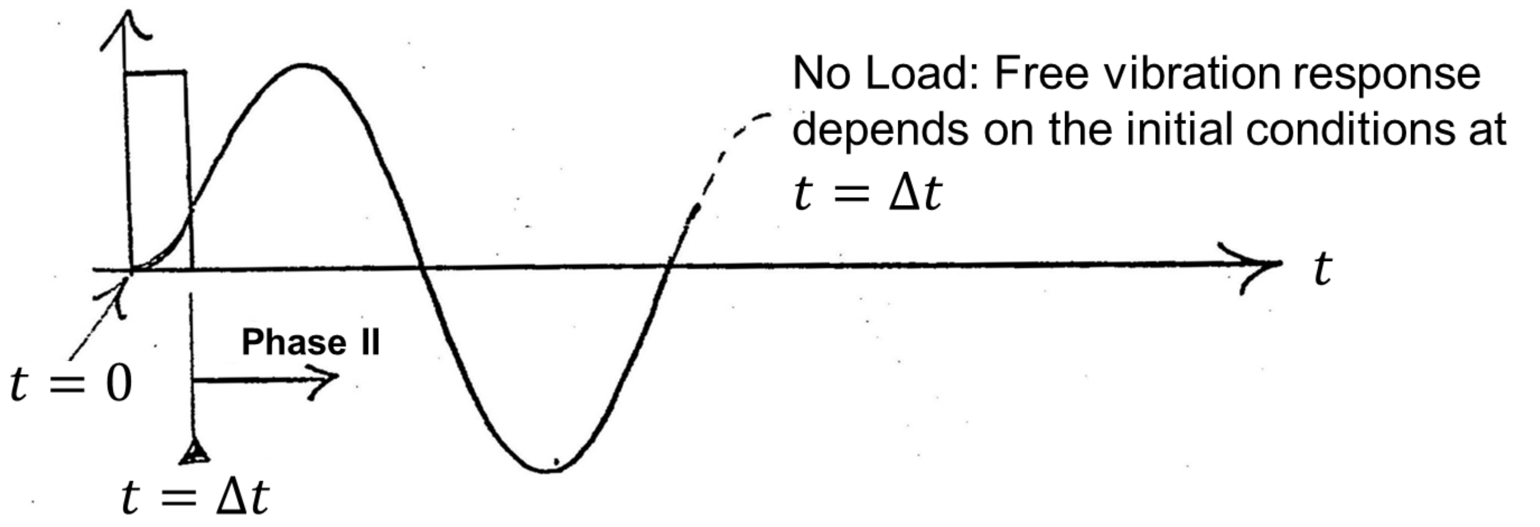
Phase 2:

$$u(t) = \frac{p_o}{k}(1 - \cos(\omega t))$$

$$u(\Delta t) = \frac{p_o}{k}(1 - \cos(\omega\, \Delta t))$$

$$\dot{u}(t) = \frac{p_o}{k}(\omega \sin(\omega t))$$

$$\dot{u}(\Delta t) = \frac{p_o}{k}(\omega \sin(\omega\, \Delta t)) \qquad \text{………..} \ (6)$$



Response to Impulse Loading

Phase 2:

Employing Tylor's expansion:

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \ldots\ldots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} \ldots\ldots$$

For small θ, by neglecting the second order term and higher terms, we get

$$\sin\theta \approx \theta, \qquad \cos\theta \approx 1 \qquad \ldots\ldots\ldots\ldots (7)$$

Response to Impulse Loading

Phase 2:

Introducing this approximation in Equation (7) into Equation (6), we obtain

$$\left.\begin{aligned} u(\Delta t) &= \frac{p_o}{k}(1-\cos(\omega\,\Delta t)) \cong 0 \\ \dot{u}(\Delta t) &= \frac{p_o}{k}(\omega\ \sin(\omega\,\Delta t)) \cong \frac{p_o\,\omega^2\,\Delta t}{k} = \frac{p_o\,\Delta t}{m} \end{aligned}\right\} \quad \ldots\ldots\ldots.. (8)$$

Note that $p_o\,\Delta t$ is an impulse.

Equation (8) says that **impulse ≈ the change in momentum (of the mass)**

The impulse introduces "**momentum**" into a structure **but the duration of impulse is so short that the displacement has not been developed yet**.

Response to Impulse Loading

Phase 2 ($t > \Delta t$):

Using the Equation (8) as the initial conditions for free vibration in Phase 2,

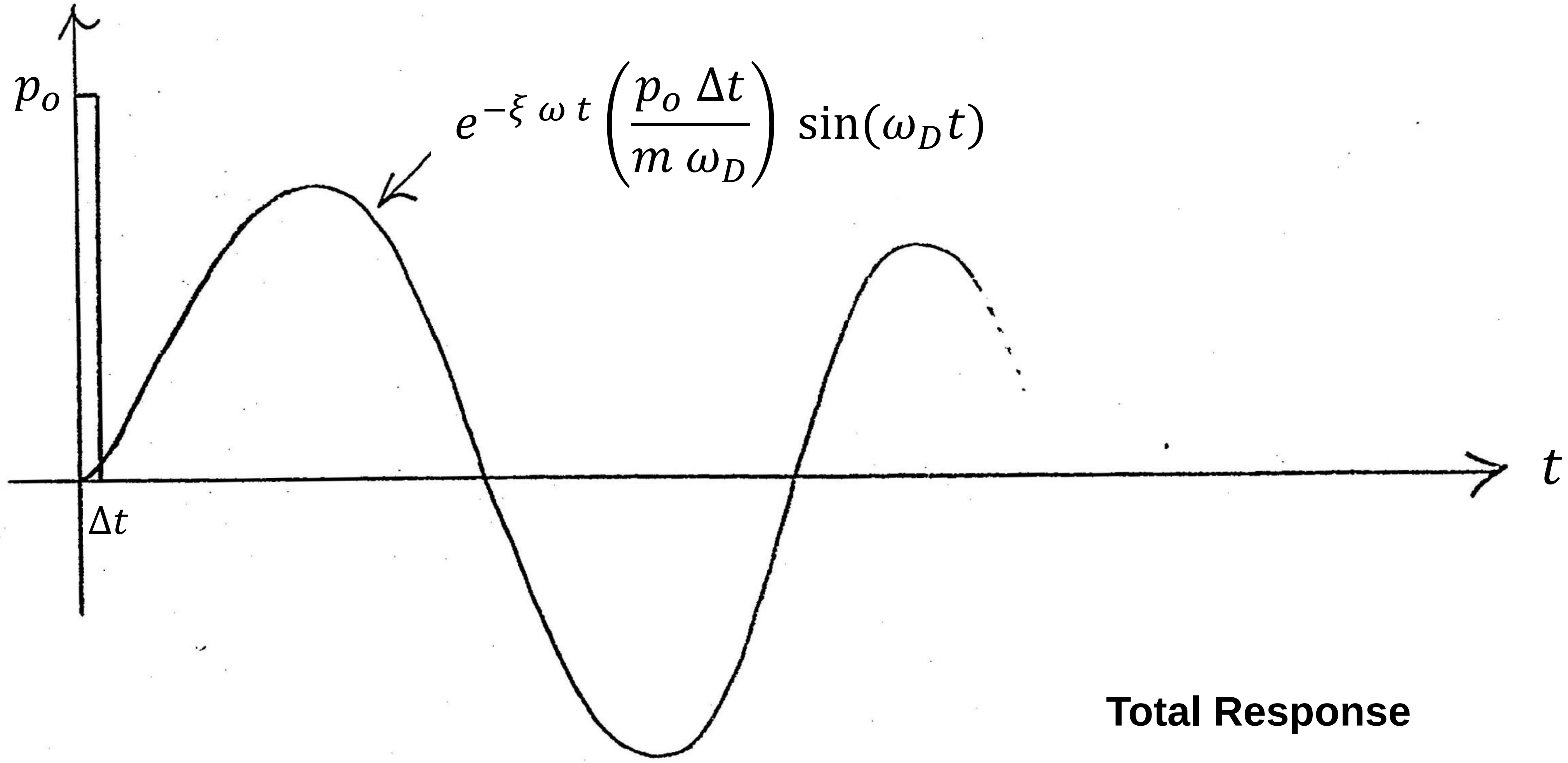
$$u(t) = e^{-\xi\,\omega\,(t-\Delta t)}\left[\frac{\dot{u}(\Delta t)}{\omega_D}\sin\big(\omega_D(t-\Delta t)\big)\right] \quad \text{……….. (9)}$$

Since $\Delta t/t << 1$, it is justified to let $t - \Delta t \approx t,$ that is

$$u(t) \approx e^{-\xi\,\omega\,t}\left(\frac{p_o\,\Delta t}{m\,\omega_D}\right)\sin(\omega_D t) \quad \text{……….. (10)}$$

Equation (10) will be used when we analyze the response to arbitrary loading in the next section.

Response to Impulse Loading



Total Response

Thank you