

# CE 75.03 – Earthquake Engineering for Tall Buildings

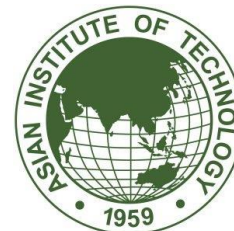
In-class Session

Semester – January 2022



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## **Prof. Dr. Pennung Warnitchai**

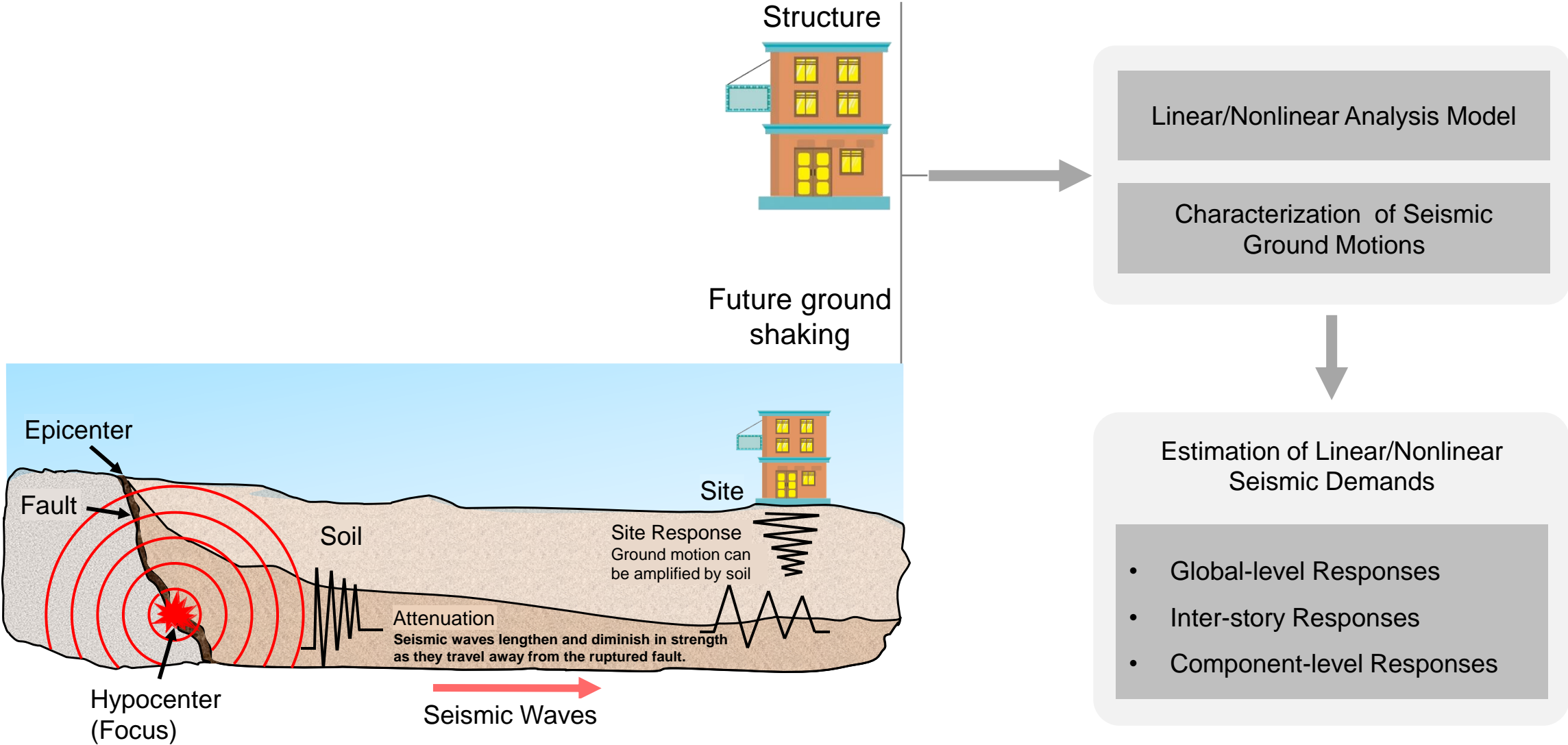
Head, Department of Civil and Infrastructure Engineering  
School of Engineering and Technology (SET)  
Asian Institute of Technology (AIT)  
Bangkok, Thailand

# Topics for this Session

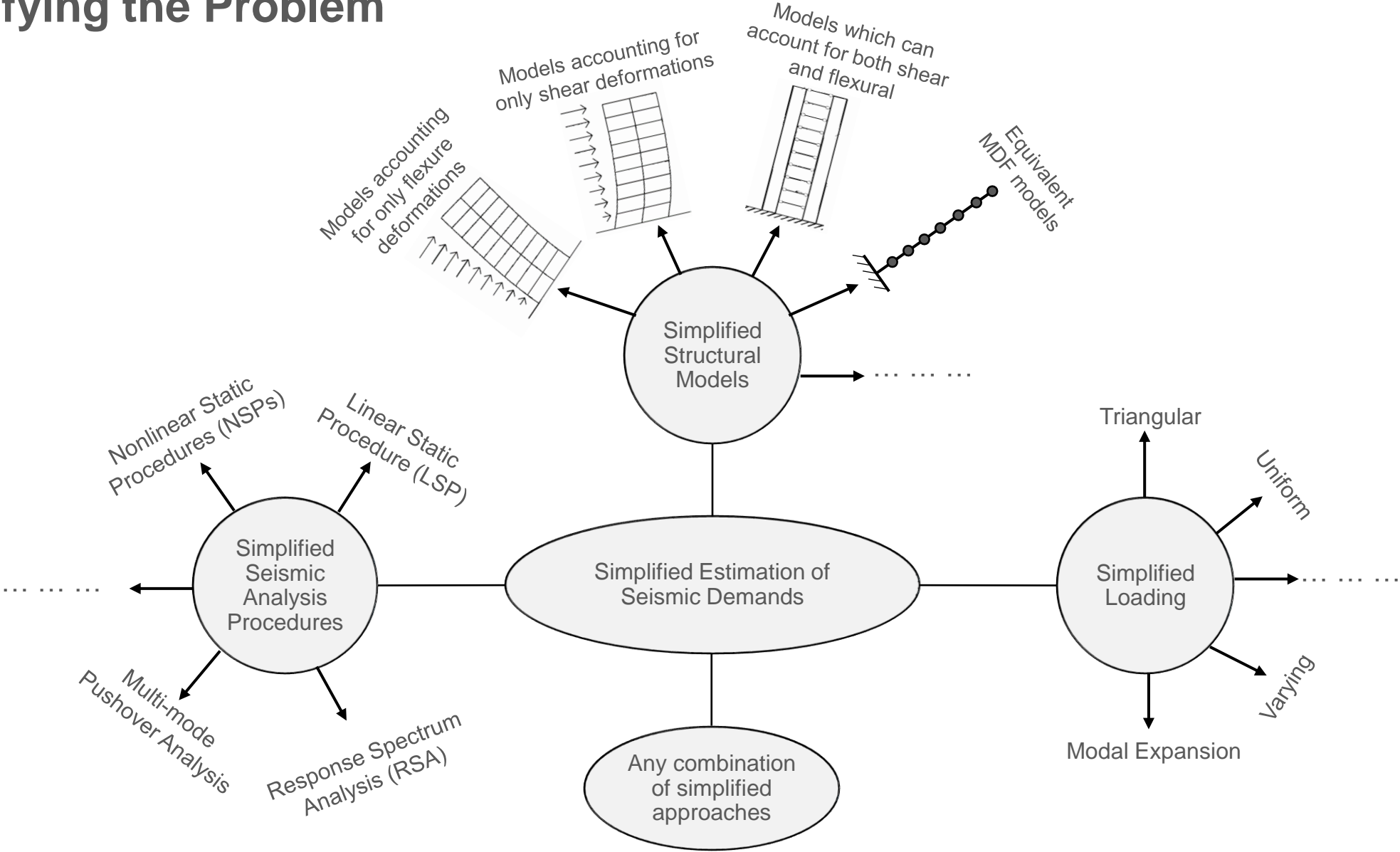
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- 1) Introduction to the Nonlinear Static (Pushover) Analysis Procedures
- 2) Approximate Multi-mode based Seismic Analysis Procedures
  - a) The Modal Pushover Analysis (MPA) Procedure
  - b) The Uncoupled Modal Response History Analysis (UMRHA) Procedure
  - c) The Modified Response Spectrum Analysis (MRSA) Procedure



# The Earthquake Problem



# Simplifying the Problem

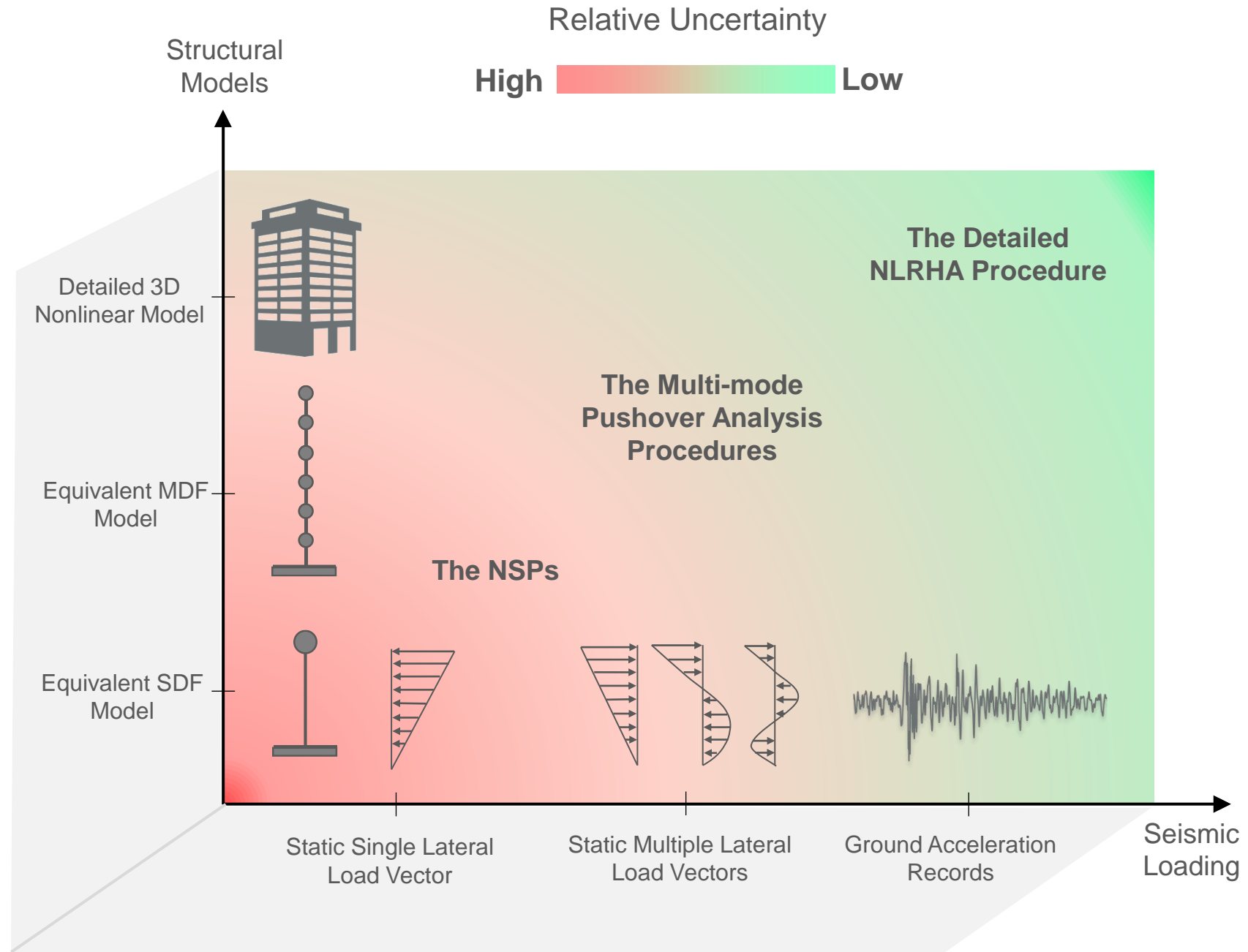



# Seismic Analysis Procedures

Structural Model	 <b>Linear</b>		 <b>Nonlinear</b>	
Seismic Loading	E, A, I, L, G etc. = Constant, K= Constant		E ≠ Constant, EI ≠ Constant, K≠ Constant	
 <b>Static</b>	<ol style="list-style-type: none"> <li>1. <span style="border: 1px solid black; padding: 2px;">Equivalent Lateral Force (ELF) Procedure</span></li> <li>2. <span style="border: 1px solid black; padding: 2px;">Response Spectrum Analysis (RSA) Procedure (or Mode Spectral Analysis)</span></li> </ol>		<ol style="list-style-type: none"> <li>5. <span style="border: 1px solid black; padding: 2px;">Several Pushover Analysis Methods or Nonlinear Static Procedures (NSPs)</span></li> </ol>	
<b>Dynamic</b>	<ol style="list-style-type: none"> <li>3. <span style="border: 1px solid black; padding: 2px;">Modal Response History (or Time History) Analysis Procedure (Modal RHA/THA)</span></li> <li>4. <span style="border: 1px solid black; padding: 2px;">Linear Response History (or Time History) Analysis Procedure (Direct Integration Linear RHA /LTHA)</span></li> </ol>		<ol style="list-style-type: none"> <li>6. <span style="border: 1px solid black; padding: 2px;">Nonlinear Modal Response History Analysis or Fast Nonlinear Analysis (FNA)</span></li> <li>7. <span style="border: 1px solid black; padding: 2px;">Nonlinear Response History (or Time History) Analysis Procedure (Direct Integration Nonlinear RHA/THA)</span></li> </ol>	

# Determination of Nonlinear Seismic Demands

## Analysis Procedures





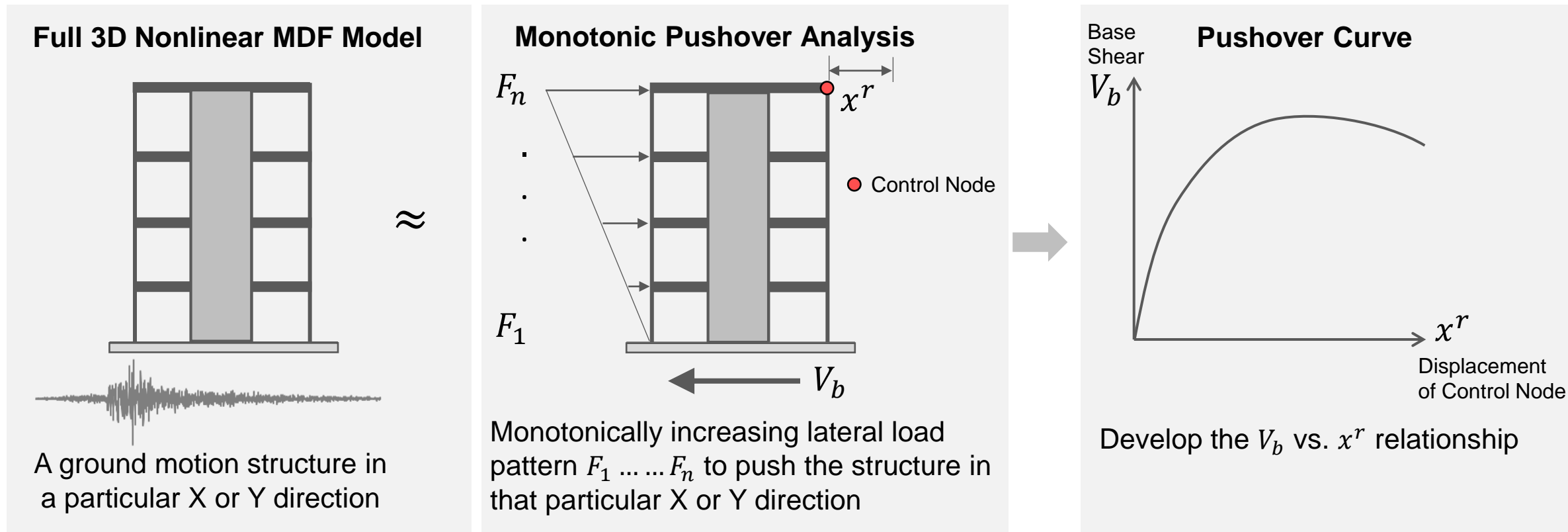
# **Introduction to the Nonlinear Static (Pushover) Analysis Procedures**

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# Nonlinear Static Analysis Procedures (NSPs) – Pushover Analysis Procedures

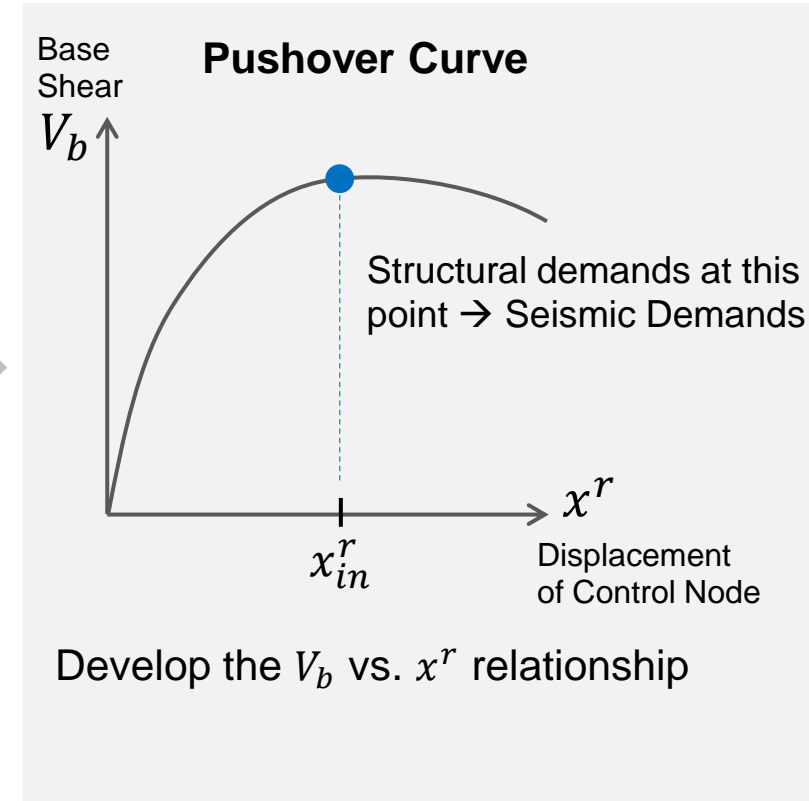
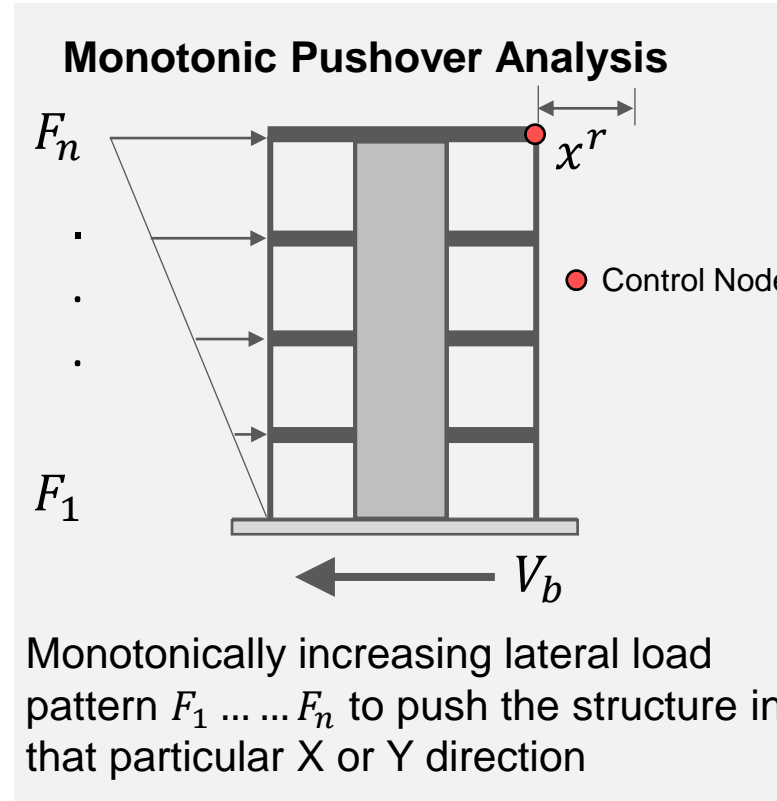
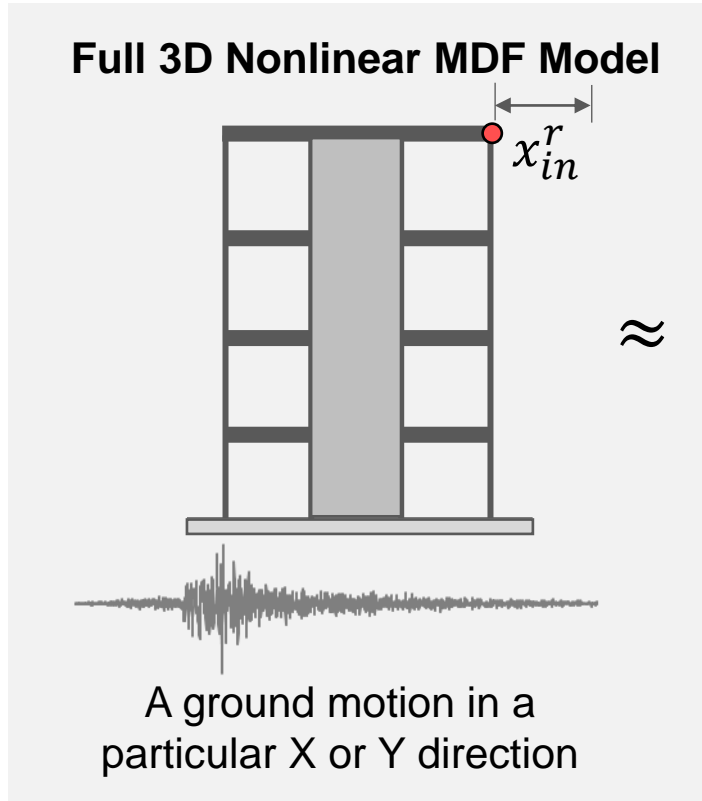
- The nonlinear analysis recommended for the first generation of performance-based seismic design methodology. Currently, it can be regarded as an alternate method of analysis for carrying out the PBD.

## Basic Idea of Pushover Analysis





# Nonlinear Static Analysis Procedures (NSPs) – Pushover Analysis Procedures



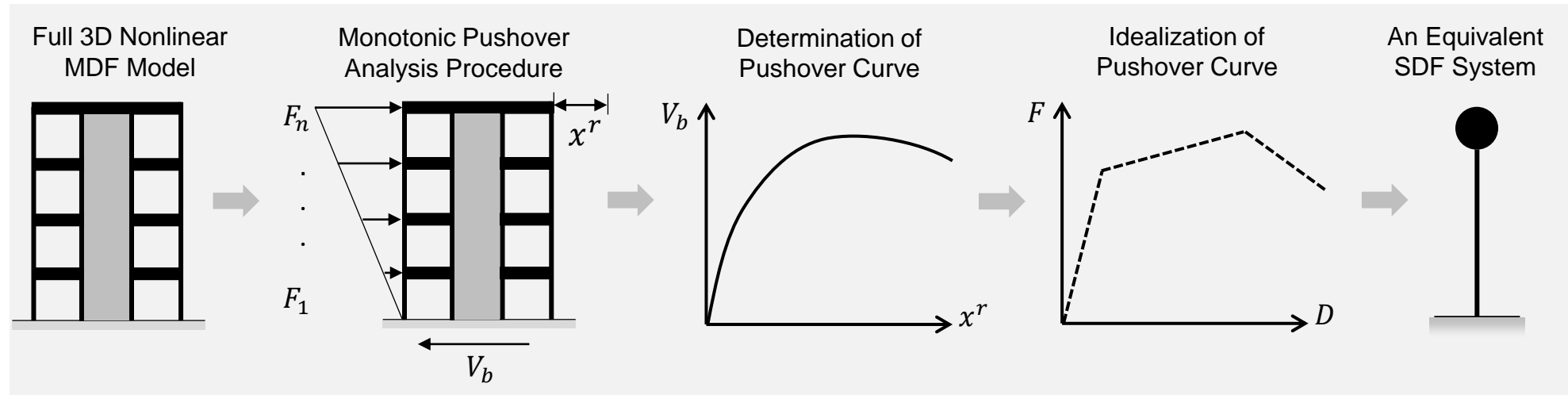
Let's set the control node as a roof node and denote the peak inelastic roof displacement occurred during the ground motion as  $x_{in}^r$ .



During the push, when  $x^r = x_{in}^r$ , the force and displacement demands of the structure are the peak seismic demands (produced by the ground motion).

How to Determine  $x_{in}^r$ ?

# Nonlinear Static Procedures



The Concept of an “Equivalent SDF System”

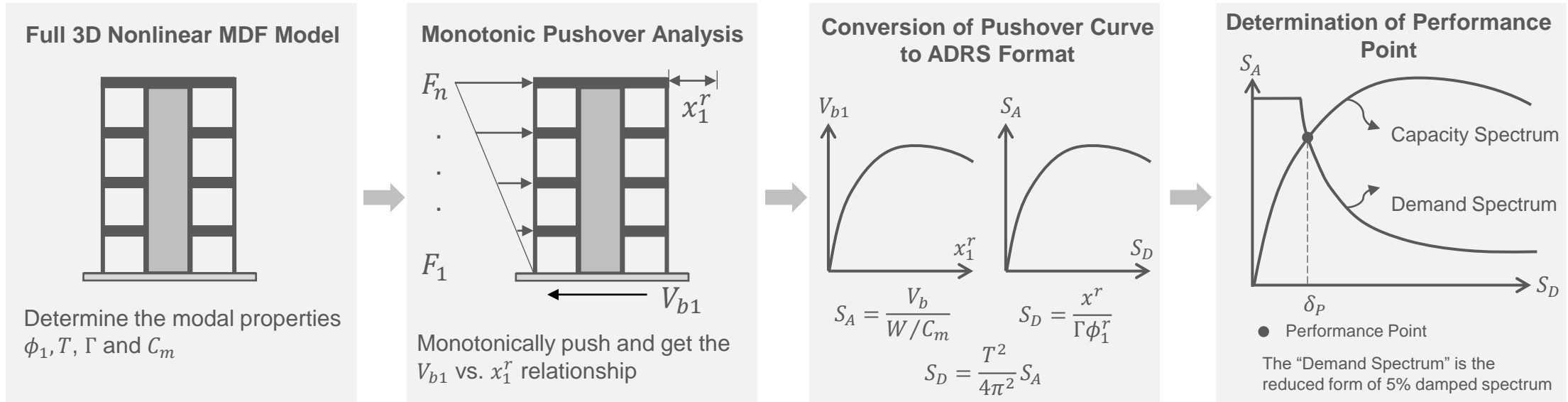
## Equivalent Linearization

- Several EL Procedures (Individual studies)
- Capacity Spectrum Method (CSM) (ATC 40, FEMA 273, FEMA 356)
- FEMA 440 Improved EL Procedure

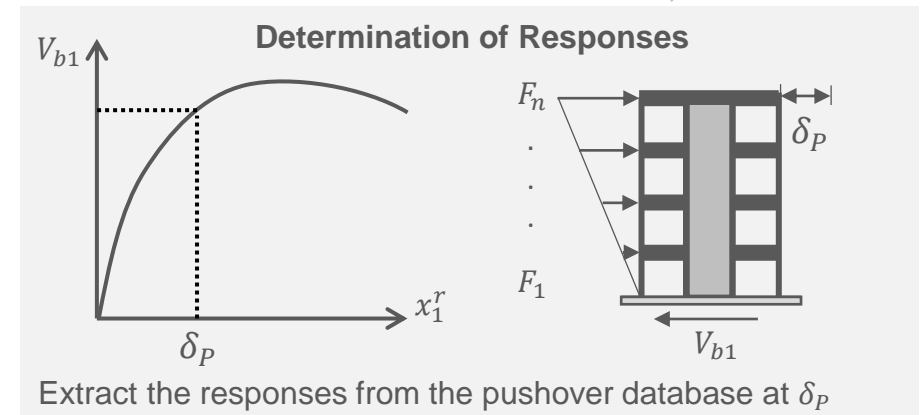
## Displacement Modification

- Several expressions for displacement modification (Individual studies)
- Displacement Coefficient Method (ATC 40, FEMA 273, FEMA 356, FEMA 440, ASCE 41-06, ASCE 41-13)

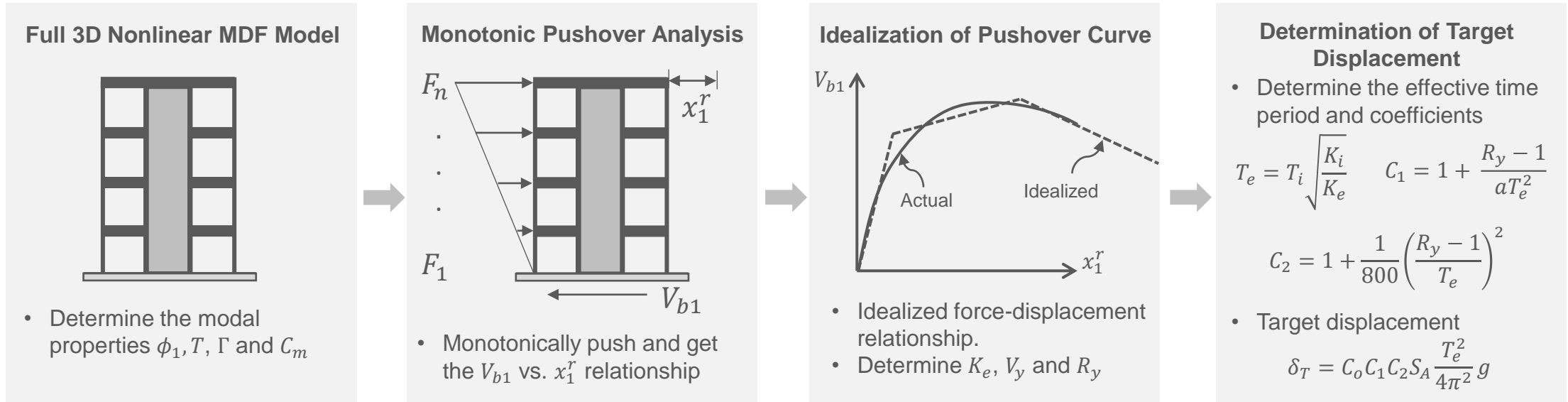
# Capacity Spectrum Method (ATC 40, 1996)



- **Capacity Spectrum** → Another form of pushover curve (SA-SD form).
- **Demand Spectrum** → Another form of response spectrum (ADRS Form) **reduced based on effective damping** (i.e. original inherent damping + additional hysteretic damping)



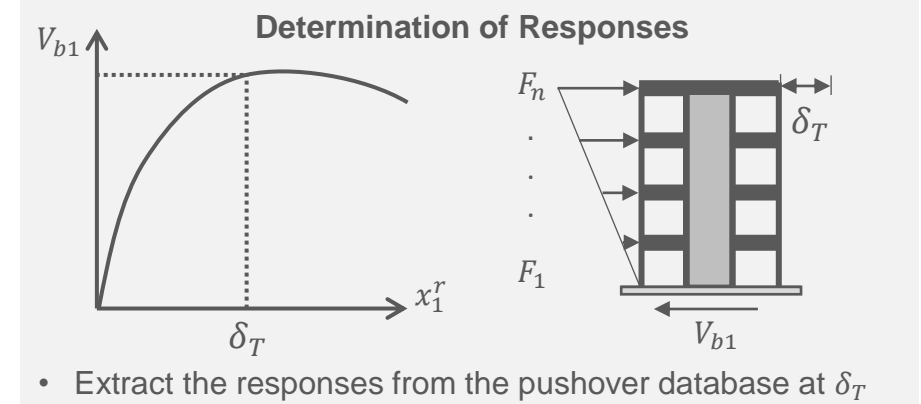
# FEMA 440 (2005) Displacement Coefficient Method



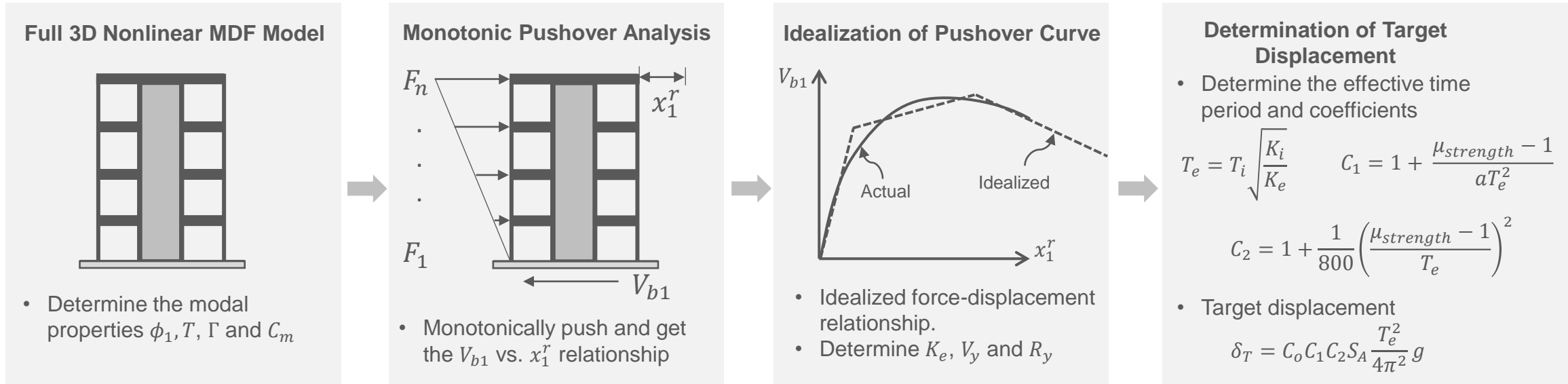
Target displacement:

$$T_e = T_i \sqrt{\frac{K_i}{K_e}}, \quad C_1 = 1 + \frac{R_y - 1}{aT_e^2}, \quad C_2 = 1 + \frac{1}{800} \left( \frac{R_y - 1}{T_e} \right)^2$$

$$\delta_T = C_o C_1 C_2 S_A \frac{T_e^2}{4\pi^2} g$$



# ASCE/SEI 41-17 (2017) Nonlinear Static Procedure

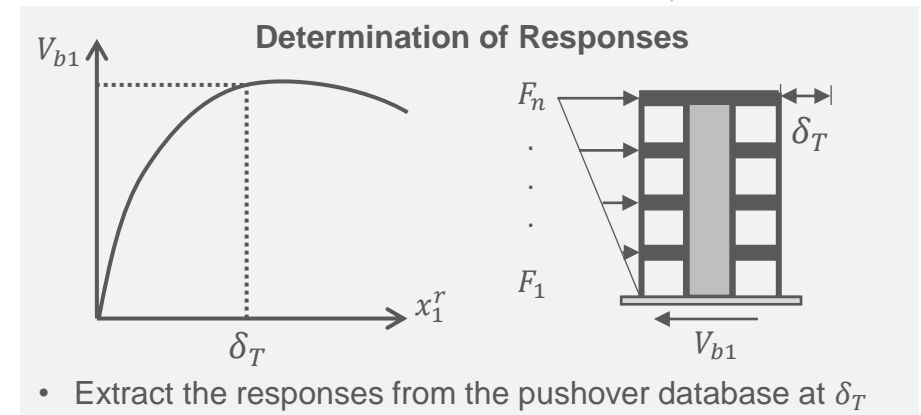


Target displacement:

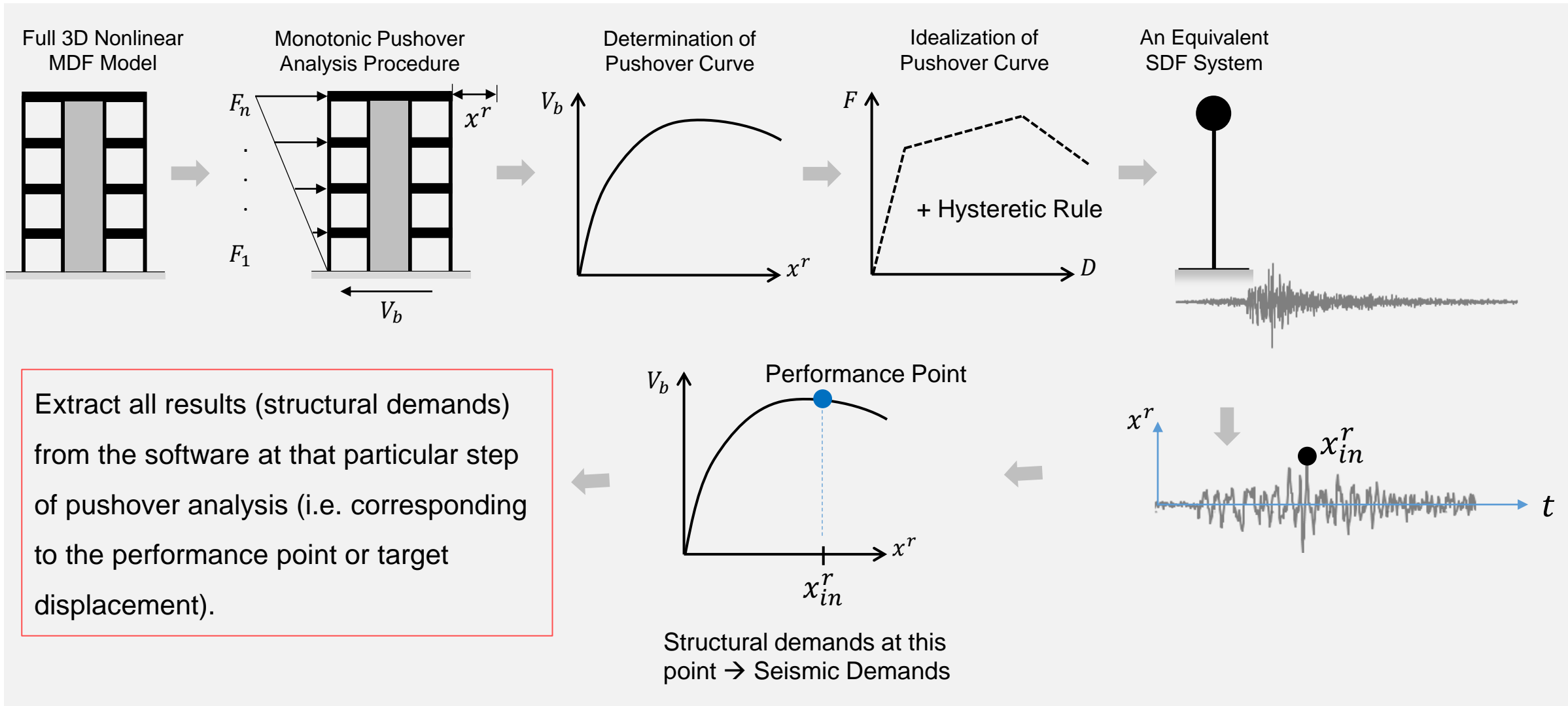
$$\delta_T = C_o C_1 C_2 S_A \frac{T_e^2}{4\pi^2} g$$

$$T_e = T_i \sqrt{\frac{K_i}{K_e}}, \quad C_1 = 1 + \frac{\mu_{strength} - 1}{aT_e^2}$$

$$C_2 = 1 + \frac{1}{800} \left( \frac{\mu_{strength} - 1}{T_e} \right)^2$$



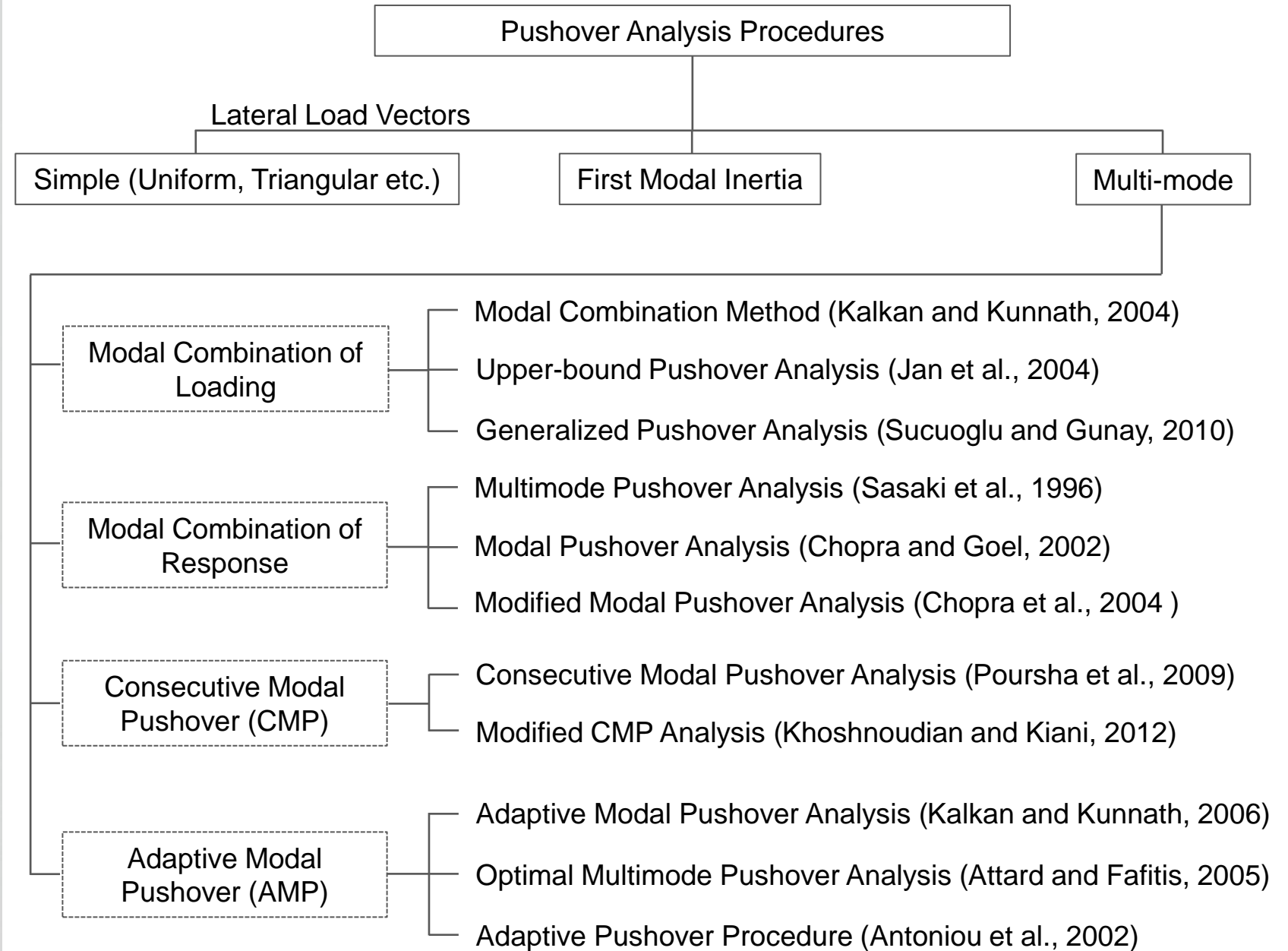
# Determining the Performance Point using NLRHA of the Equivalent SDF System



# **Approximate Multi-mode based Seismic Analysis Procedures**

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## Multi-mode Pushover Analysis Procedures





# Why We Still Need Approximate Seismic Analysis Procedures?

Despite the development of fast computing tools, software and other advancements, the detailed nonlinear RHA is still a difficult task for several reasons.

- **Ground motions** compatible with the seismic design spectrum for the site must be selected.
- **Computationally demanding**, inelastic modeling, 3D analysis to account for coupling between lateral and torsional motions, subjected to 2 horizontal components of ground motions.
- Must be repeated for **several excitations**.
- **Structural model** must be sophisticated enough to represent a building realistically, especially deterioration in its strength at large displacements.

So, approximate methods are still an attractive option as an alternate to the rigorous NLRHA procedure.

# Effective Earthquake Forces on an MDF System

The governing equation of motion for an elastic MDF system subjected to the earthquake ground motion is

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{M} \mathbf{1} \ddot{u}_g(t) = \mathbf{P}_{eff}(t)$$

Where the effective earthquake forces are

$$\mathbf{P}_{eff}(t) = -\mathbf{M} \mathbf{1} \ddot{u}_g(t)$$

The spatial distribution of these forces over the structure is defined by the vector

$$\mathbf{s} = \mathbf{M} \mathbf{1}$$

## The idea of modal expansion of excitation vector $P(t)$ of the form $P(t) = s p(t)$

$$P(t) = s p(t)$$

The primary idea is to expand the vector  $s$  as

$$s = \sum_{r=1}^N s_r = \sum_{r=1}^N \Gamma_r M \phi_r$$

This equation may be viewed as an expansion of the distribution  $s$  of applied forces in terms of inertia force distributions  $s_r$  associated with natural modes.

Pre-multiplying both sides of above equation by  $\phi_n^T$  and utilizing the orthogonality property of modes gives

$$\Gamma_n = \frac{\phi_n^T s}{M_n}$$

The contribution of the  $n$ th mode to  $s$  is

$$s_n = \Gamma_n M \phi_n$$

# Modal Expansion of the Effective Earthquake Forces

$$\mathbf{P}_{eff}(t) = -\mathbf{M} \mathbf{1} \ddot{u}_g(t) = -\mathbf{s} \ddot{u}_g(t)$$

- This force distribution can be expanded as a summation of modal inertia force distributions

$$\mathbf{s} = \mathbf{M} \mathbf{1} = \sum_{n=1}^N \mathbf{s}_n = \sum_{n=1}^N \Gamma_n \mathbf{M} \boldsymbol{\phi}_n$$

Where  $\Gamma_n = \frac{L_n}{M_n}$

$$L_n = \boldsymbol{\phi}_n^T \mathbf{M} \mathbf{1}$$

$$M_n = \boldsymbol{\phi}_n^T \mathbf{M} \boldsymbol{\phi}_n$$

# Modal Expansion of the Effective Earthquake Forces

- The effective earthquake forces can then be expressed as

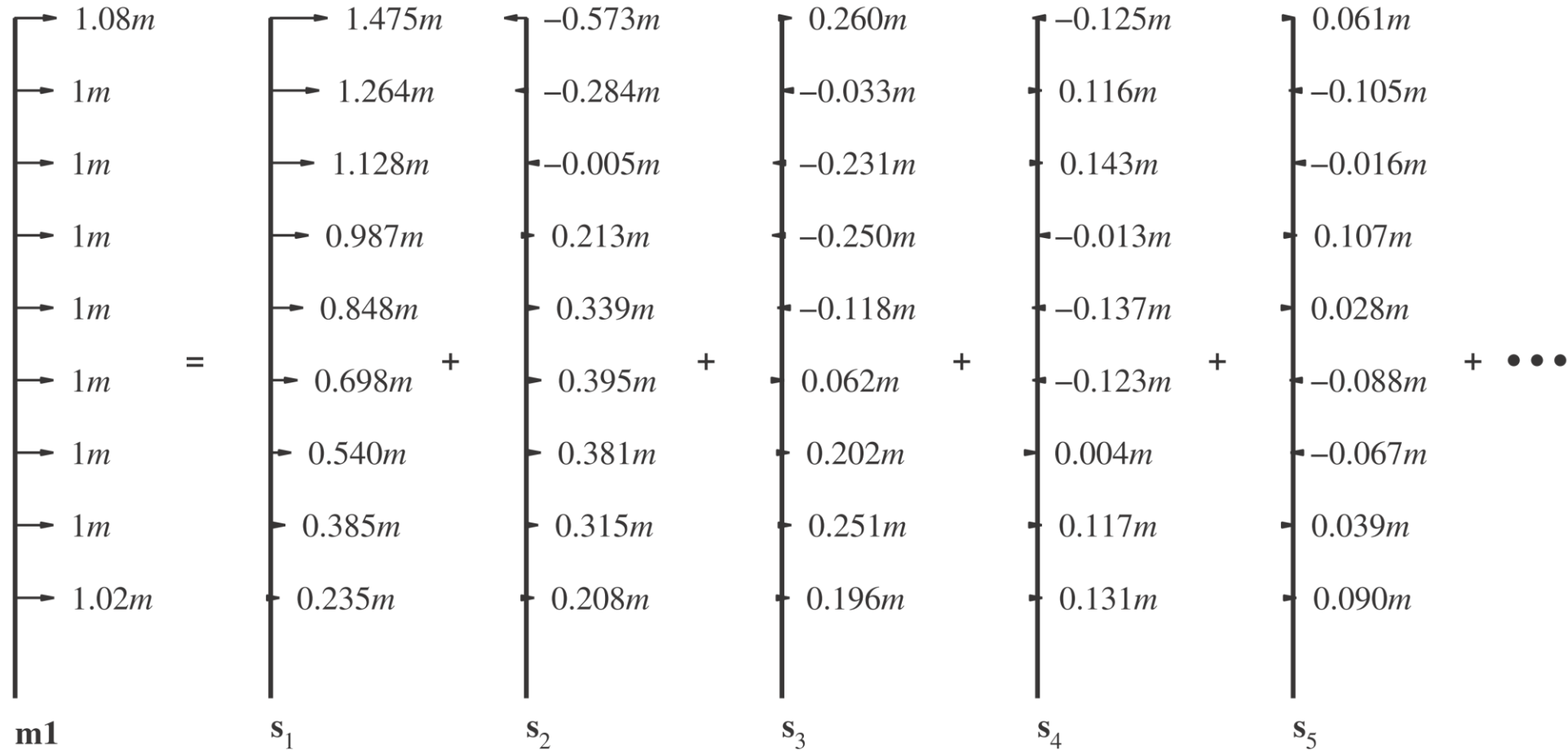
$$\mathbf{P}_{eff}(t) = \sum_{n=1}^N \mathbf{P}_{eff,n}(t) = \sum_{n=1}^N -\mathbf{s}_n \ddot{u}_g(t)$$

- The contributions of the  $n^{th}$  mode to  $\mathbf{P}_{eff}(t)$  and  $\mathbf{s}$  are

$$\mathbf{P}_{eff,n}(t) = -\mathbf{s}_n \ddot{u}_g(t)$$

$$\mathbf{s}_n = \Gamma_n \mathbf{M} \boldsymbol{\phi}_n$$

# Modal expansion of the distribution $s = M \mathbf{1}$ of effective earthquake forces



The direction of force  $s_{jn}$  at the  $j$ th floor level is controlled by the algebraic sign of

$\phi_{jn}$ , the  $j$ th-floor displacement in mode  $\phi_n$

# Modal Expansion of the Effective Earthquake Forces

- Utilizing the modal expansion of  $\mathbf{P}_{eff}(t)$  and  $\mathbf{s}$ , two procedures for approximate analysis of inelastic buildings are proposed by Chopra and Goel (2002).
  - The uncoupled modal response history analysis (UMRHA), and
  - The modal pushover analysis (MPA)
- Not intended for practical application, the UMRHA procedure is developed only to provide a rationale for the MPA procedure.
- In the **UMRHA procedure**, the response history of the building to  $\mathbf{P}_{eff,n}(t)$ , the nth-mode component of the excitation, is determined by **nonlinear RHA of an inelastic SDF system, and superposition of these “modal” responses gives the total response.**
- In the **MPA procedure**, the peak response to  $\mathbf{P}_{eff,n}(t)$  is determined by a **nonlinear static, or pushover, analysis, and the peak modal responses are combined by modal combination rules** to estimate the total response.

# The Uncoupled Modal Response History Analysis Procedure

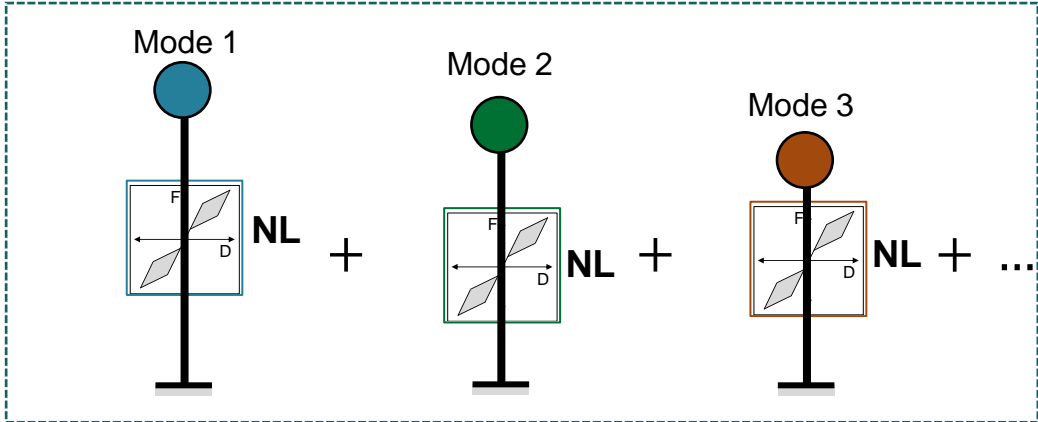
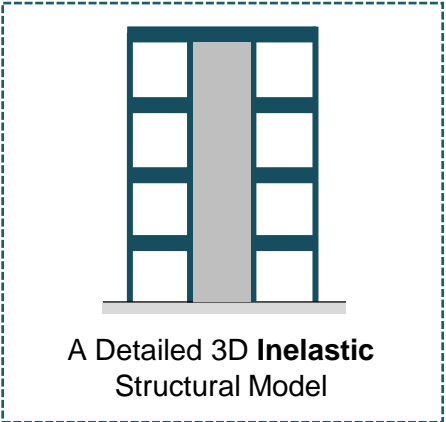
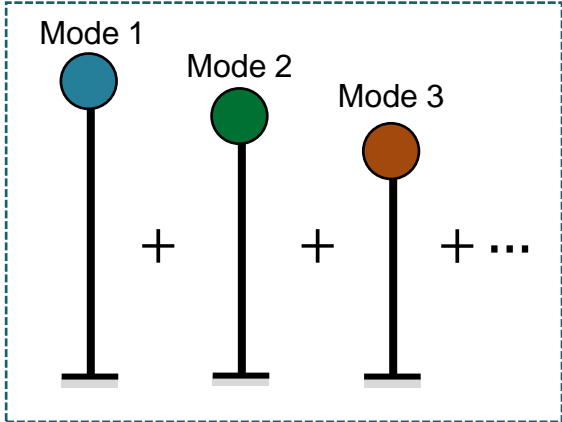
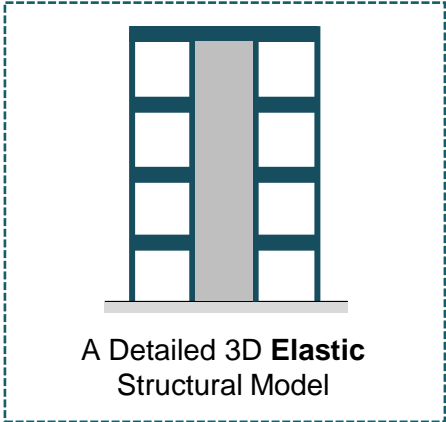


# The UMRHA Procedure

The Classical Modal Analysis Procedure



The Uncoupled Modal Response History Analysis (UMRHA) Procedure



# The Uncoupled Modal Response History Analysis (UMRHA) Procedure

## Linearly Elastic Systems

- The classical modal analysis procedure for linearly elastic systems is equivalent to finding the response of the structure to  $\mathbf{P}_{eff,n}(t)$  for each  $n$  and superposing the responses for all  $n$ .
- The response of the system to  $\mathbf{P}_{eff,n}(t)$  is entirely in the  $n$ th mode, **with no contribution from other modes**, which implies that the modes are **uncoupled**.
- The equations governing the response of the linearly elastic MDF system to  $\mathbf{P}_{eff,n}(t)$ ,

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = -\mathbf{s}_n\ddot{u}_g(t)$$

and the resulting floor displacements (using the idea of modal expansion from modal analysis) are

$$\mathbf{u}_n(t) = \boldsymbol{\phi}_n q_n(t)$$

Substituting this  $\mathbf{u}_n(t)$  in governing equation and pre-multiplying by  $\boldsymbol{\phi}_n^T$  leads to the equation governing the modal coordinate  $q_n(t)$ :

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2 q_n(t) = -\Gamma_n \ddot{u}_g(t)$$

Where  $\omega_n$  is the natural frequency,  $\xi_n$  is the damping ratio, and  $\Gamma_n$  is the modal participation factor for the  $n$ th mode.

- As demonstrated in classical modal analysis, the solution of nth-mode equation of motion is

$$q_n(t) = \Gamma_n D_n(t)$$

Where  $D_n(t)$  is deformation response of the nth mode linearly elastic SDF system governed by

$$\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t)$$

Therefore,

$$\mathbf{u}_n(t) = \Gamma_n \boldsymbol{\phi}_n D_n(t)$$

$$\Delta_{jn}(t) = \Gamma_n (\phi_{jn} - \phi_{j-1,n}) D_n(t)$$

- The above equations represent the response of the MDF system to  $\mathbf{P}_{eff,n}(t)$ , and **superposing the responses for all  $n$  gives the response of the system due to total excitation  $\mathbf{P}_{eff}(t)$ :**

$$r(t) = \sum_{n=1}^N r_n(t)$$

- **The UMRHA procedure for exact analysis of linearly elastic systems is identical to classical modal RHA.** But to derive these equations, now we have used the modal expansion of spatial distribution of effective earthquake forces.

# The Uncoupled Modal Response History Analysis (UMRHA) Procedure

## Inelastic Systems

- Although modal analysis is **not valid** for an inelastic system, its dynamic response can usefully be discussed in terms of the natural vibration modes of the corresponding linear system.
- Each structural element of this linear system is defined to have the same stiffness as its initial stiffness in the inelastic system; both systems have the same mass and damping. Therefore, the natural vibration periods and modes of the **corresponding linear system** are the same as the vibration properties of the **inelastic system undergoing small oscillation**, which are referred to as “periods” and “modes” of the inelastic system.
- Thus, the modal expansion of effective earthquake forces is also valid for inelastic systems, where  $\phi_n$  now represents the modes of the corresponding linear system.

# The Uncoupled Modal Response History Analysis (UMRHA) Procedure

## Inelastic Systems

- The equations governing the response of the inelastic MDF system to  $\mathbf{P}_{eff,n}(t)$  are

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{f}_s(\mathbf{u}) = -\mathbf{s}_n\ddot{u}_g(t)$$

- The solution of this equation will no longer be described by  $\mathbf{u}_n(t) = \boldsymbol{\phi}_n q_n(t)$  because modes other than the  $n$ th mode will also contribute to the system response, implying that the vibration modes are now coupled.
- Thus, the floor displacements are given by:

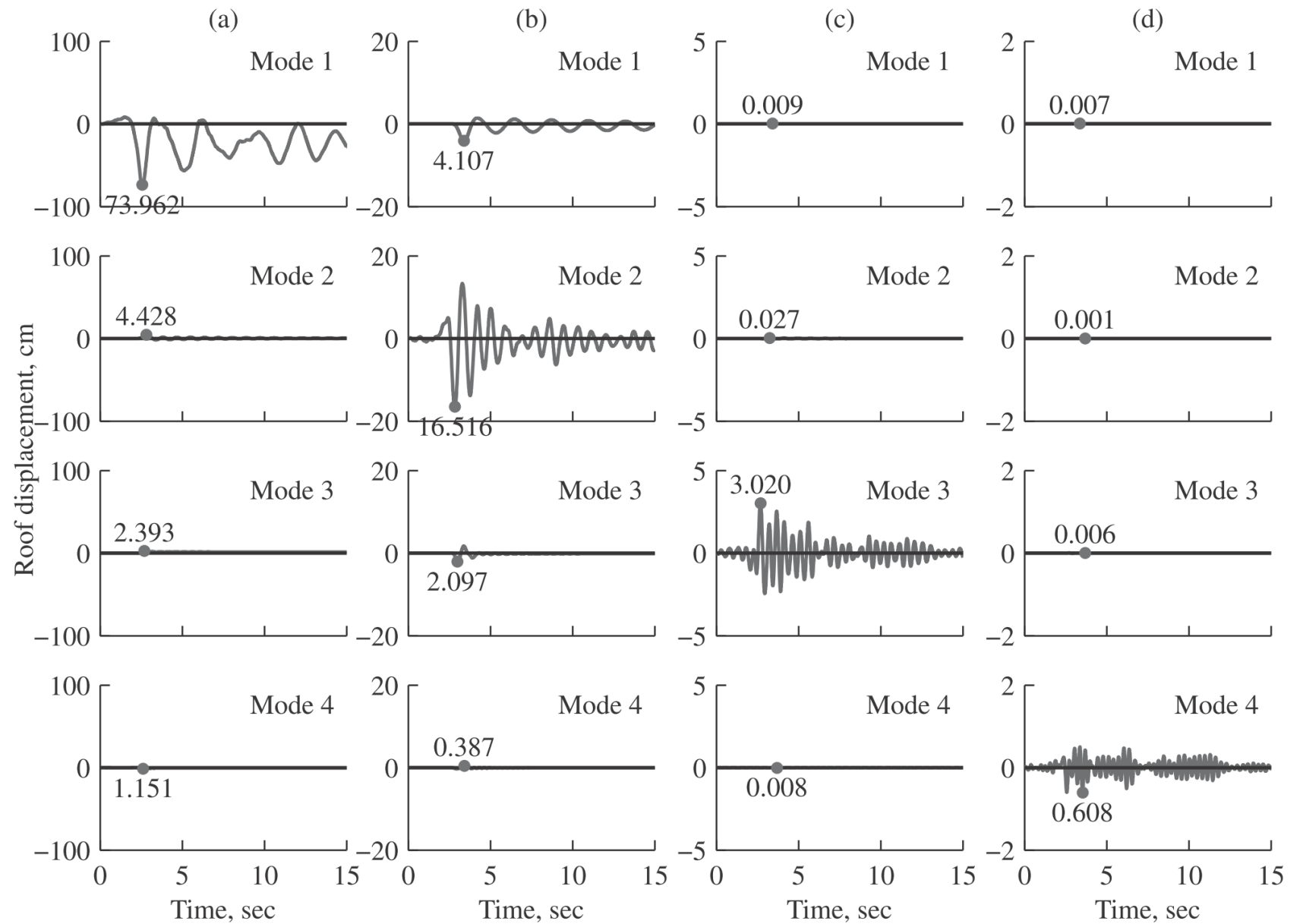
$$\mathbf{u}_n(t) = \sum_{r=1}^N \boldsymbol{\phi}_r q_r(t)$$

- However, because for linear systems  $q_r(t) = 0$  for all modes **other than the  $n$ th mode**, it is reasonable to expect that  $q_r(t)$  **may be small** for inelastic systems, implying that the elastic modes are, at most, weakly coupled. Therefore,

$$\mathbf{u}_n(t) = \sum_{r=1}^N \boldsymbol{\phi}_r q_r(t) \approx \boldsymbol{\phi}_n q_n(t)$$

Figure shows that the roof displacement due to the force vector  $\mathbf{P}_{eff,n}(t)$  is due primarily to the  $n$ th mode but that other modes contribute to the response. The second, third, and fourth modes start responding to excitation  $\mathbf{P}_{eff,1}(t)$  the instant the structure first yields.

Although the natural vibration modes are no longer uncoupled if the system responds in the inelastic range, modal coupling is weak.



**Figure 20.6.1** Modal decomposition of roof displacement due to  $\mathbf{p}_{eff,n}(t) = -\mathbf{s}_n \ddot{u}_g(t)$ ,  $n = 1, 2, 3$ , and 4, where  $\ddot{u}_g(t) = \text{LA25}$  ground motion: (a)  $\mathbf{p}_{eff,1} = -\mathbf{s}_1 \times \text{LA25}$ ; (b)  $\mathbf{p}_{eff,2} = -\mathbf{s}_2 \times \text{LA25}$ ; (c)  $\mathbf{p}_{eff,3} = -\mathbf{s}_3 \times \text{LA25}$ ; (d)  $\mathbf{p}_{eff,4} = -\mathbf{s}_4 \times \text{LA25}$ .

- This weak coupling of modes implies that the structural response due to excitation  $\mathbf{P}_{eff,n}(t)$  may be approximated b

$$\mathbf{u}_n(t) \simeq \boldsymbol{\phi}_n q_n(t)$$

- Substituting this approximation into governing equation and pre-multiplying by  $\boldsymbol{\phi}_n^T$  gives

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \frac{F_{sn}}{M_n} = -\Gamma_n\ddot{u}_g(t)$$

where  $F_{sn}$  is a nonlinear hysteretic function of the  $n$ th modal coordinate  $q_n$ :

$$F_{sn} = F_{sn}(q_n) = \boldsymbol{\phi}_n^T \mathbf{f}_s(q_n)$$

- If the smaller contributions of other modes had not been neglected,  $F_{sn}$  would depend on all modal coordinates, and the set of equations would be coupled because of yielding of the structure.
- By comparing the  $n$ th-mode governing equation of motion with linear counterpart, the solution can be expressed as  $q_n(t) = \Gamma_n D_n(t)$ , where  $D_n(t)$  is now governed by

$$\ddot{D}_n(t) + 2\xi_n\omega_n\dot{D}_n(t) + \frac{F_{sn}}{L_n} = -\ddot{u}_g(t)$$

- $D_n$  may be interpreted as the deformation response of the nth-mode **inelastic SDF system**, an SDF defined by
  - 1) small-oscillation vibration properties—natural frequency  $\omega_n$  (natural period  $T_n$ ) and damping ratio  $\xi_n$ —of the nth mode of the MDF system; and
  - 2) The force–deformation ( $F_{sn}/L_n - D_n$ ) relation. Introducing the nth-mode inelastic SDF system permitted the extension to inelastic systems of the well-established concepts for elastic systems.
- The solution of the nonlinear modal equation provides  $D_n(t)$ , which can be substituted into following (same) equations to obtain floor displacements and story drifts.

$$\mathbf{u}_n(t) = \Gamma_n \boldsymbol{\phi}_n D_n(t)$$

$$\Delta_{jn}(t) = \Gamma_n (\phi_{jn} - \phi_{j-1,n}) D_n(t)$$

- These equations approximate the response of the inelastic MDF system to  $\mathbf{P}_{eff,n}(t)$ , the nth-mode contribution to  $\mathbf{P}_{eff}(t)$ .
- Superposition of responses to  $\mathbf{P}_{eff,n}(t)$ — $n = 1, 2, \dots, N$ —provides the total response to  $\mathbf{P}_{eff}(t)$ .

$$r(t) = \sum_{n=1}^N r_n(t)$$

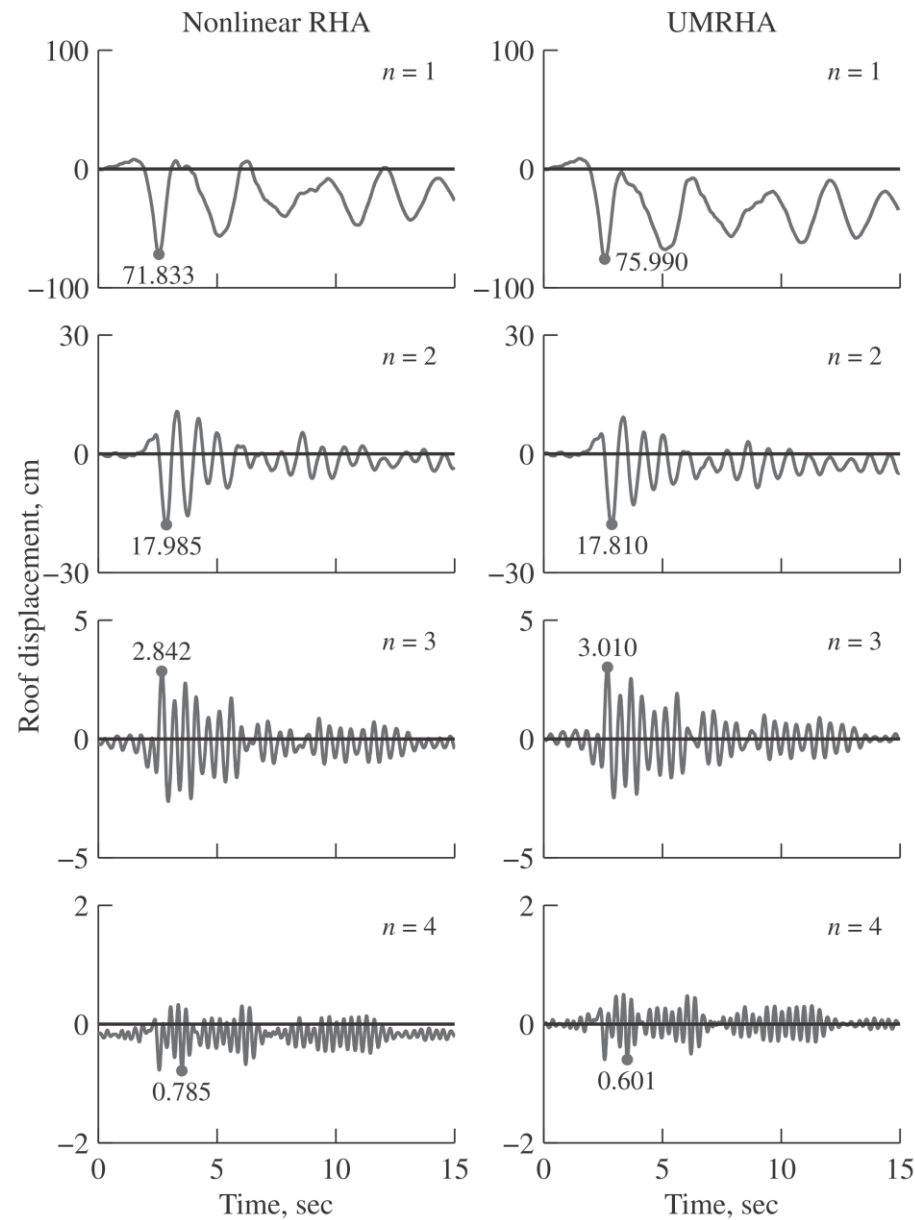
- This is the UMRHA procedure for approximate analysis of inelastic systems.



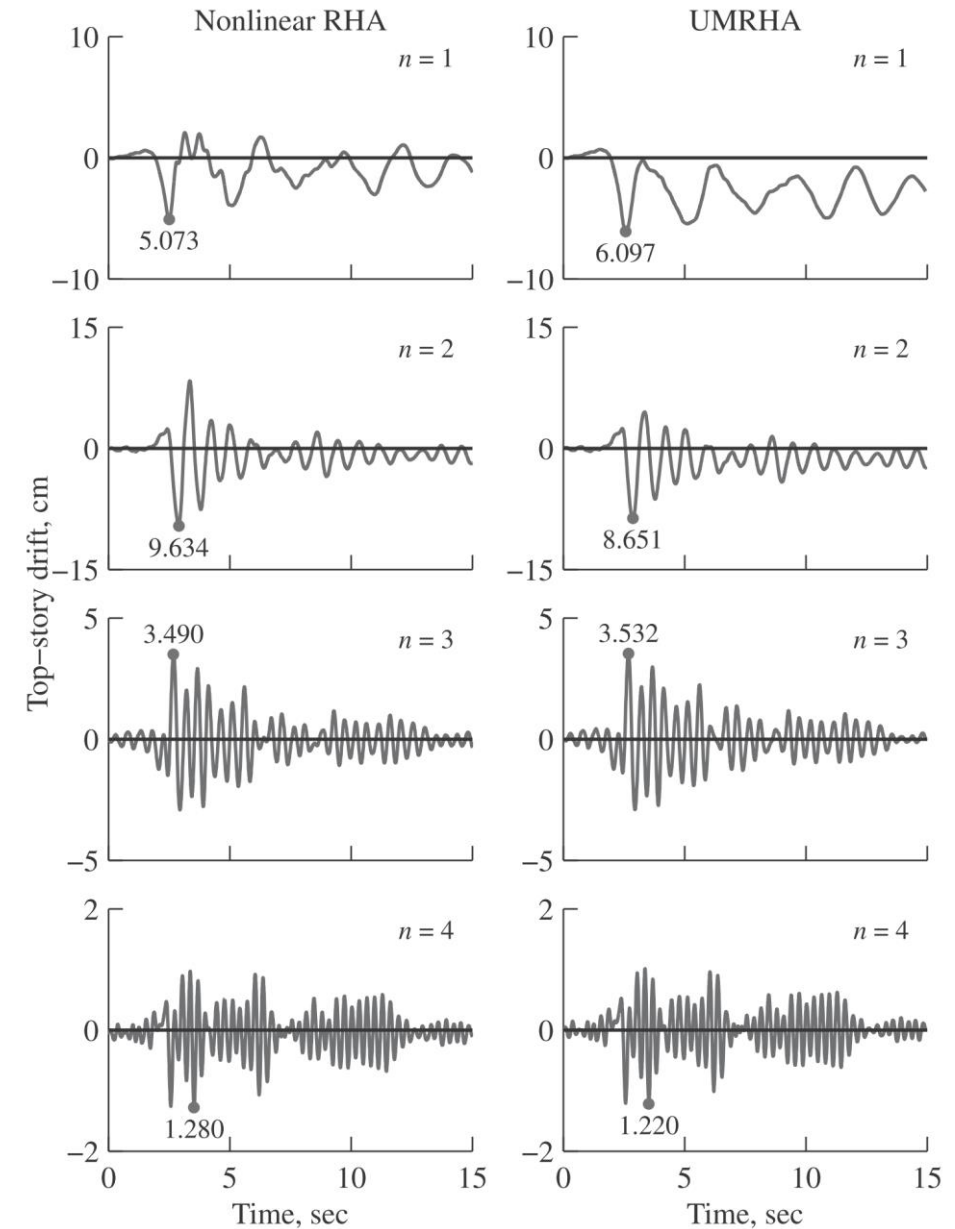
To test the modal uncoupling approximation in UMRHA, the response of the a 9-story building to  $\mathbf{P}_{eff,n}(t) = -\mathbf{s}_n \ddot{u}_g(t)$  is determined by two methods and compared.

Left: Roof Displacement

Right: Roof Drift

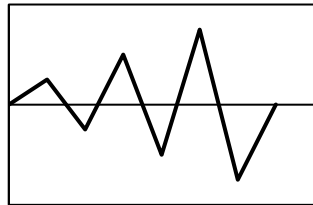


**Figure 20.6.2** Comparison of approximate roof displacement from UMRHA and exact result from nonlinear RHA due to  $\mathbf{p}_{eff,n}(t) = -\mathbf{s}_n \ddot{u}_g(t)$ ,  $n = 1, 2, 3,$  and  $4$ , where  $\ddot{u}_g(t) = \sim$  LA25 ground motion.

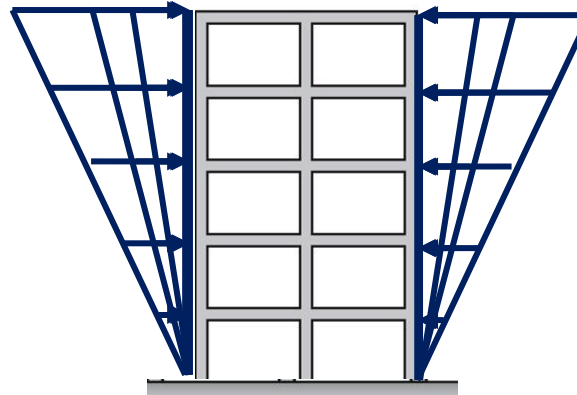


**Figure 20.6.3** Comparison of approximate top-story drift from UMRHA and exact result from nonlinear RHA due to  $\mathbf{p}_{eff,n}(t) = -\mathbf{s}_n \ddot{u}_g(t)$ ,  $n = 1, 2, 3,$  and  $4$ , where  $\ddot{u}_g(t) = \sim$  LA25 ground motion.

# The UMRHA Procedure

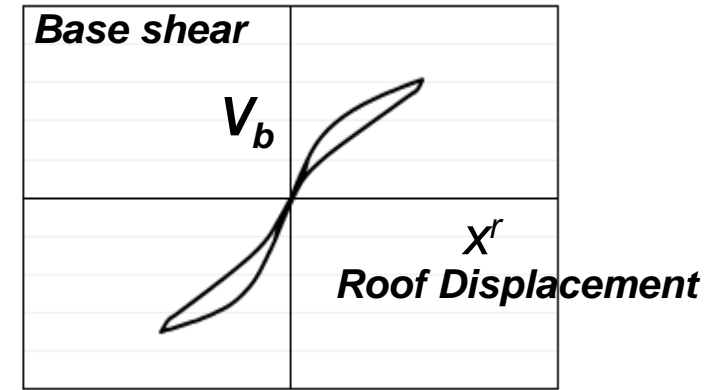


Cyclic Modal Load



Cyclic Modal Pushover

**MDF**



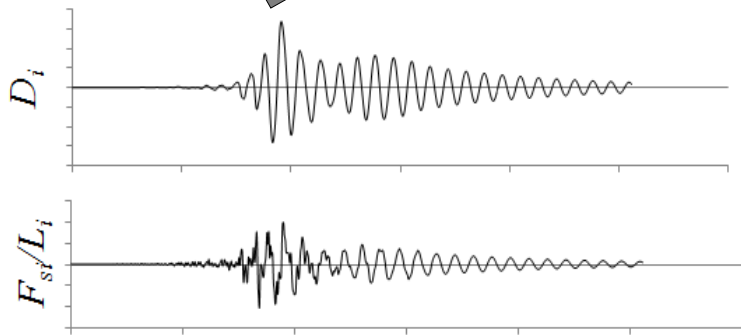
*Base shear*

$V_b$

$x^r$

*Roof Displacement*

## Displacement-related responses



Time (Sec)

## Force-related responses

$m$

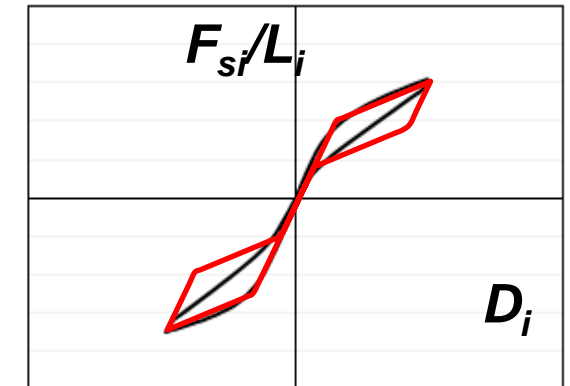
$\xi_i$

**SDOF**

$$\ddot{D}_i + 2\xi_i\omega_i\dot{D}_i + F_{si}(D_i, \dot{D}_i)/L_i = -\ddot{x}_g(t)$$

Acceleration(g)

Time (Sec)



$F_{si}/L_i$

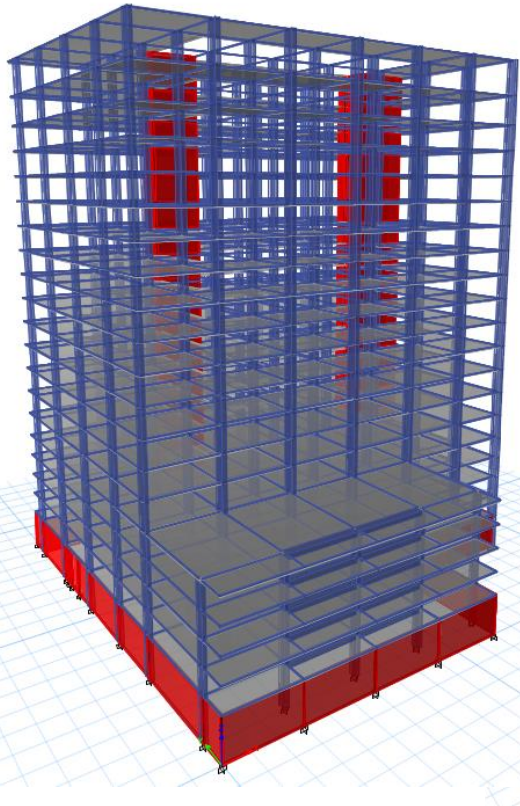
$D_i$

Idealized Hysteretic Model

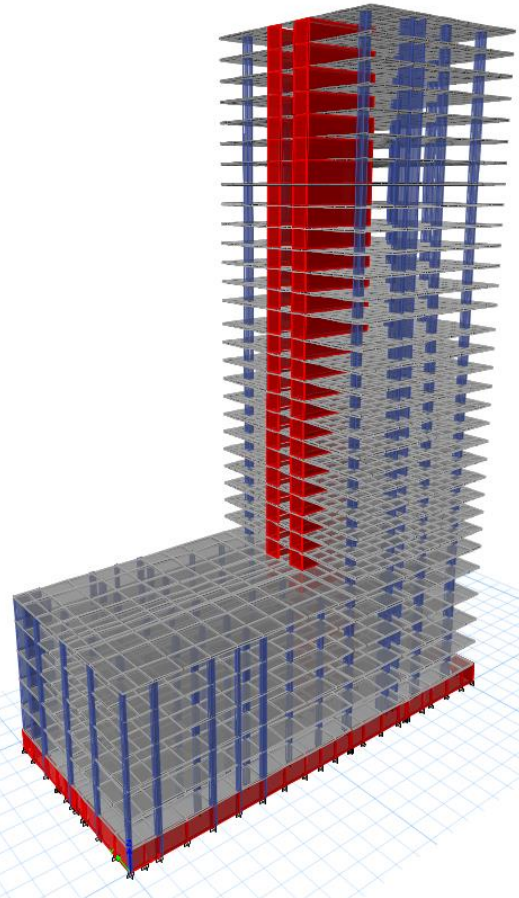
# The Uncoupled Modal Response History Analysis (UMRHA) Procedure

- For elastic systems → UMRHA = **Classical Modal RHA (an exact analysis procedure)**.
- For inelastic systems → UMRHA = **Approximate analysis procedure**.
- The UMRHA for inelastic systems is based on two approximations
  - 1) **Superposition of responses** (Strictly valid for only elastic systems. Approximately valid for inelastic systems)
  - 2) **Neglecting the coupling of modal coordinates**, which permitted computing the response of inelastic MDF system to  $\mathbf{P}_{eff,n}(t)$  from that of an SDF system. This approximation is reasonable only because the excitation is  $\mathbf{P}_{eff,n}(t)$ , the nth-mode contribution to the total excitation  $\mathbf{P}_{eff}(t)$ . It would not be valid for an excitation with lateral force distribution different than  $\mathbf{s}_n$  [e.g., the total excitation  $\mathbf{P}_{eff}(t)$ ], pointing out that the modal expansion of effective earthquake forces is a key concept underlying the UMRHA.

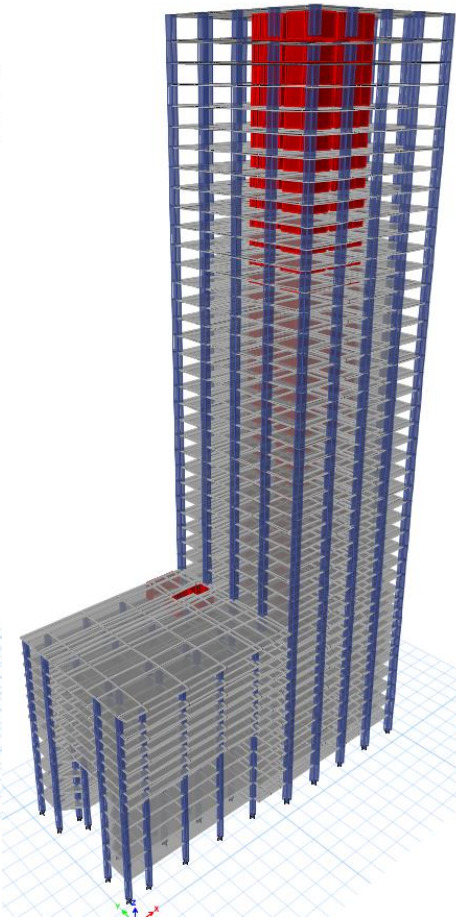
# Case study Buildings



20-story  
B1



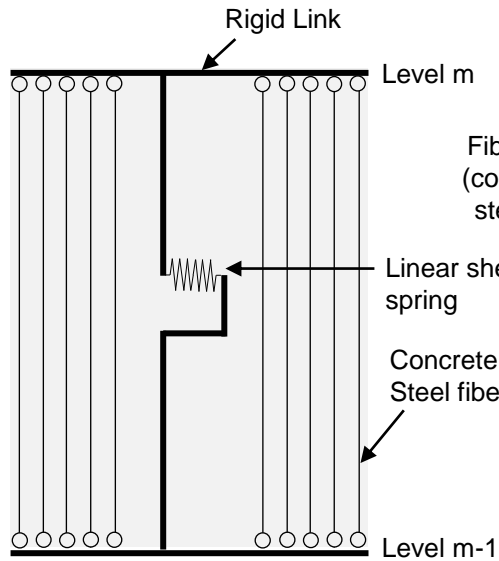
33-story  
B2



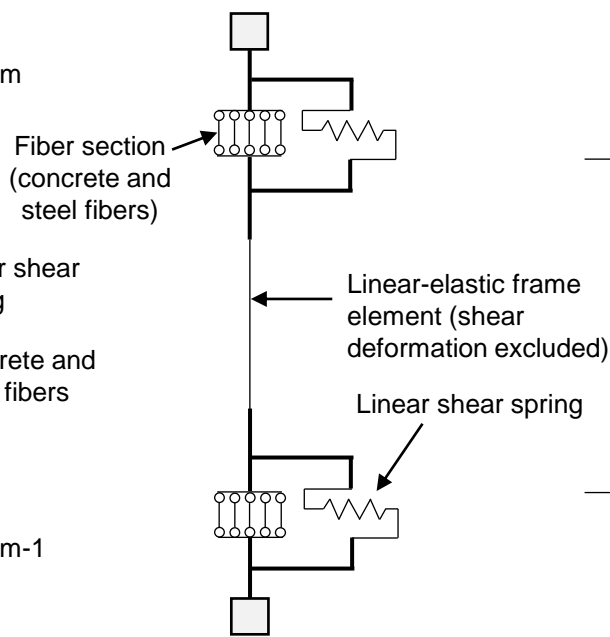
44-story  
B3

- Located in Bangkok, Thailand
- Heights vary from 20 to 44 stories
- RC slab-column frames carry gravity loads
- RC walls & cores resist lateral loads
- Masonry infill walls extensively used
- Designed for wind loads, but not for seismic effects
- Possess irregular features commonly found in typical tall buildings, e.g. podium and non-symmetrical arrangement of RC walls, etc.

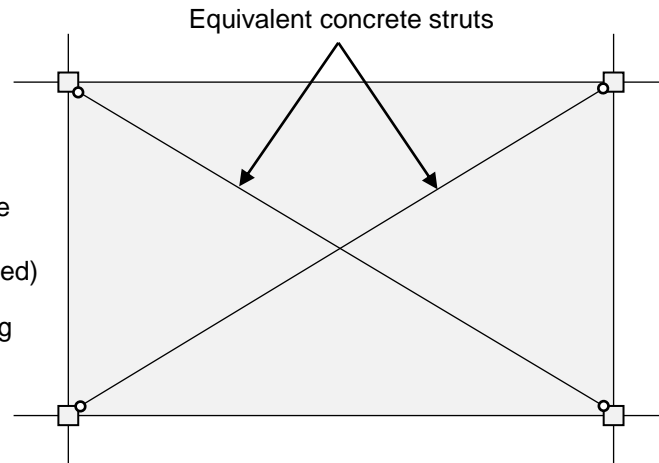
# Nonlinear Modeling of Case Study Buildings



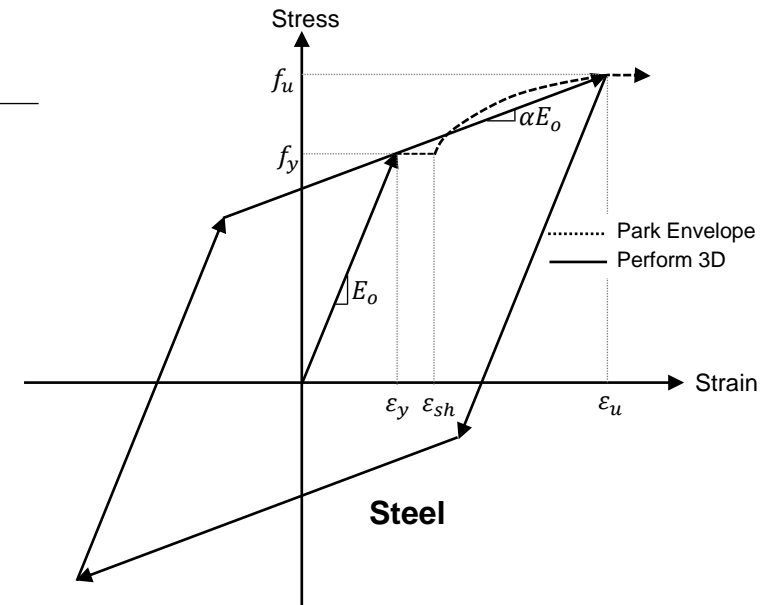
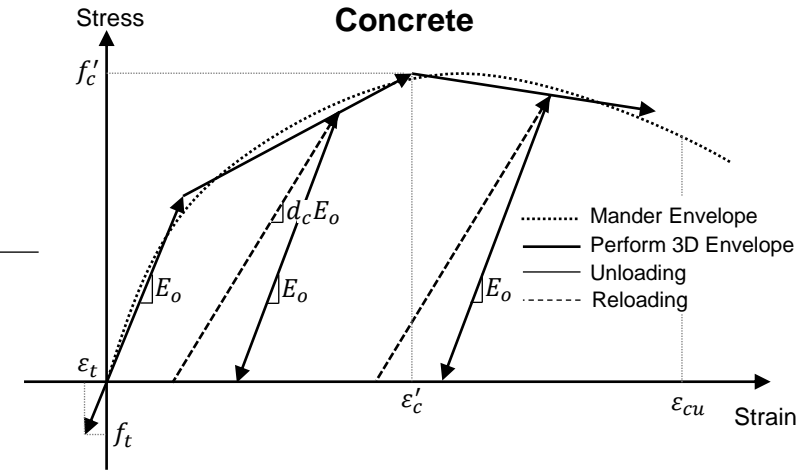
MVLEM for RC Walls



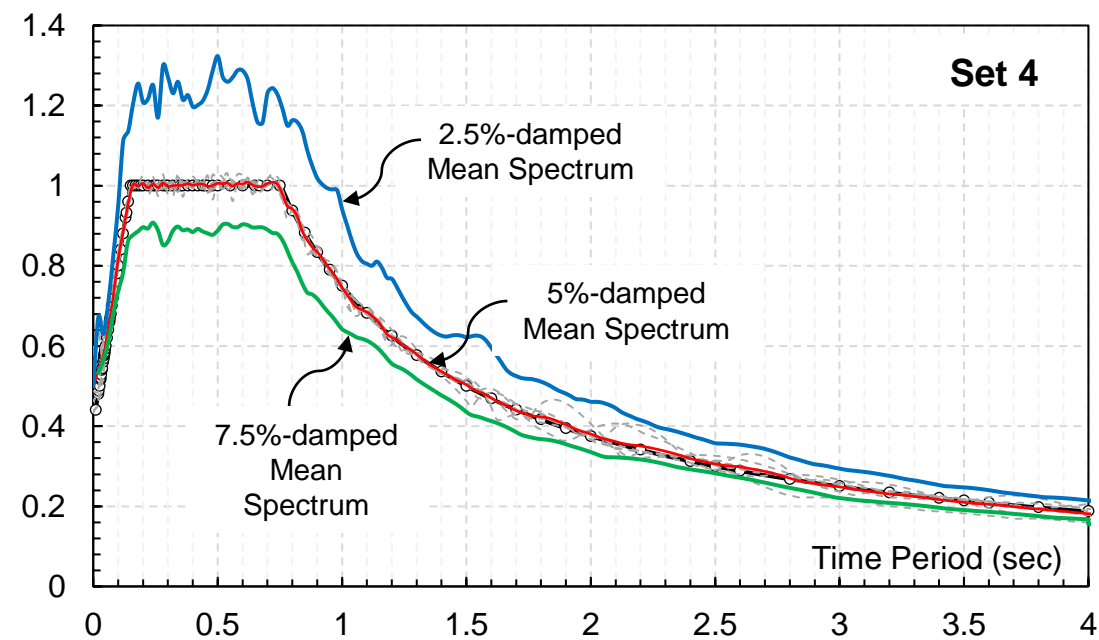
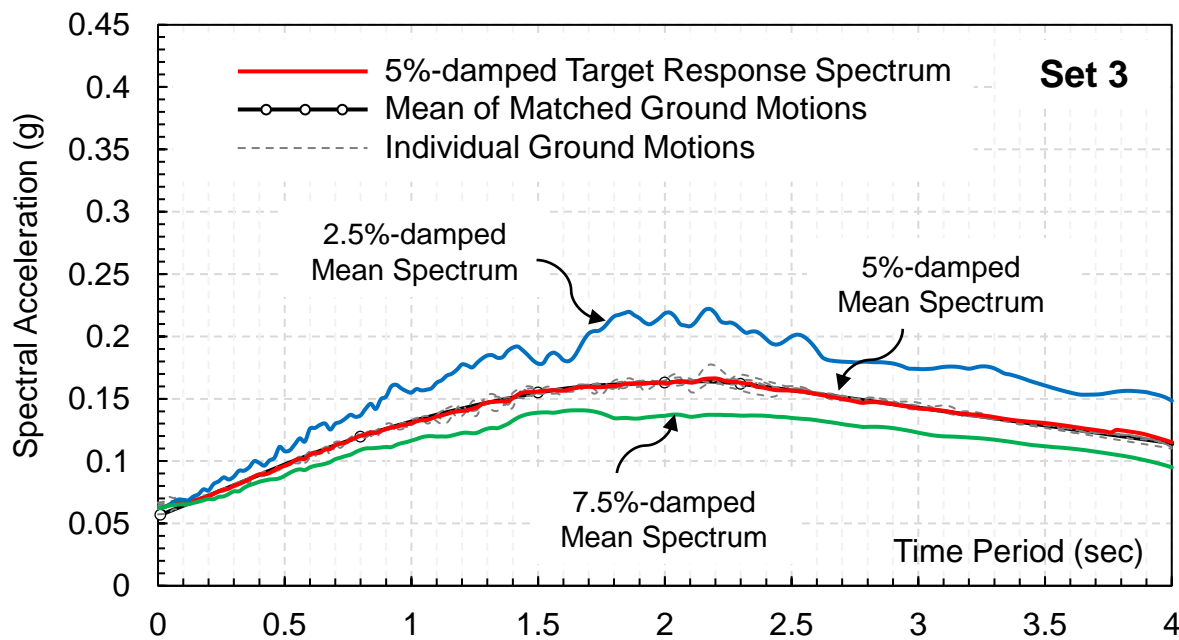
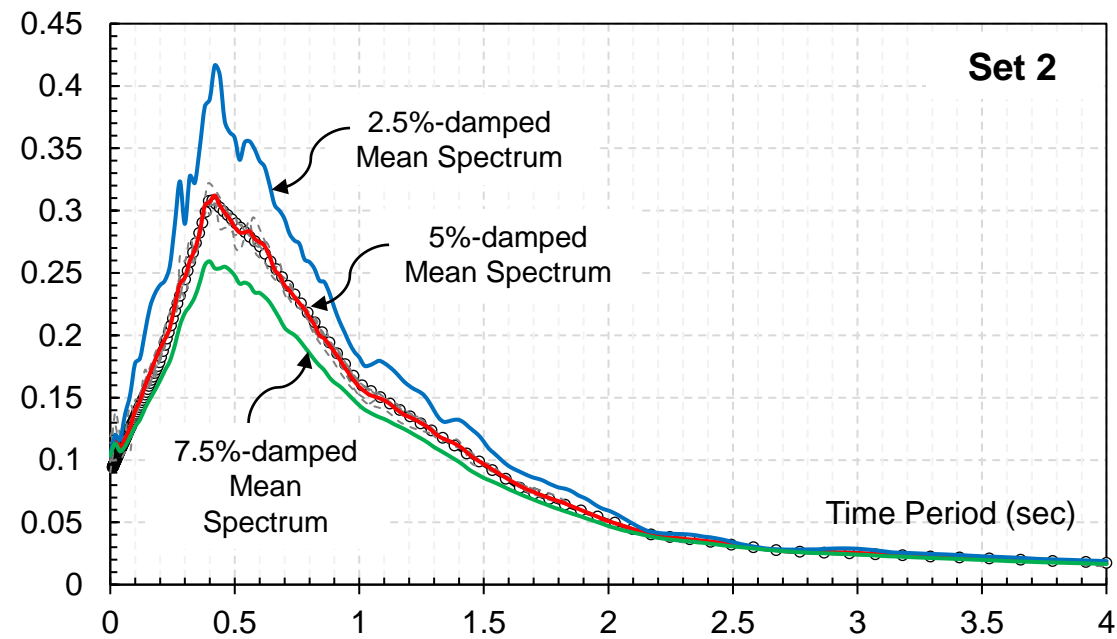
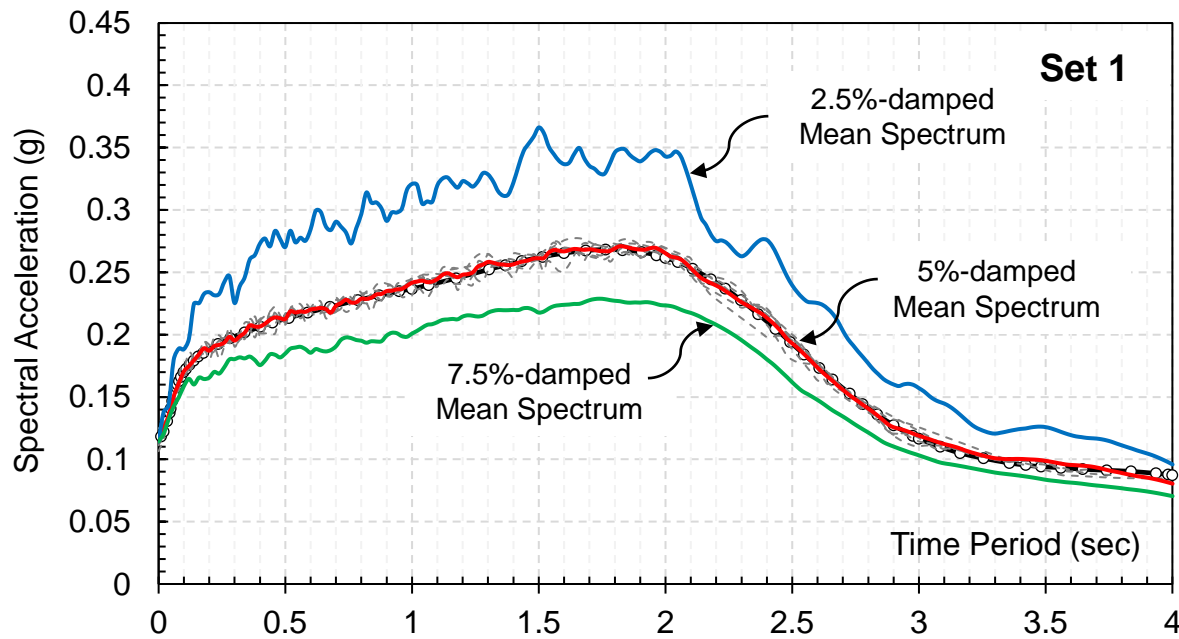
Lumped Fibers for RC Columns



Masonry Infill Wall Model

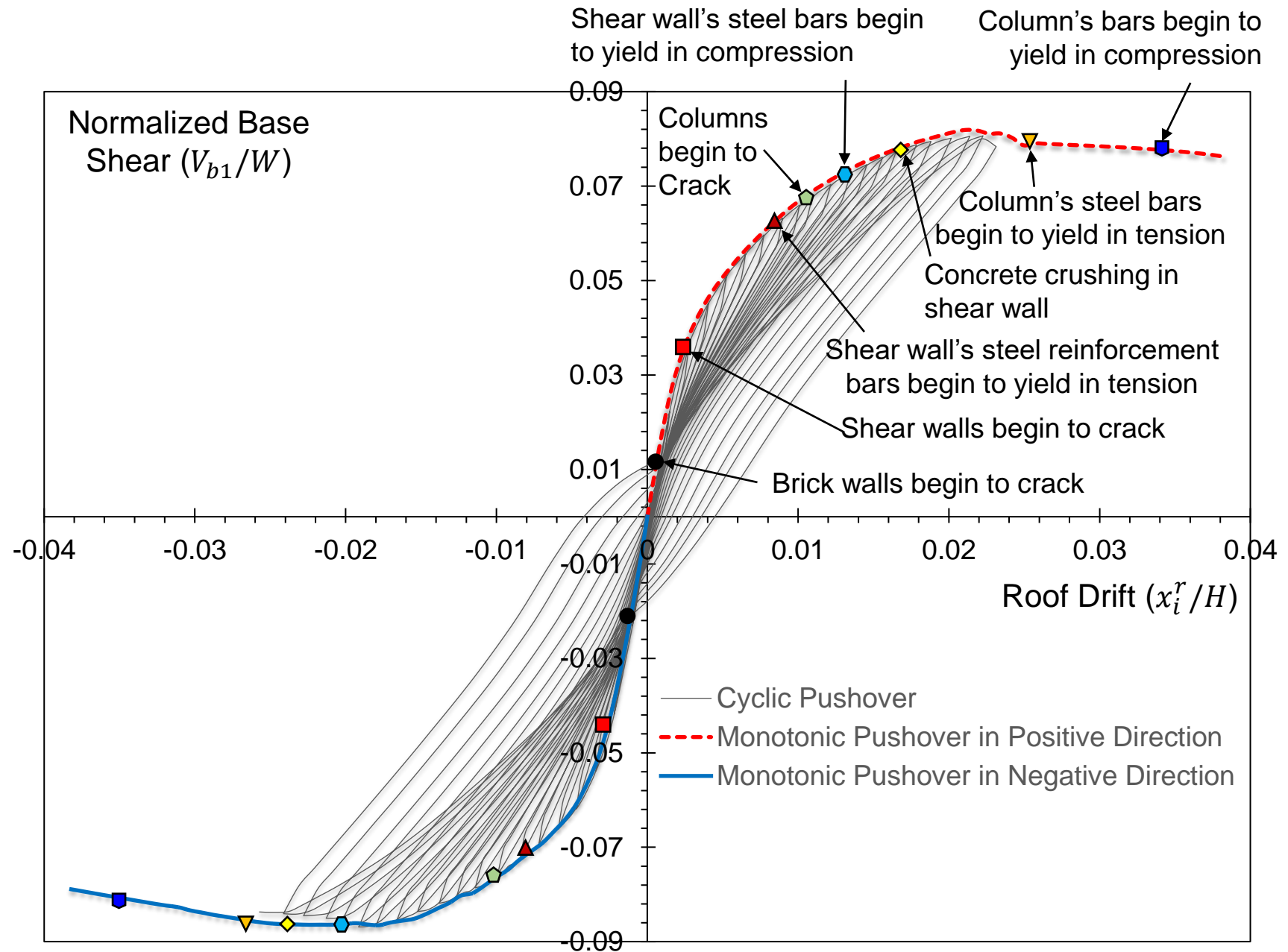


Steel

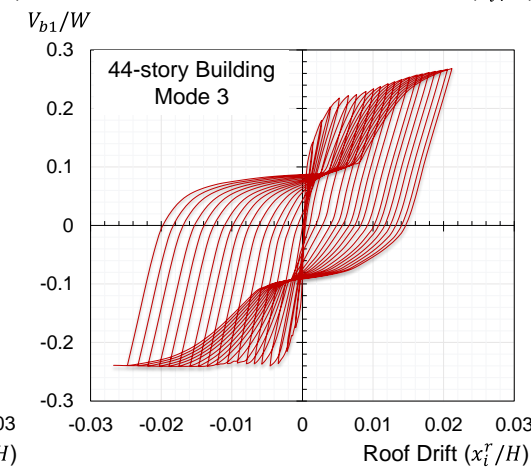
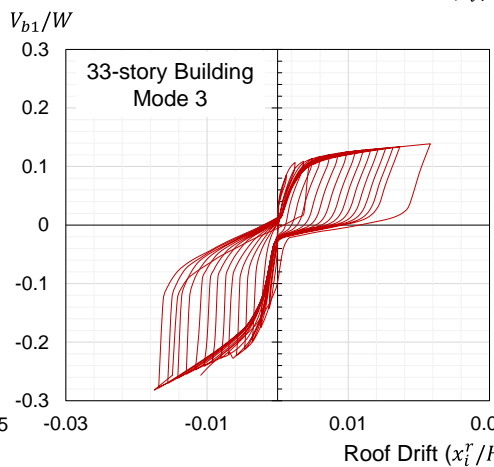
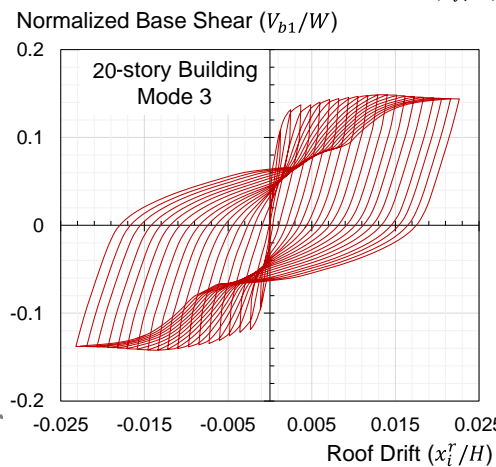
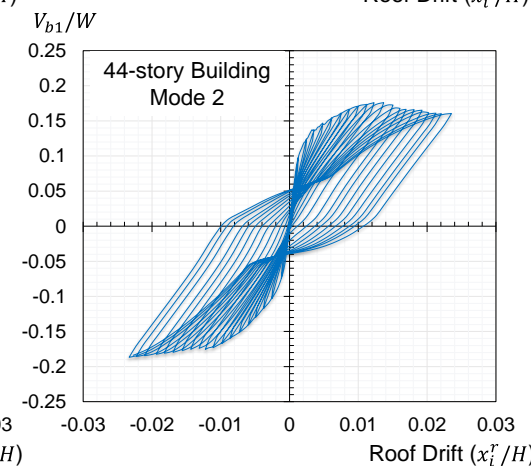
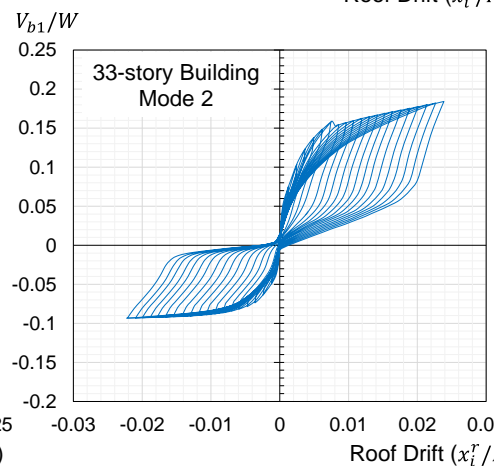
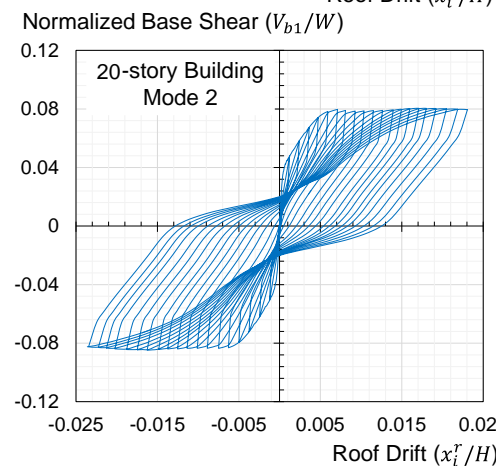
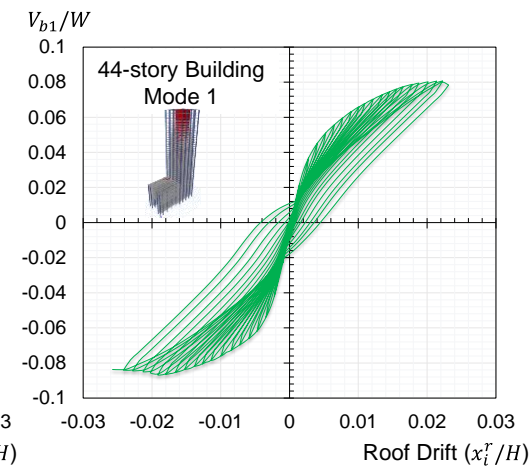
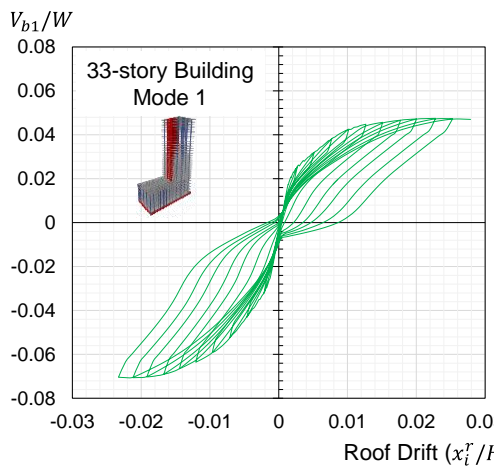
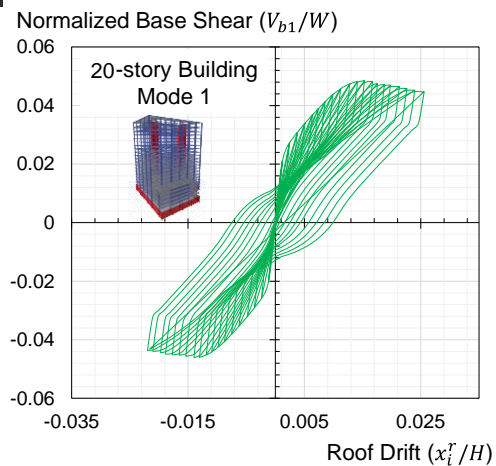


# The Cyclic Behavior of Case Study Buildings

44-story case study building in Strong Direction

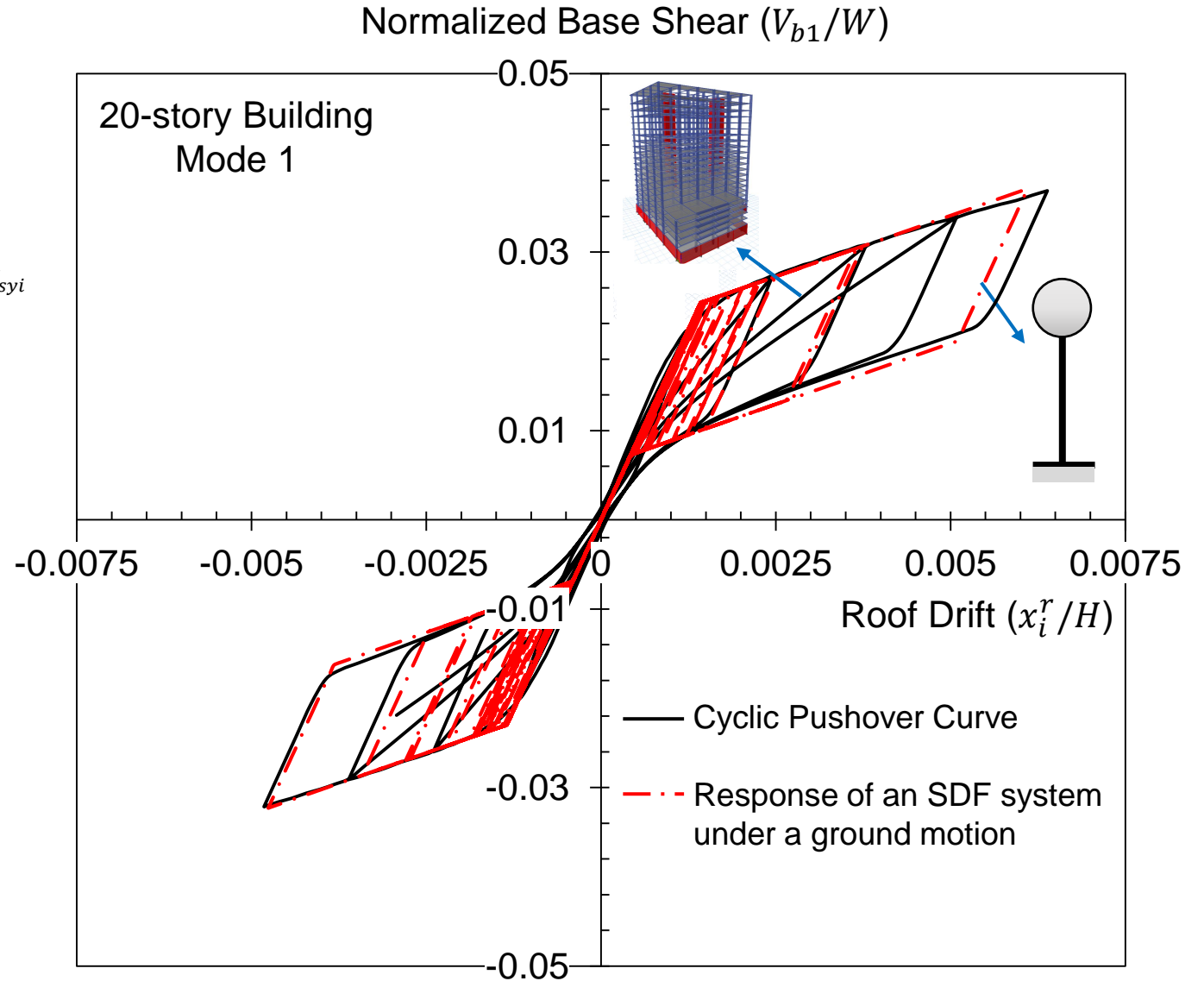
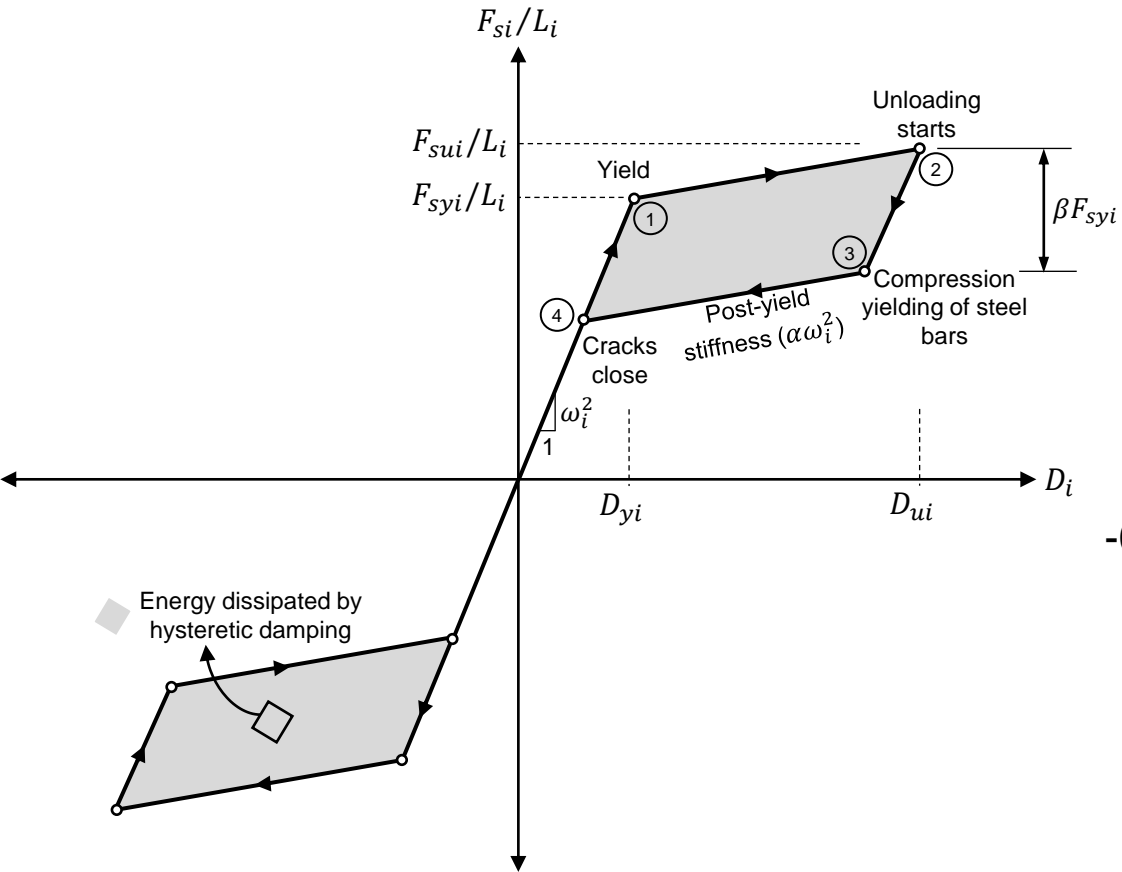


# The Cyclic Behavior of Case Study Buildings



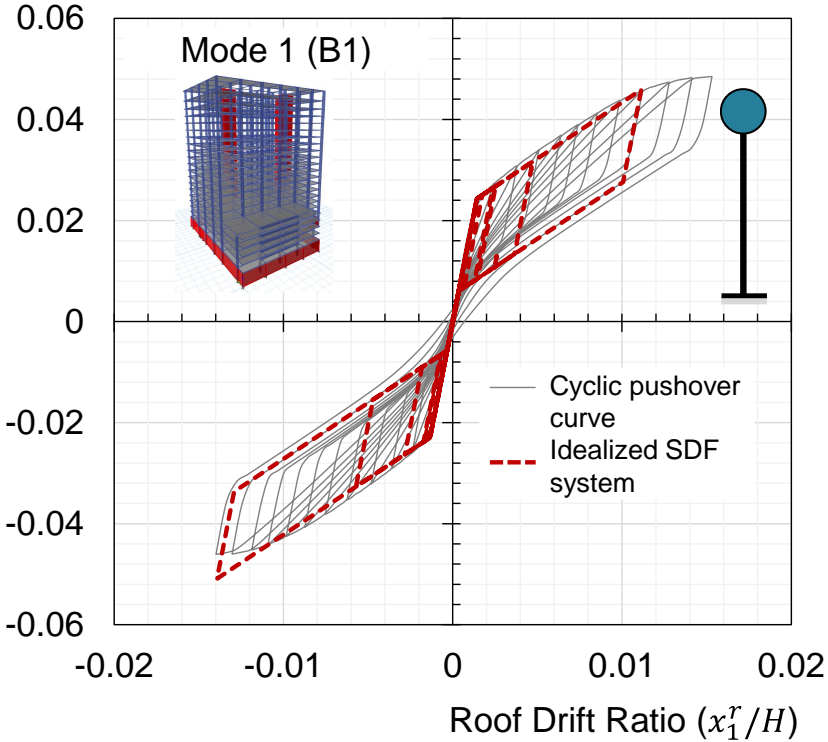


# Idealization of Cyclic Pushover Curves

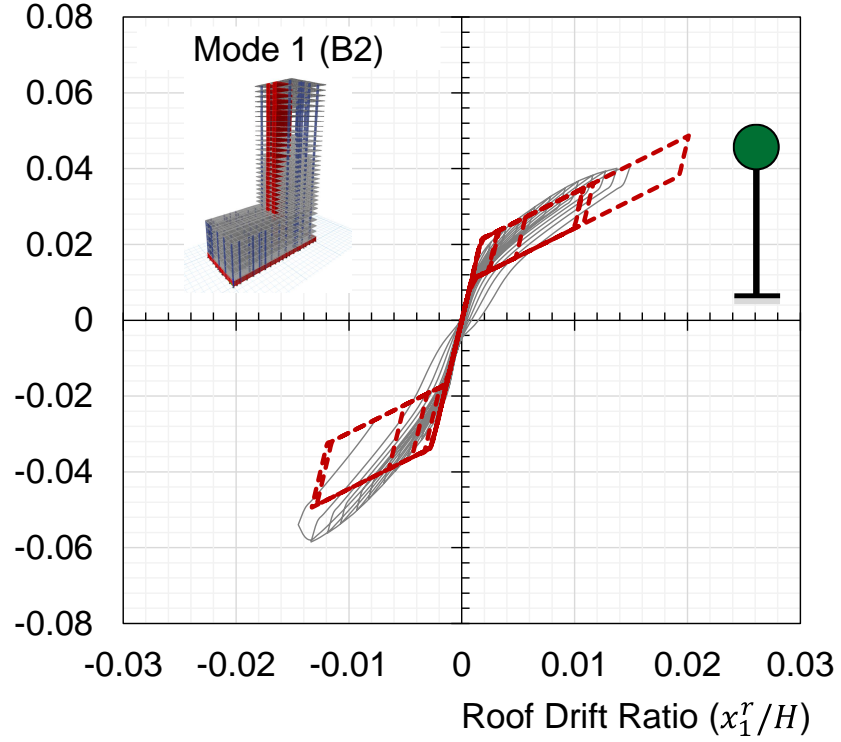


# Idealization of Cyclic Pushover Curves

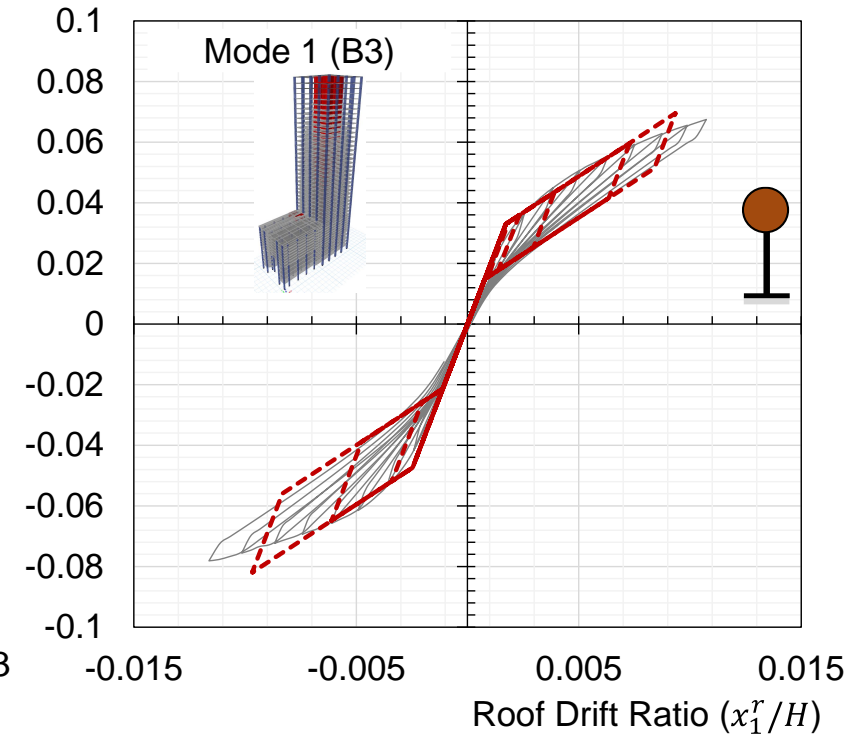
Normalized Base Shear ( $V_{b1}/W$ )



Normalized Base Shear ( $V_{b1}/W$ )



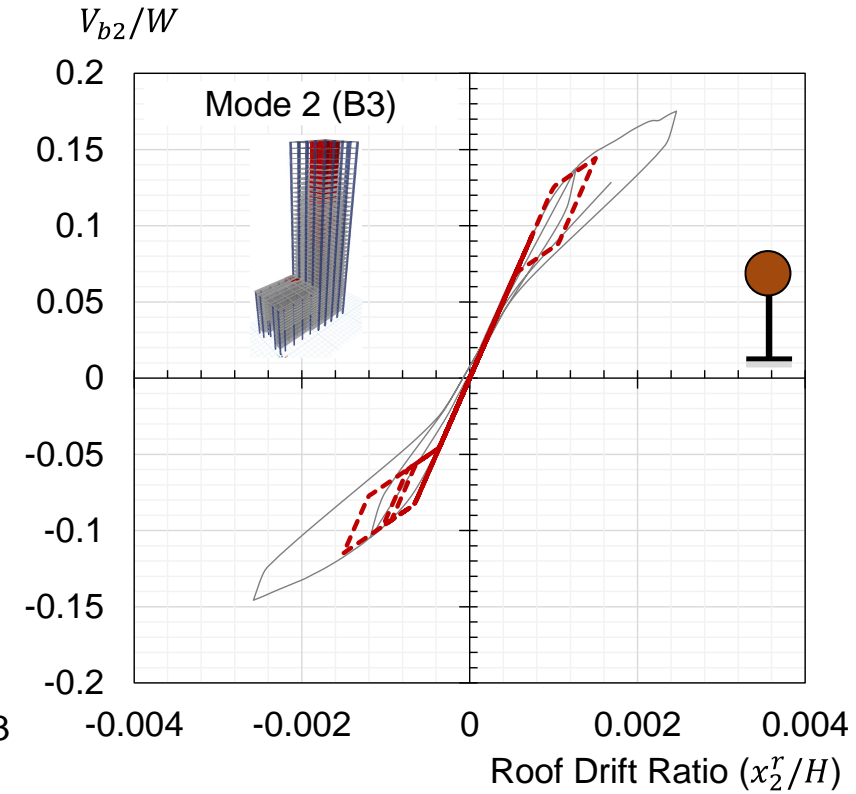
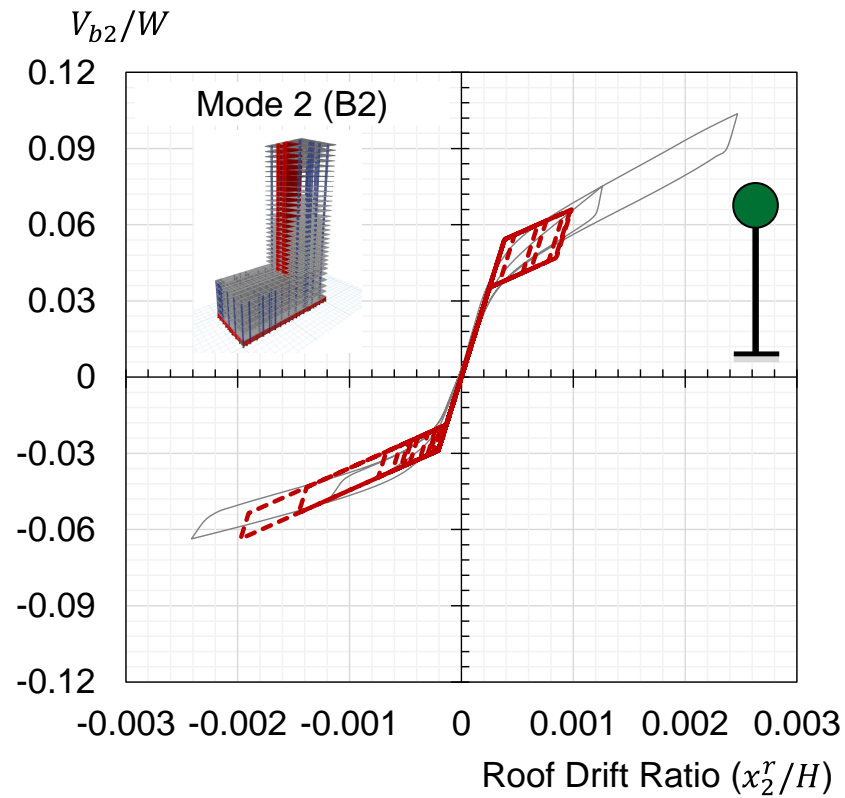
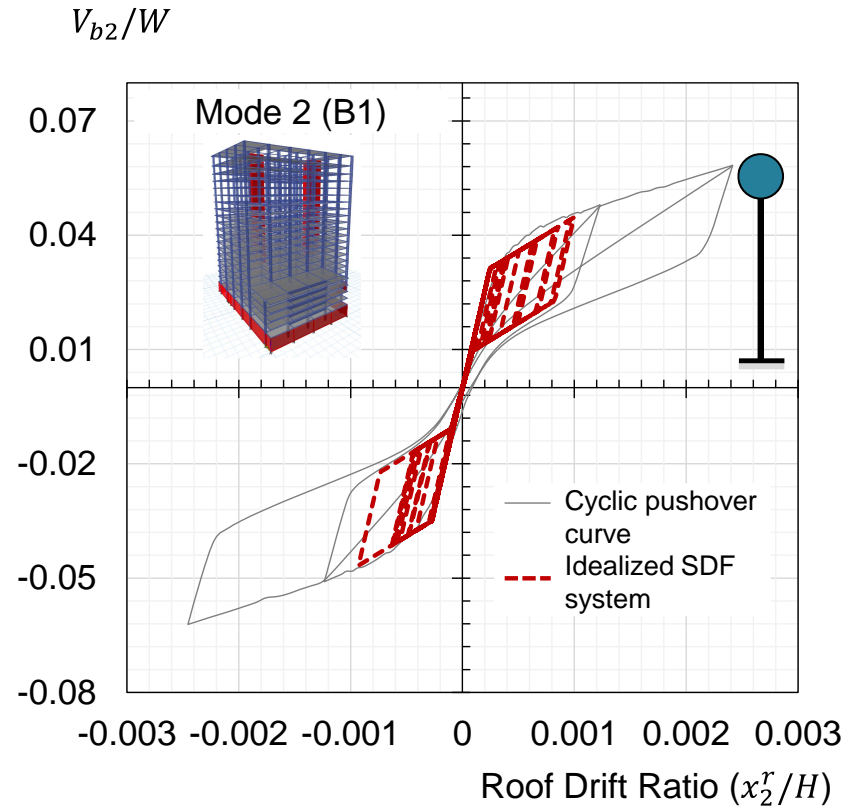
Normalized Base Shear ( $V_{b1}/W$ )



**Ground Motion Set 4**

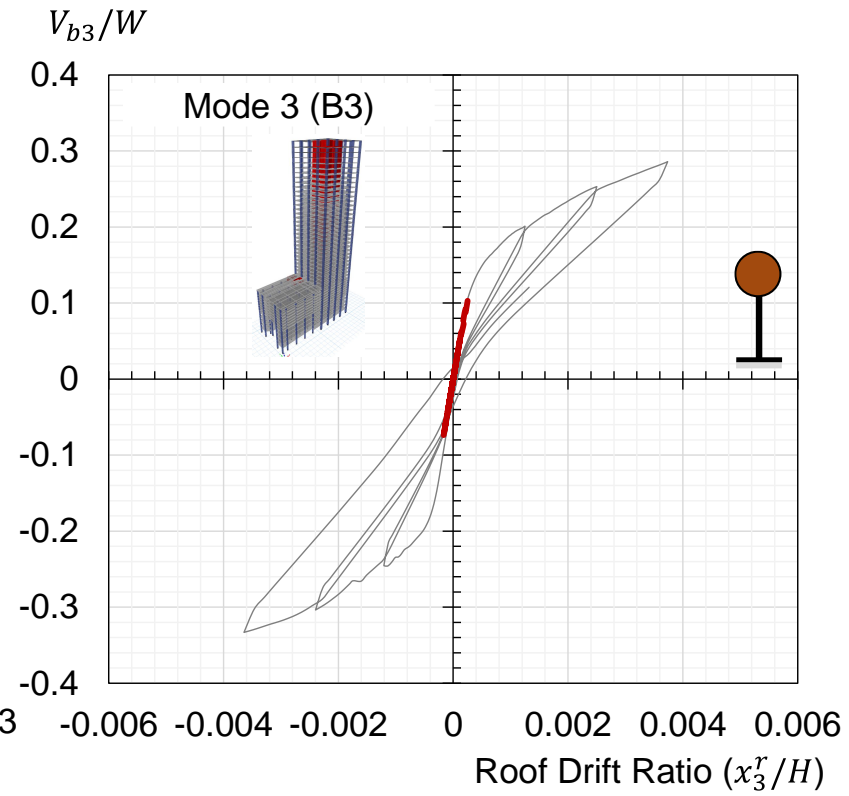
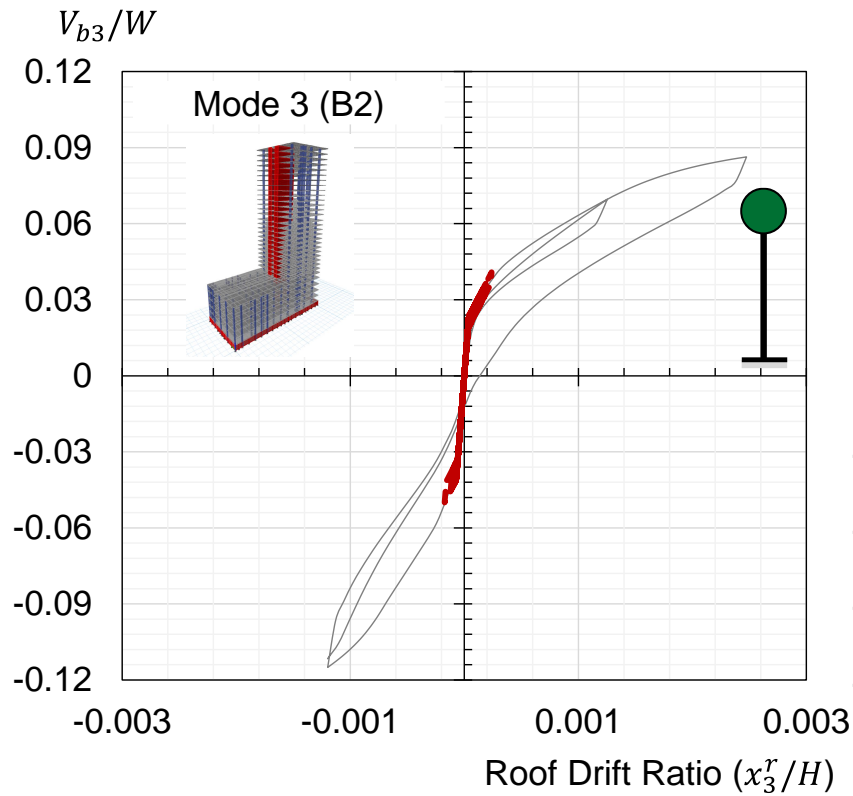
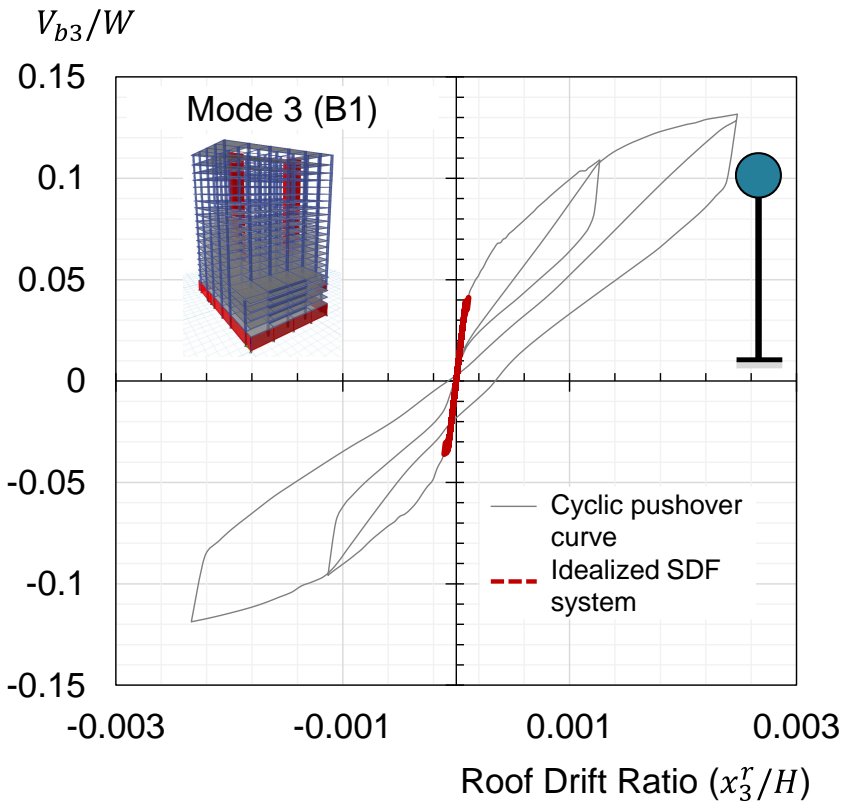


# Idealization of Cyclic Pushover Curves



**Ground Motion Set 4**

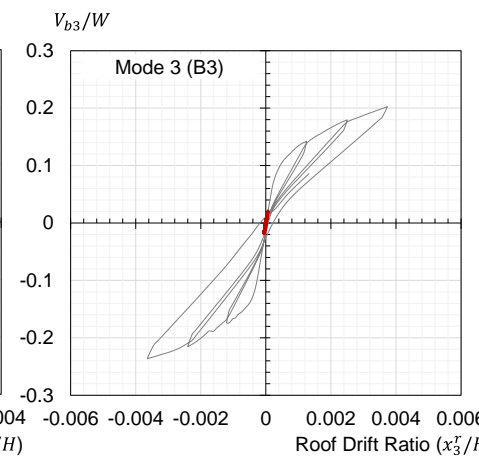
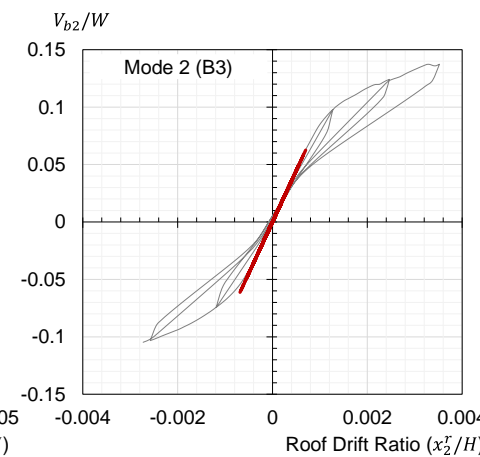
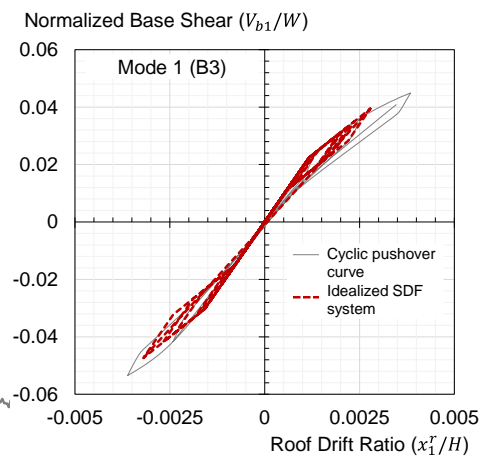
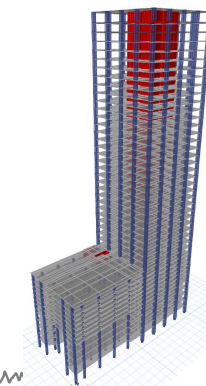
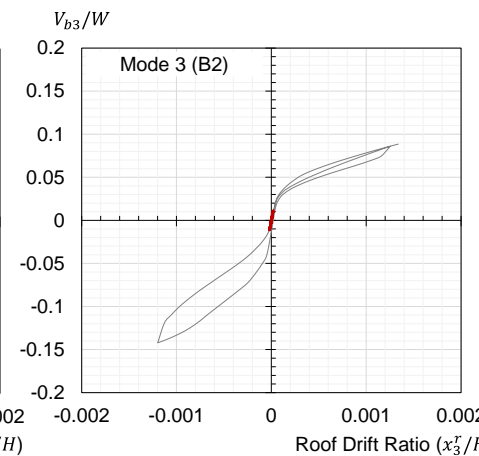
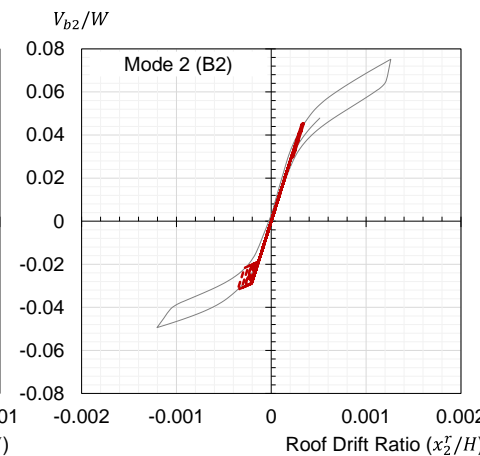
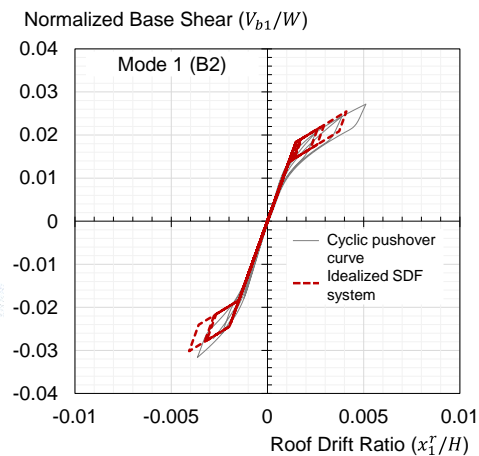
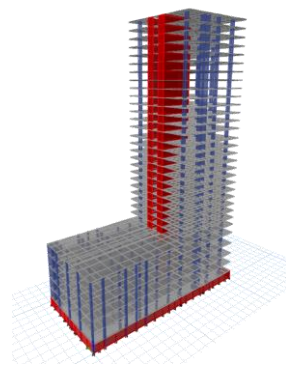
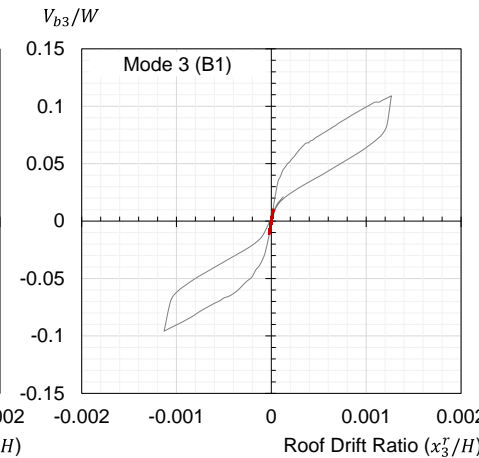
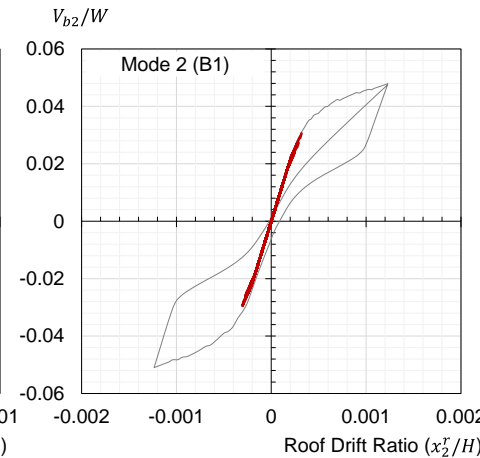
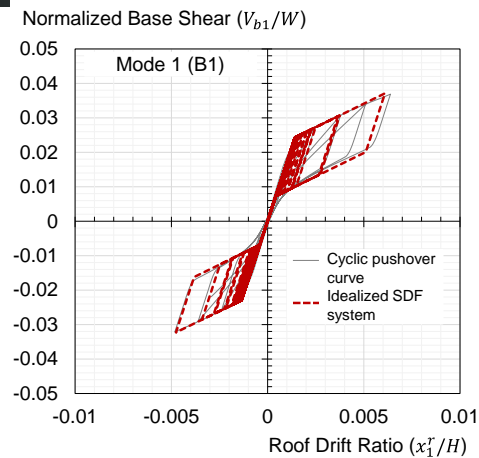
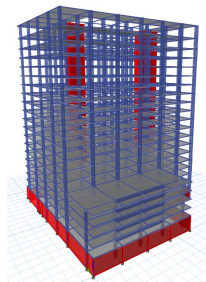
# Idealization of Cyclic Pushover Curves



Ground Motion Set 4

# The Idealization of Cyclic Pushover Curves

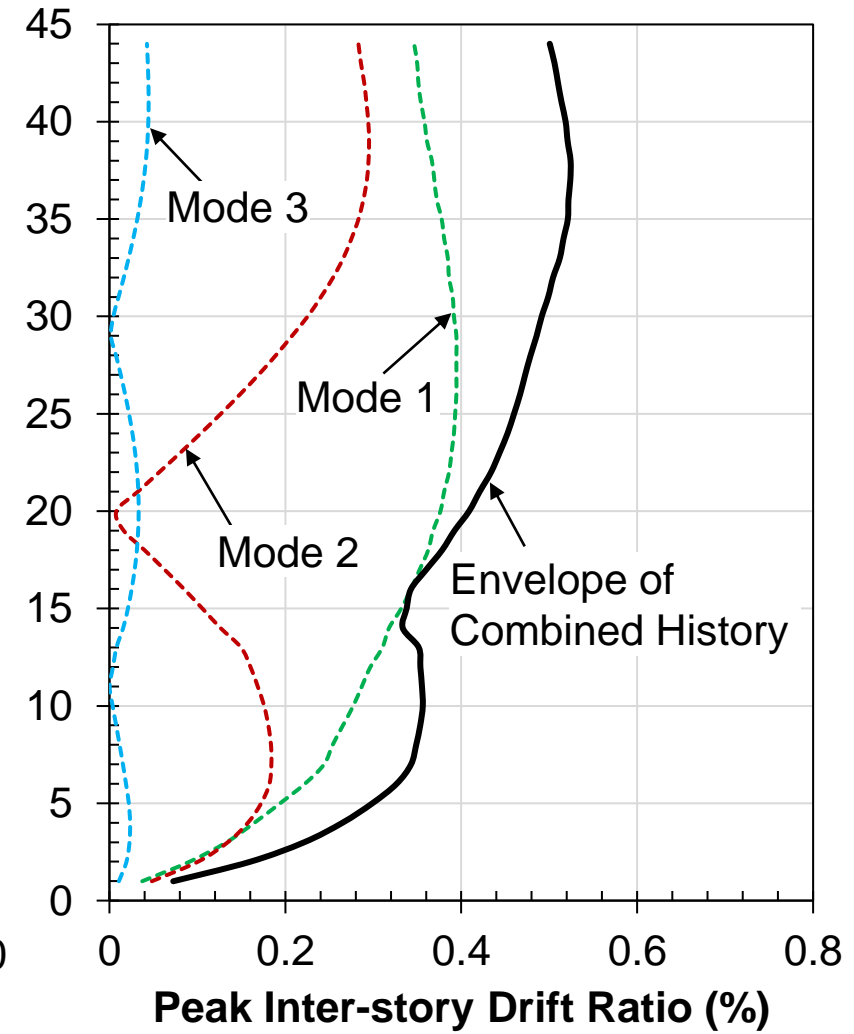
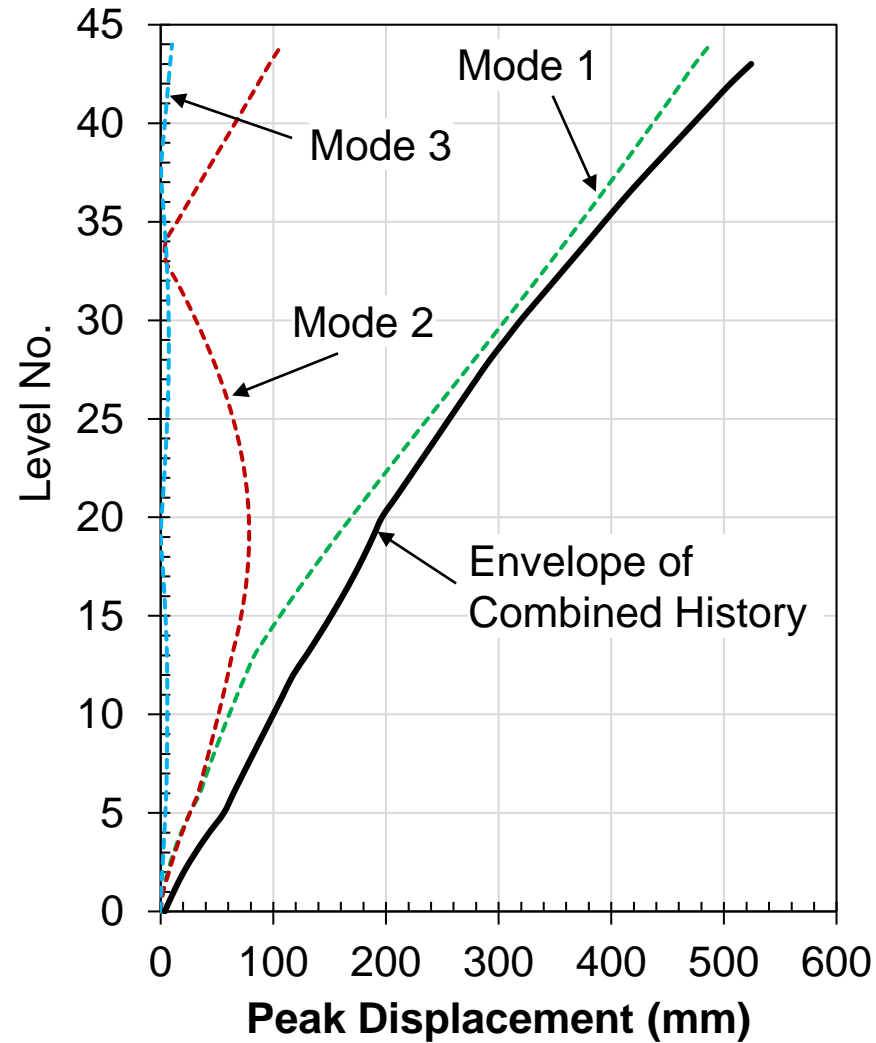
## Ground Motion Set 1



# Modal Decomposition of Nonlinear Responses

44-story case study building in Strong Direction

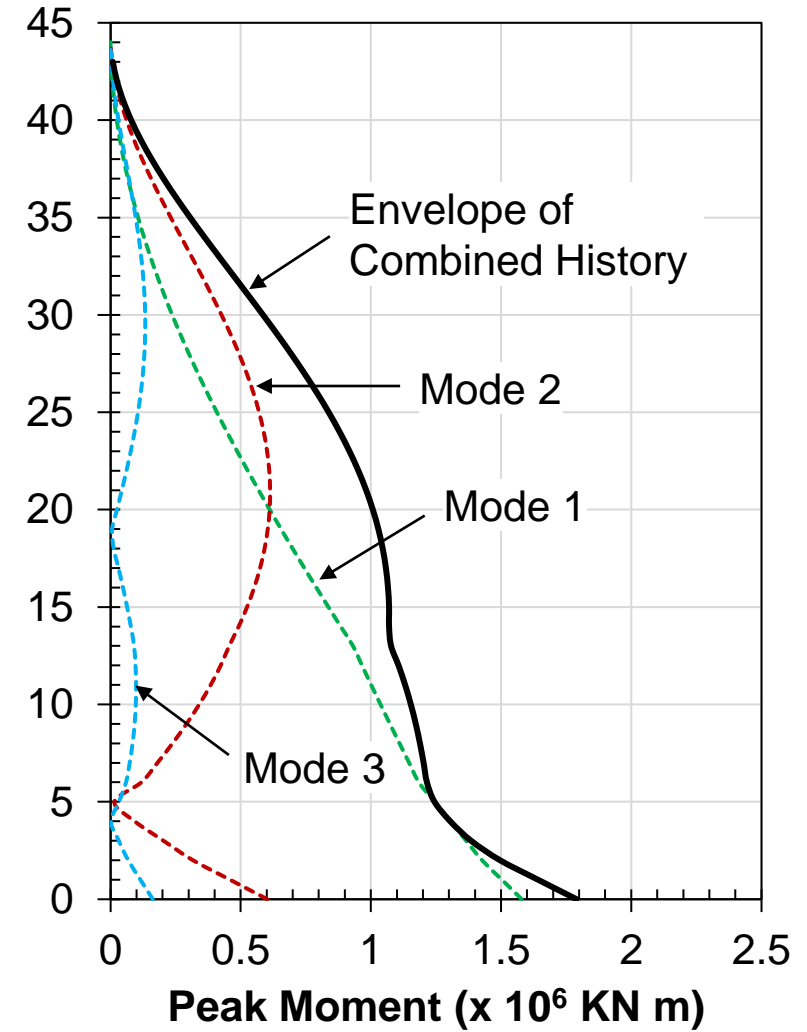
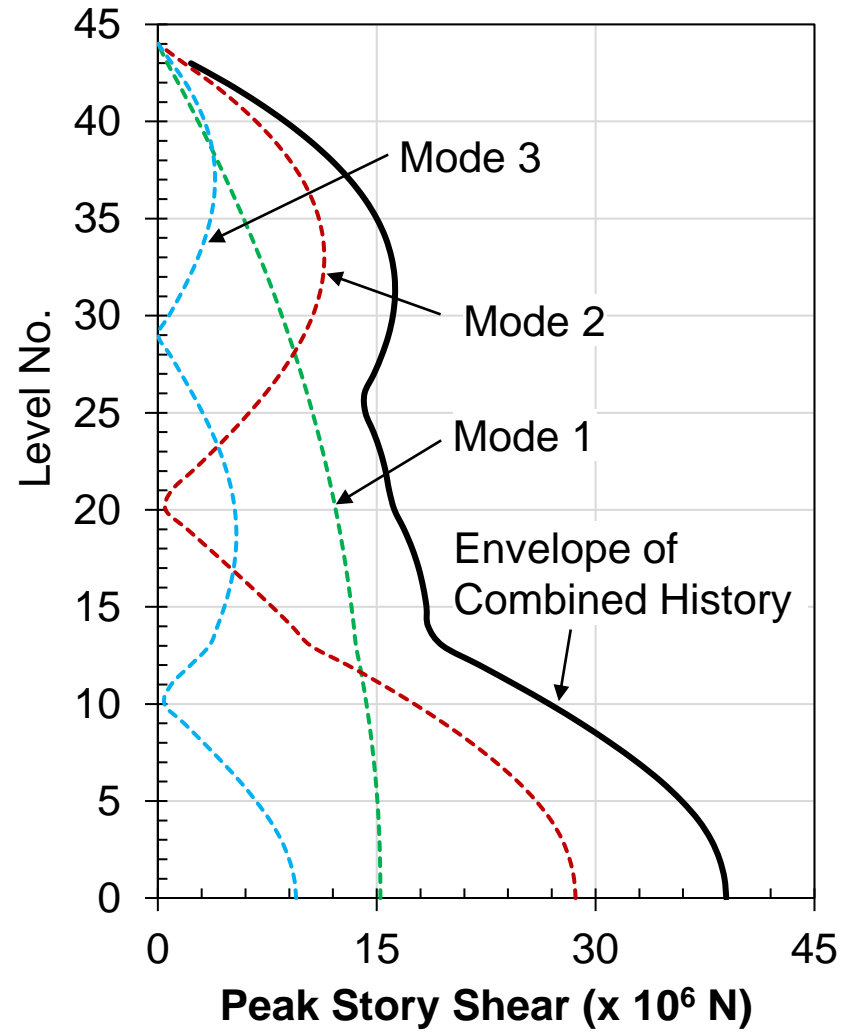
Ground Motion Set 4



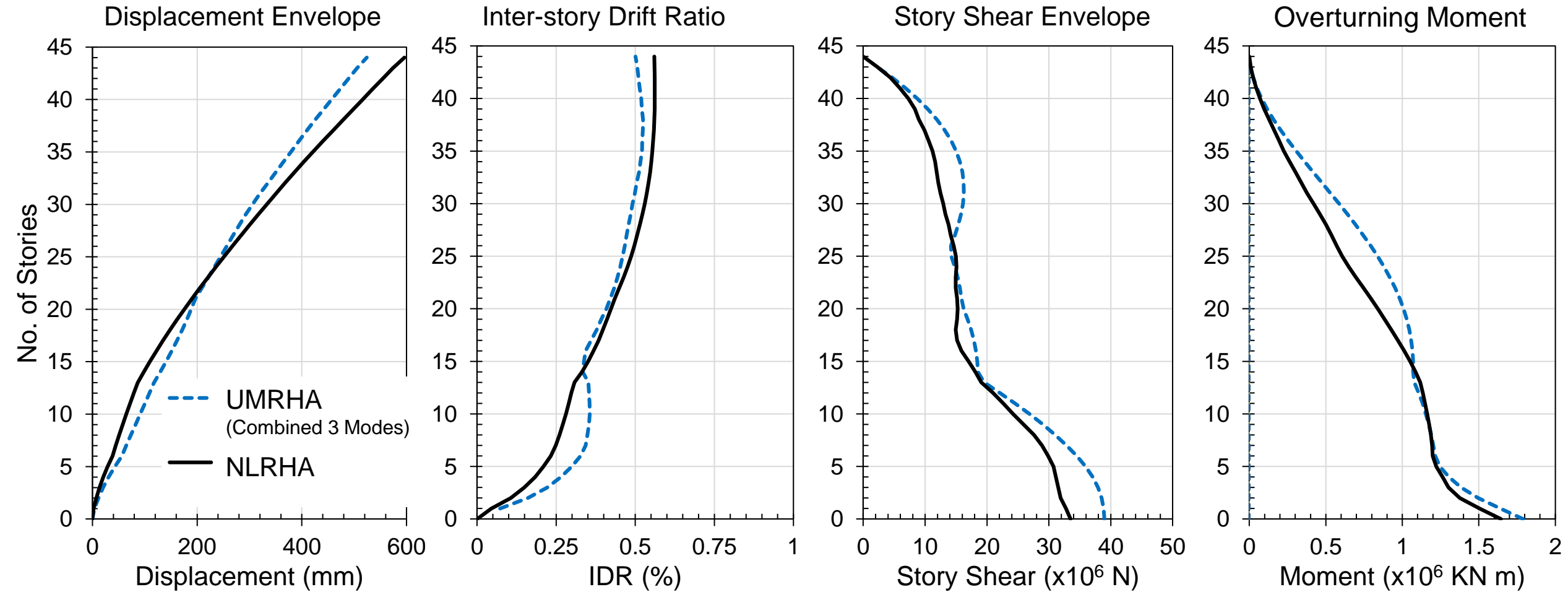
# Modal Decomposition of Nonlinear Responses

44-story case study building in Strong Direction

Ground Motion Set 4



# UMRHA vs. NLRHA



44-story case study building in Strong Direction - Ground Motion Set 4

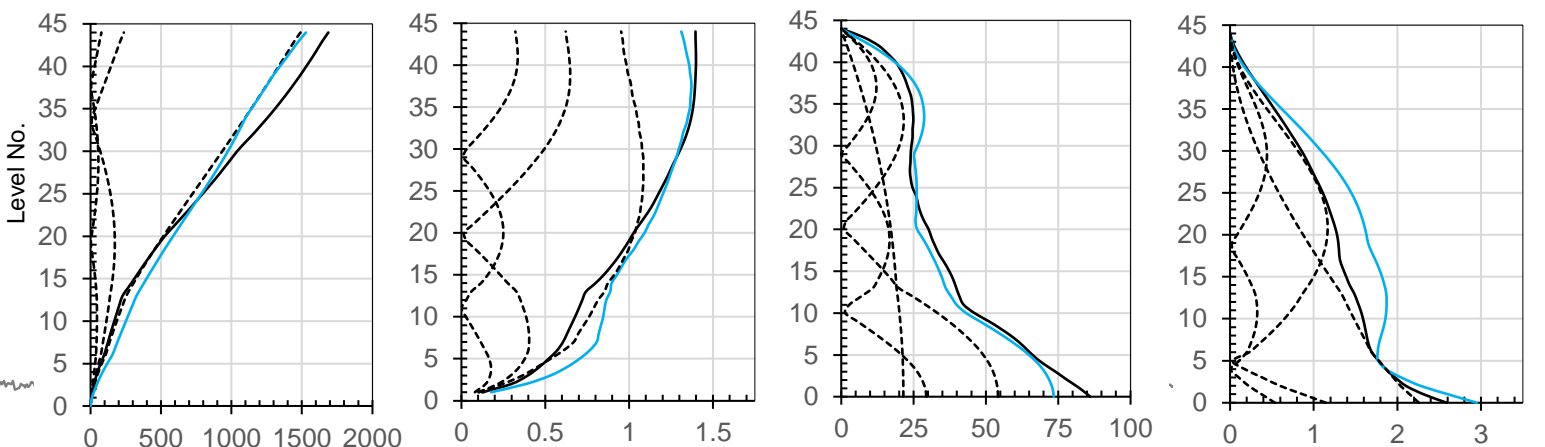
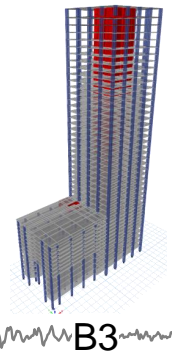
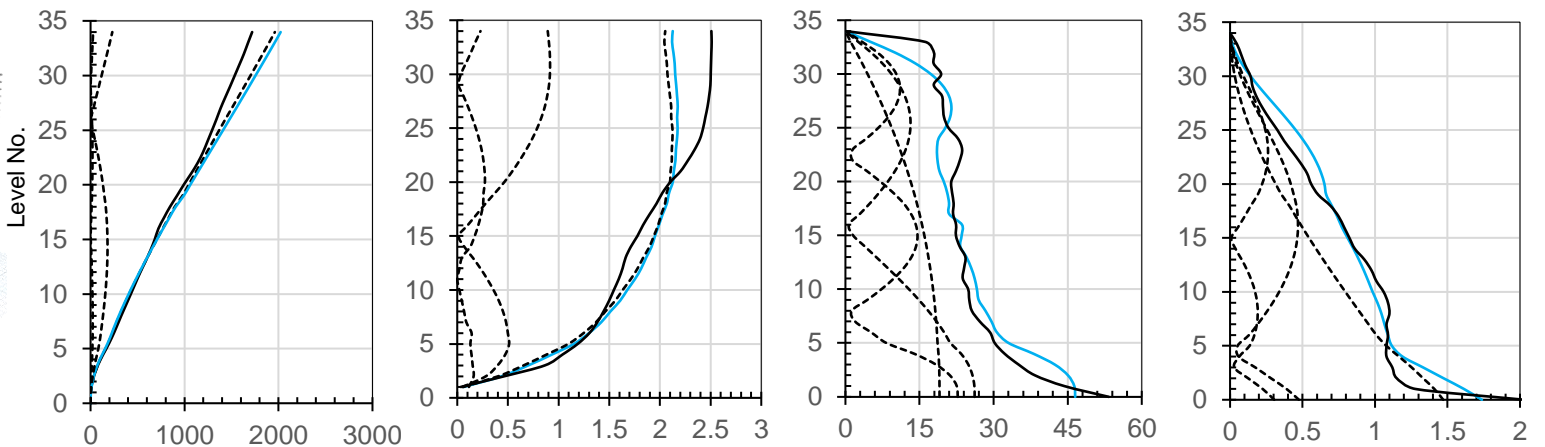
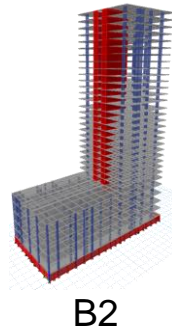
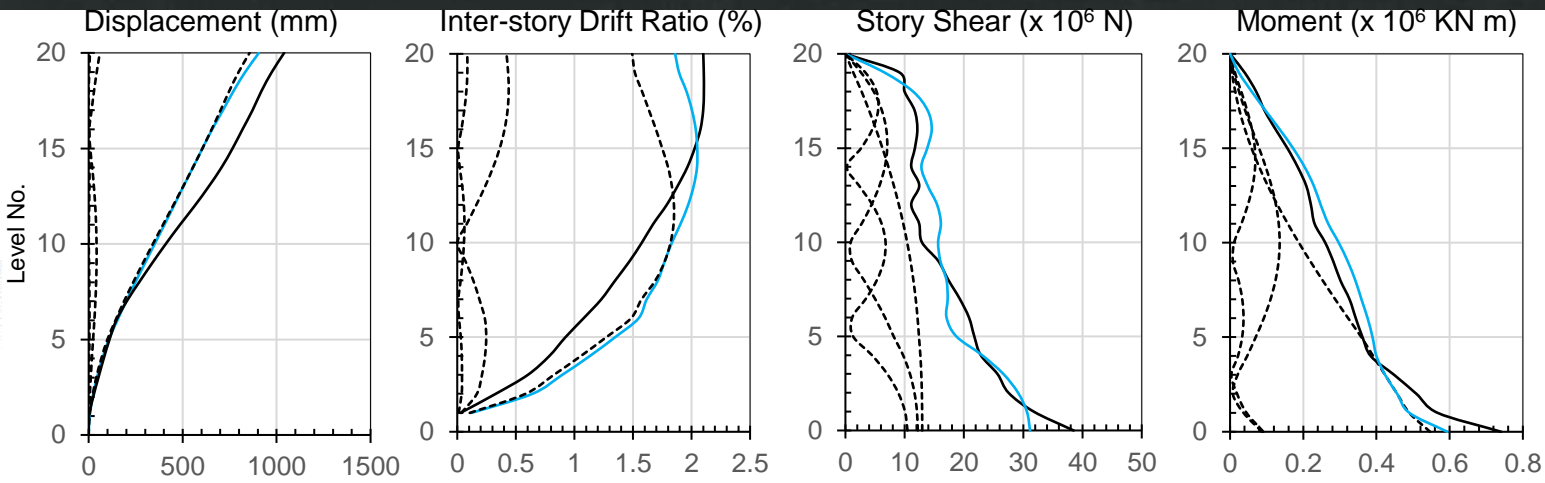
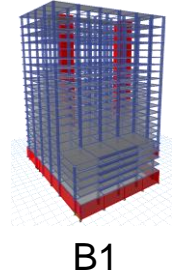


# Modal Decomposition of Nonlinear Responses

44-story case study building in Strong Direction

Ground Motion Set 4

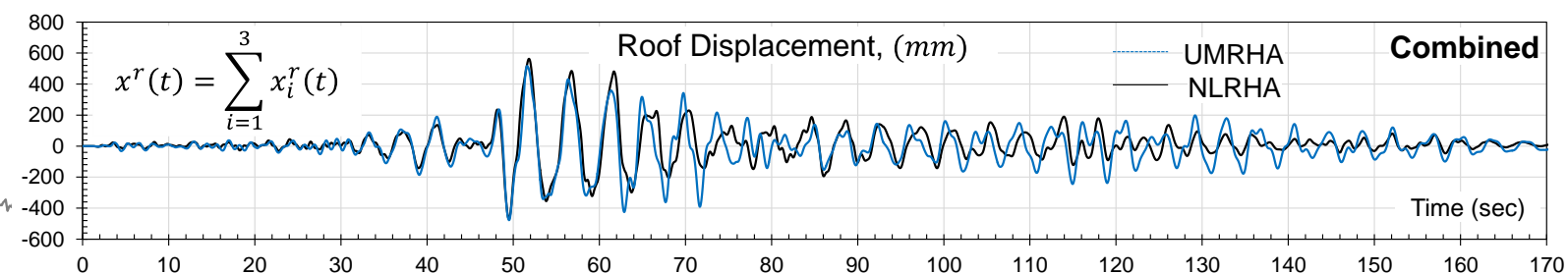
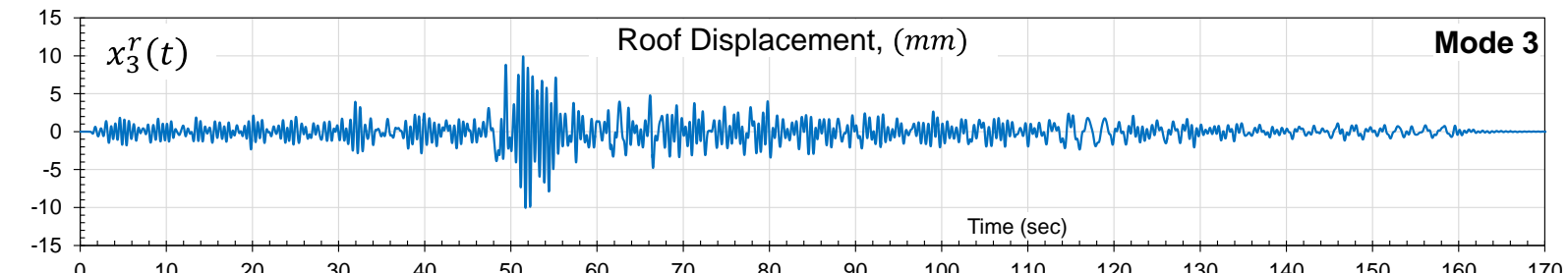
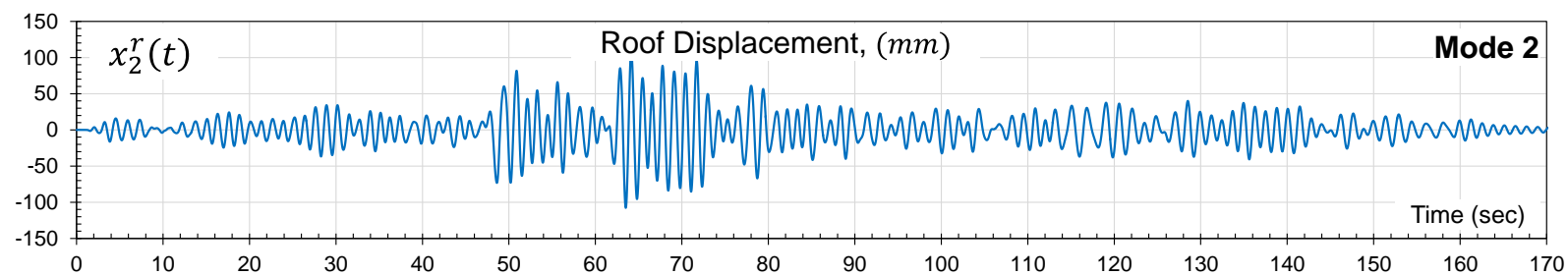
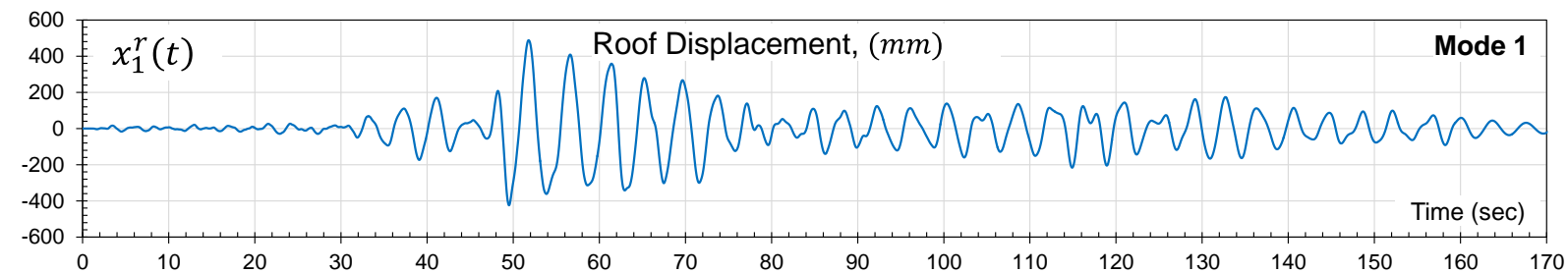
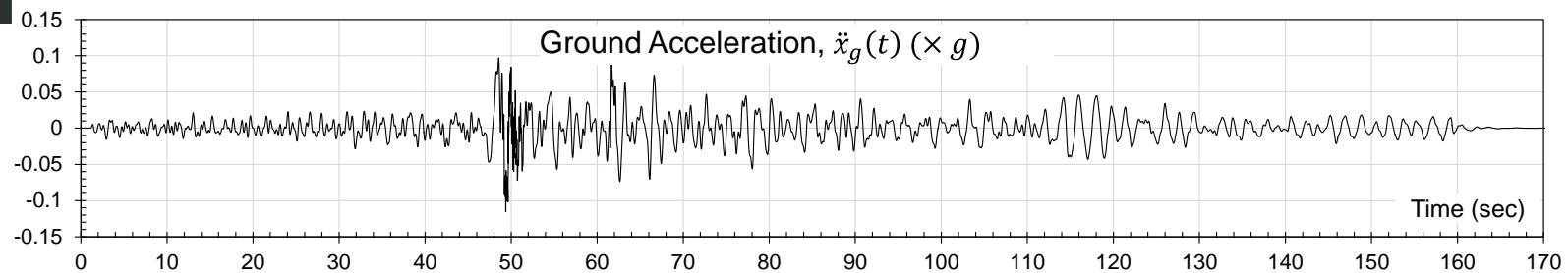
- NLRHA
- UMRHA (Combined)
- - - Individual Modes



# Modal Decomposition of Nonlinear Responses

44-story case study building in Strong Direction

Ground Motion Set 4

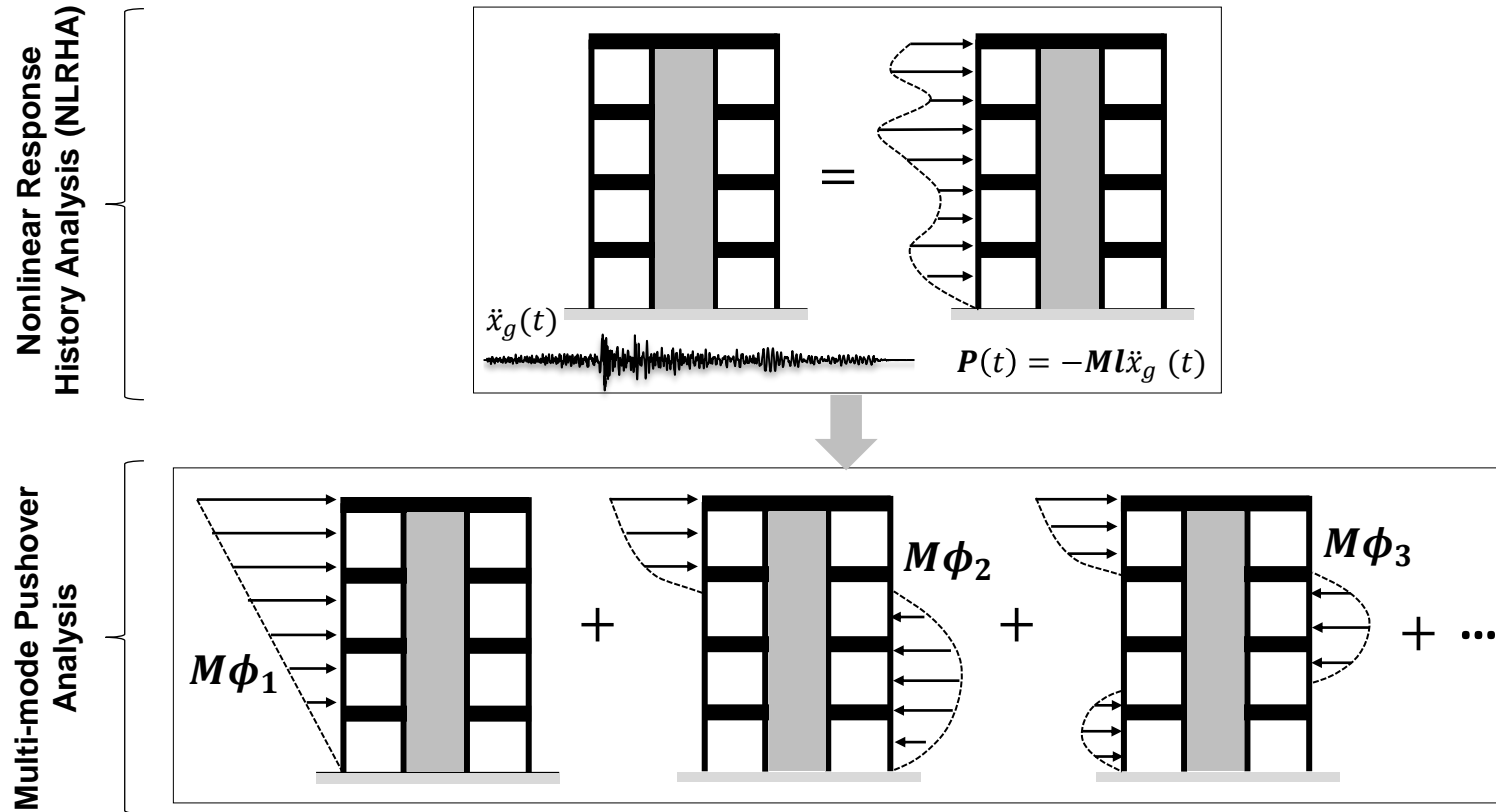


# Conclusions

- ❑ UMRHA + Accurate Modal Hysteretic Model is able to compute non-linear seismic responses of RC tall buildings with reasonable accuracy.
- ❑ The required computational effort is very low compared to that of NLRHA.
- ❑ More understanding in complex non-linear dynamic responses of tall buildings can be gained by 'Modal Decomposition' of responses.
- ❑ This allows engineers to develop effective strategies to improve the seismic performance of these buildings.

# **The Modal Pushover Analysis (MPA) Procedure (Chopra and Goel, 2002)**

# The Modal Pushover Analysis Procedure (MPA)



# The Modal Pushover Analysis Procedure (MPA)

## Linearly Elastic Systems

- The response spectrum analysis (RSA) procedure, which is a dynamic analysis procedure, can be interpreted in two ways: as static analysis or as pushover analysis.
  - a) **Static analysis** of the building subjected to lateral forces  $\mathbf{f}_n = \mathbf{s}_n A_n = \Gamma_n \mathbf{M} \boldsymbol{\phi}_n A_n$  will provide the same value of  $r_n$ , the peak value of the  $n^{th}$ -mode response  $r_n(t)$ , as obtained from the RSA procedure. (Where  $A_n = A(T_n, \xi_n)$ , the pseudo-acceleration spectrum ordinate corresponding to the natural vibration period  $T_n$  and damping ratio  $\xi_n$  of the  $n$ th mode).
  - b) Alternatively, this peak modal response can be obtained by linear static analysis of the structure subjected to **monotonically increasing lateral forces** with an invariant height-wise distribution:  $\mathbf{s}_n^* = \mathbf{M} \boldsymbol{\phi}_n$ , pushing the structure up to the roof displacement,  $u_{rn}$ .

$u_{rn}$  is the peak value of the roof displacement due to the  $n$ th mode, and is given by

$$u_{rn} = \Gamma_n \phi_{rn} D_n$$

where  $D_n \equiv D(T_n, \xi_n)$  is the ordinate of the deformation response spectrum corresponding to the period  $T_n$  and damping ratio  $\xi_n$  of the  $n$ th mode.

# The Modal Pushover Analysis (MPA) Procedure

## Linearly Elastic Systems

- The peak modal responses,  $r_n$ , each determined by one pushover analysis, can be combined according to the [modal combination rules](#) (as used in the Response Spectrum Analysis, RSA [SRSS or CQC]) to obtain an estimate of the peak value  $r$  of the total response.
- Being equivalent to the standard RSA procedure, the MPA procedure [offers no advantage for linearly elastic systems](#), but this interpretation of RSA permits extension of MPA to inelastic systems.
- Note that  $r_n$  determined by pushover analysis can also be interpreted as the peak response of the linearly elastic system to  $\mathbf{P}_{eff,n}(t)$ , the  $n$ th-mode component of the effective earthquake forces. This interpretation is valid because, the system responds only in its  $n$ th mode when subjected to this excitation.

# The Modal Pushover Analysis (MPA) Procedure

## Inelastic Systems

- The peak response  $r_n$  of the inelastic system to  $\mathbf{P}_{eff,n}(t)$  is also determined by a **pushover analysis**, which is now a **nonlinear static analysis** instead of a linear static analysis, of the structure subjected to lateral forces distributed over the building height according to  $\mathbf{s}_n^*$  with the forces increased to push the structure up to roof displacement  $u_{rn}$ .

$$u_{rn} = \Gamma_n \phi_{rn} D_n$$

$D_n$  is now the peak deformation of the **nth-mode inelastic SDF system** (instead of the nth-mode elastic SDF system).

$$\ddot{D}_n(t) + 2\xi_n \omega_n \dot{D}_n(t) + \frac{F_{sn}}{L_n} = -\ddot{u}_g(t)$$

- At  $u_{rn}$ , the results of **nonlinear static analysis** provide an estimate of the peak value  $r_n$  of the response quantity  $r_n(t)$ : floor displacements, story drifts, and other deformation quantities.



# The Modal Pushover Analysis (MPA) Procedure

## Inelastic Systems

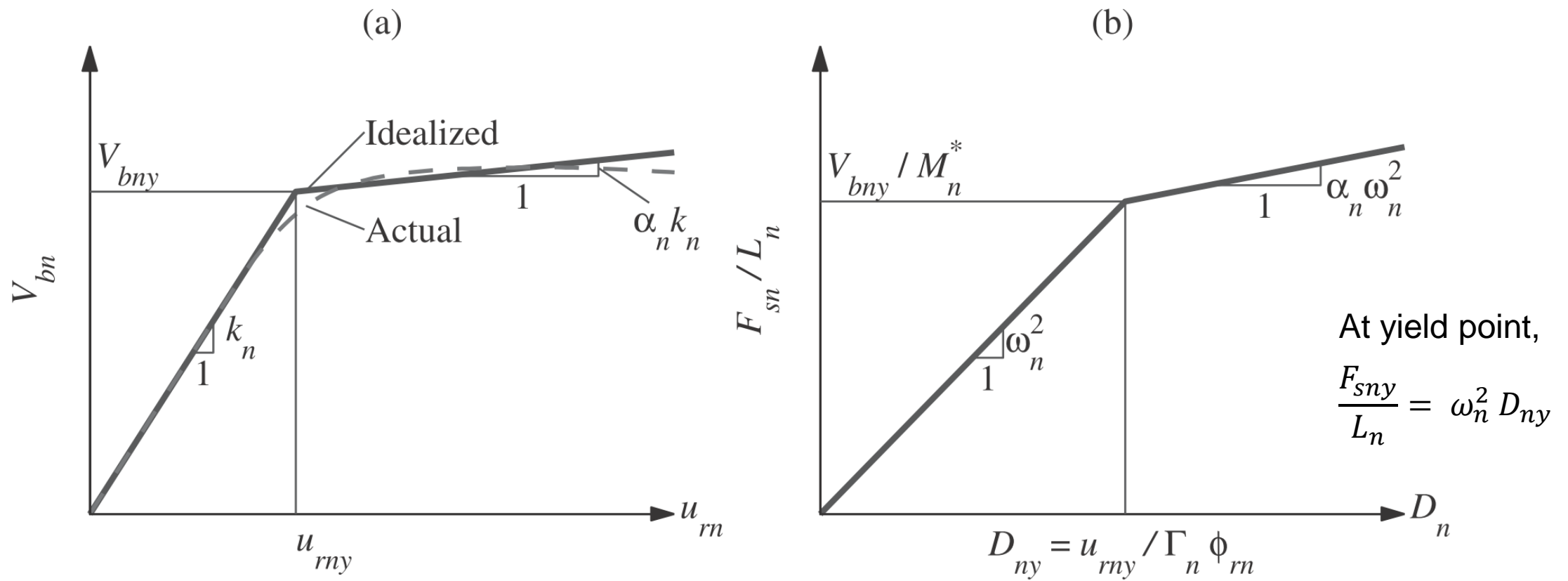
- Nonlinear static analysis using force distribution  $s_n^*$  leads to the nth-mode pushover curve, a plot of **base shear**  $V_{bn}$  versus roof displacement  $u_{rn}$ .
- From the nth-mode pushover curve is obtained the force–deformation  $(F_{sn}/L_n - D_n)$  curve for the nth-mode inelastic SDF system, which is required to determine  $D_n$  from the following equation.

$$\ddot{D}_n(t) + 2\xi_n\omega_n\dot{D}_n(t) + \frac{F_{sn}}{L_n} = -\ddot{u}_g(t)$$

- The forces and displacements in the two sets of curves are related as follows:

$$D_n = \frac{u_{rn}}{\Gamma_n\phi_{rn}} \qquad \frac{F_{sn}}{L_n} = \frac{V_{bn}}{M_n^*}$$

Where,  $M_n^* = L_n\Gamma_n$  is the effective modal mass.



**Figure 20.7.2** (a) An  $n$ th-mode pushover curve and its bilinear idealization; (b) force–deformation relation for the  $n$ th-mode inelastic SDF system.

## UMRHA vs. MPA

- The response value  $r_n$  determined by pushover analysis is an estimate of the peak value of the response  $r_n(t)$  of the inelastic structure to  $\mathbf{P}_{eff, n}(t)$ , but it is **not identical** to another estimate determined by UMRHA.
- For elastic systems UMRHA = Modal RHA, and MPA = RSA.
- For inelastic systems the two—UMRHA and MPA—estimates of the peak modal response are both approximate and **different from each other**; the only exception is the roof displacement because it is deliberately matched in the two analyses.

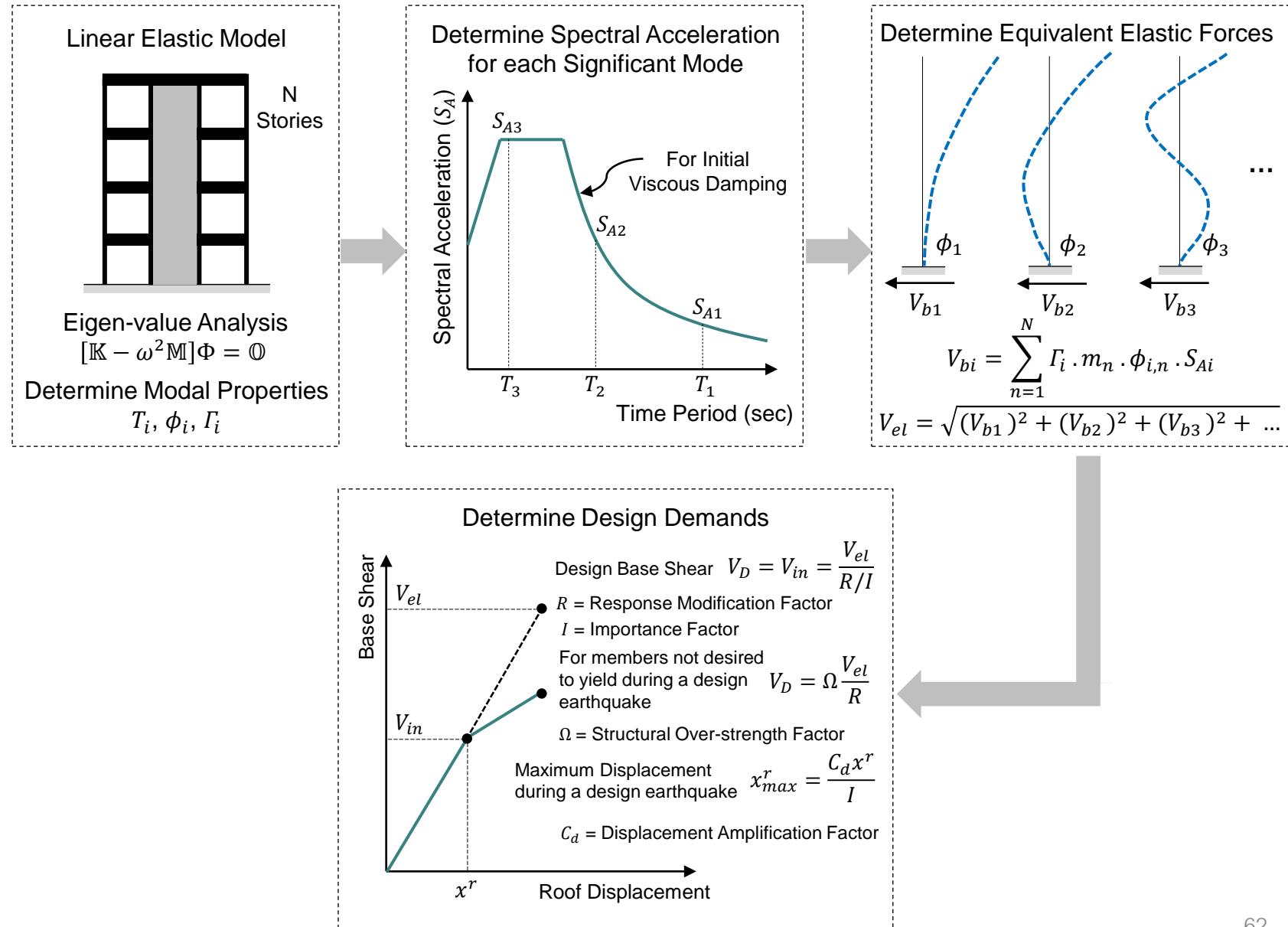
## UMRHA vs. MPA Procedures

- The two estimates **differ** because the underlying analyses involve different assumptions.
- The UMRHA is based on the approximation contained in  $\mathbf{u}_n(t) \simeq \boldsymbol{\phi}_n q_n(t)$ , which is avoided in MPA because the floor displacements, story drifts, and other deformation quantities are determined by nonlinear static analysis using force distribution  $\mathbf{s}_n^*$ . As a result, the floor displacements of the inelastic system are **no longer proportional** to the nth-mode shape, in contrast to  $\mathbf{u}_n(t) \simeq \boldsymbol{\phi}_n q_n(t)$ . **In this sense, the MPA procedure represents the nonlinear behavior of the structure better than UMRHA.**
- However, the MPA procedure contains a different source of approximation, which does not exist in UMRHA. The peak modal responses  $r_n$ , each determined by one nonlinear static analysis, are combined by a **modal combination rule**, just as in RSA of linearly elastic systems. This application of modal combination rules to inelastic systems lacks a rigorous theoretical basis, but seems reasonable because the modes are only weakly coupled.

# A Modified Response Spectrum Analysis (MRSA) Procedure

# The Standard RSA Procedure (ASCE 7-10, IBS 2012, EC 8)

## Primary Motivation



# Criticisms on the RSA Procedure

(a) Inelastic Demands =  $\frac{\text{Elastic Demands}}{\text{Some Factor}}$

(b) Inelastic Demands of Each Mode =  $\frac{\text{Elastic Demands of That Mode}}{\text{Same Factor}}$

(c) Statistical Combination of Modal Demands

(d) Static Analysis Procedure (or Pseudo-dynamic, to say the least)

Newmark & Hall (1982)  
Eibl & Keintzel (1988)  
Bertero (1986)  
Miranda & Bertero (1994)  
ATC 19 (1995)  
ATC-34 (1995)  
Cuesta & Aschheim (2001)  
Priestley & Amaris (2002)  
Foutch & Wilcoski (2005)  
Sullivan, Priestley & Calvi (2008)  
Maniatakis et al. (2013)

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. .  
. . .

# Criticisms on the RSA Procedure

*“Ray Clough and I regret we created the approximate response spectrum method for seismic analysis of structures in 1962.”*

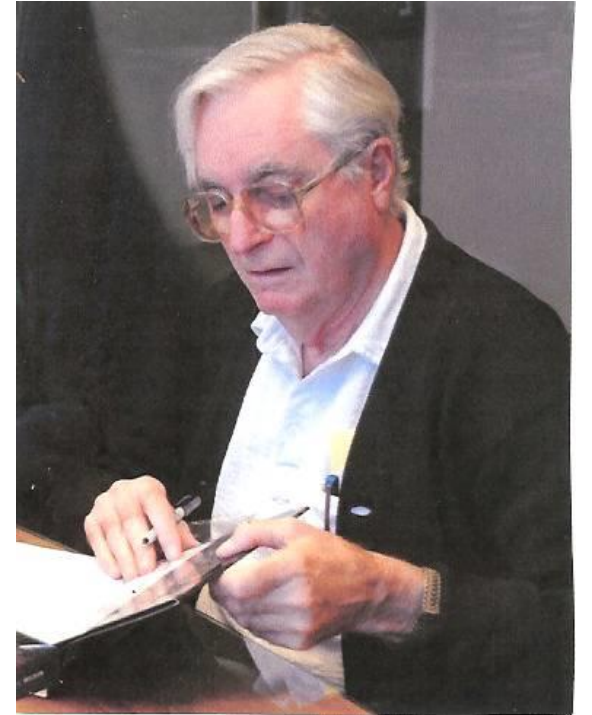
*“... allowed engineers to produce meaningless positive numbers of little or no value.”*

*“Do not be called a Neanderthal man.”*

## **CASE CLOSED**

*The use of the Response Spectrum Method in Earthquake Engineering must be terminated.*

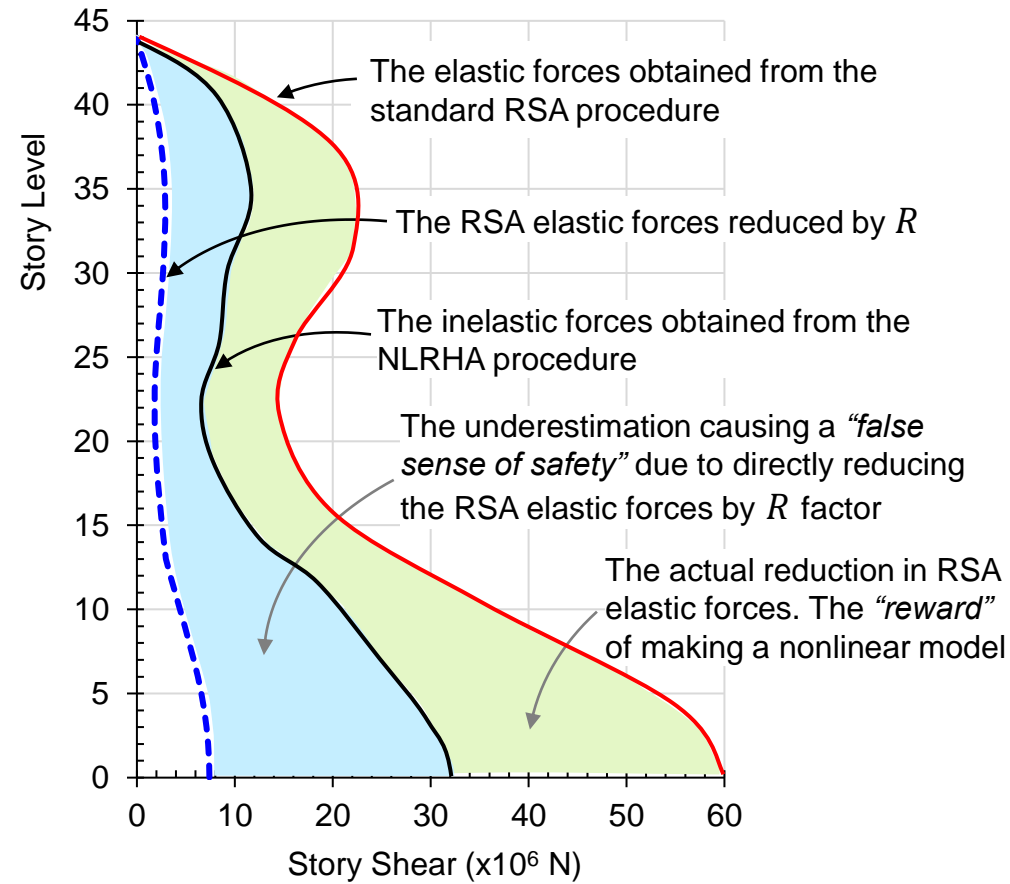
*It is not a dynamic analysis method – The results are not a function of time.*



– Edward L. Wilson  
Professor Emeritus (CEE, UC Berkeley)



# The Problem with R Factor



# The Basic Concept of the Modified Response Spectrum Analysis (MRSA)

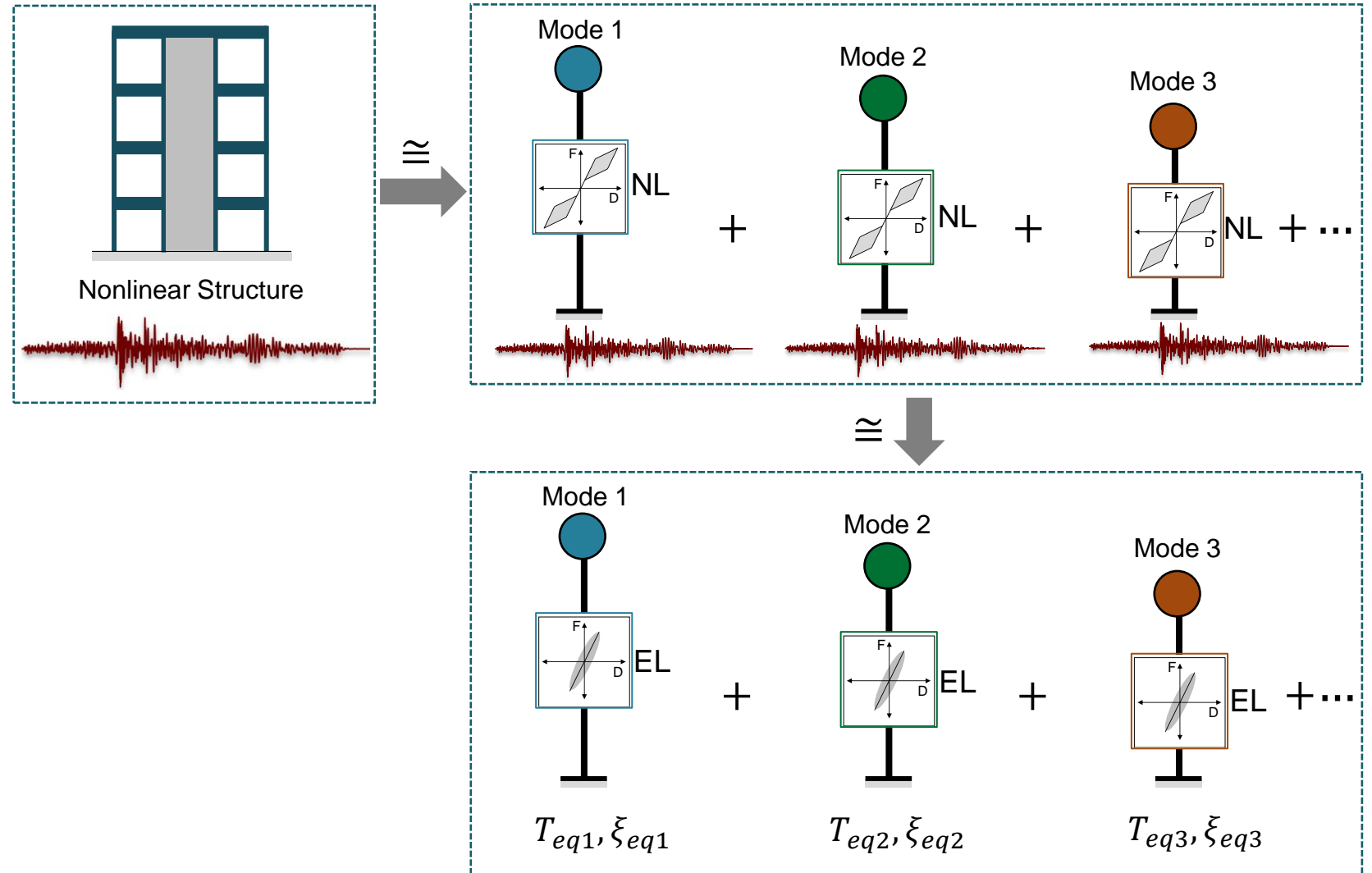
NLRHA



UMRHA

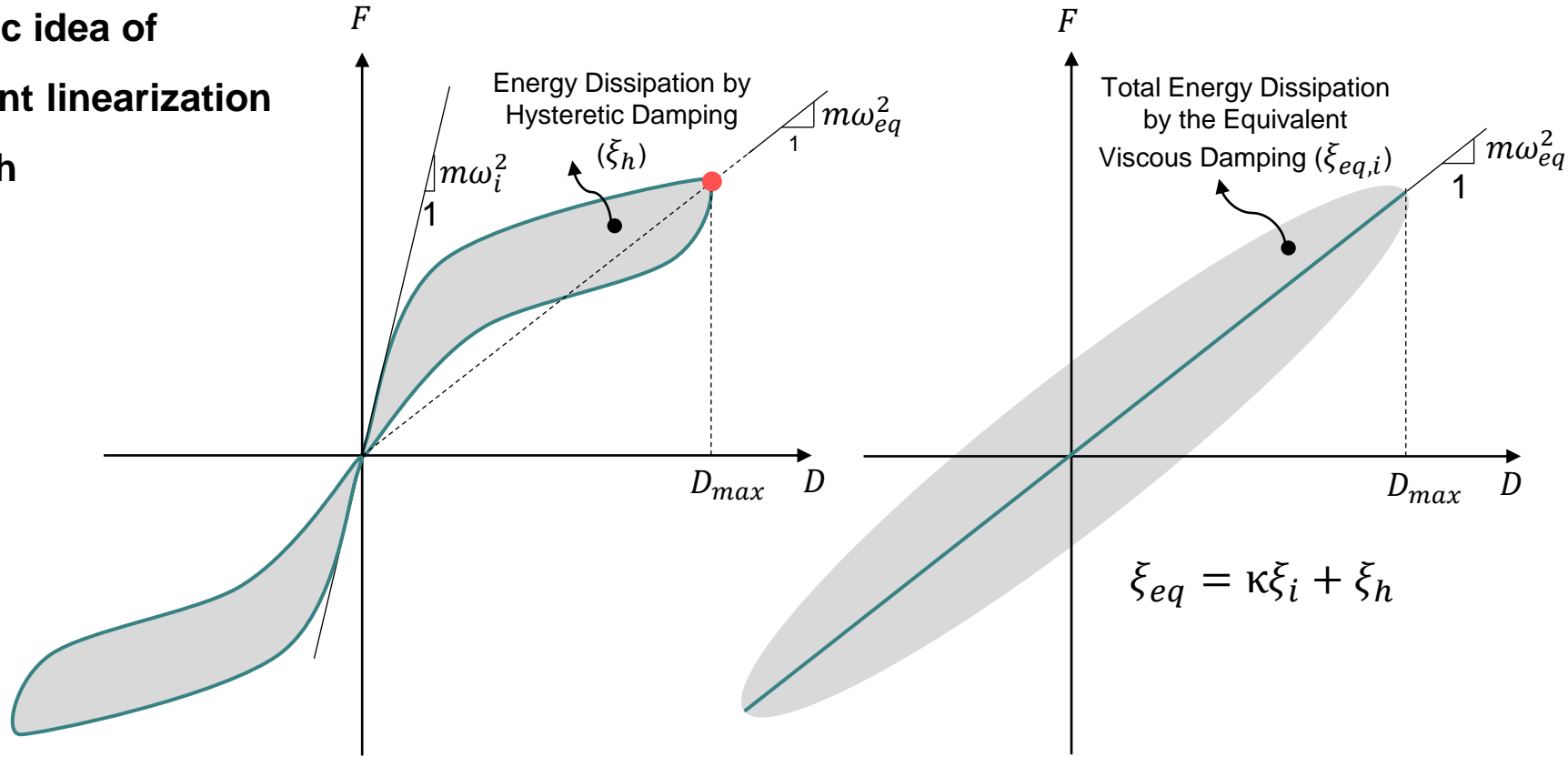


MRSA



# Conversion of a Nonlinear SDF System into an “Equivalent Linear” SDF System

The basic idea of equivalent linearization approach



A nonlinear SDF system with initial circular natural frequency  $\omega_i$ , and with initial inherent viscous damping  $\xi_i$

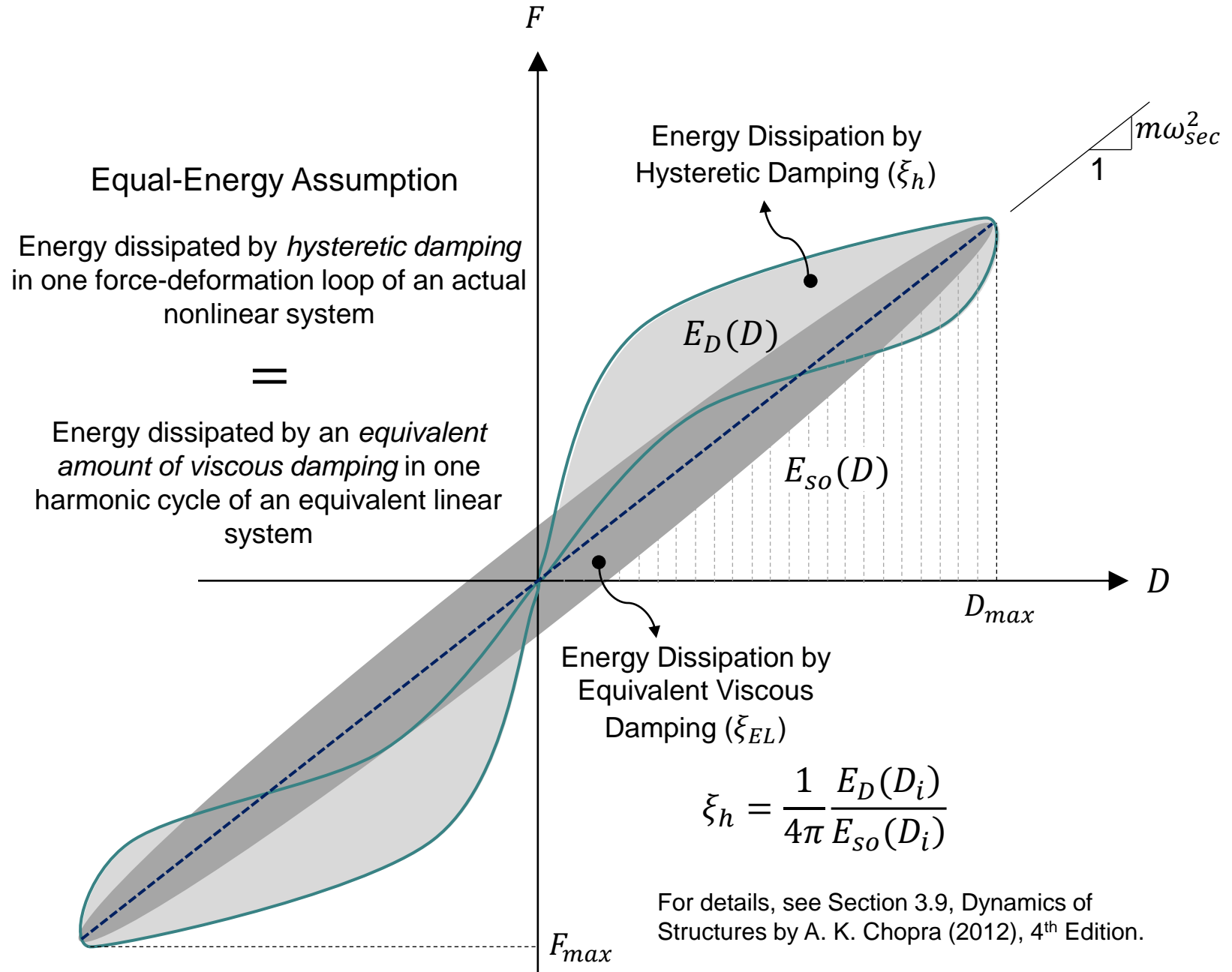


An Equivalent Linear System with Elongated Period and Additional Damping

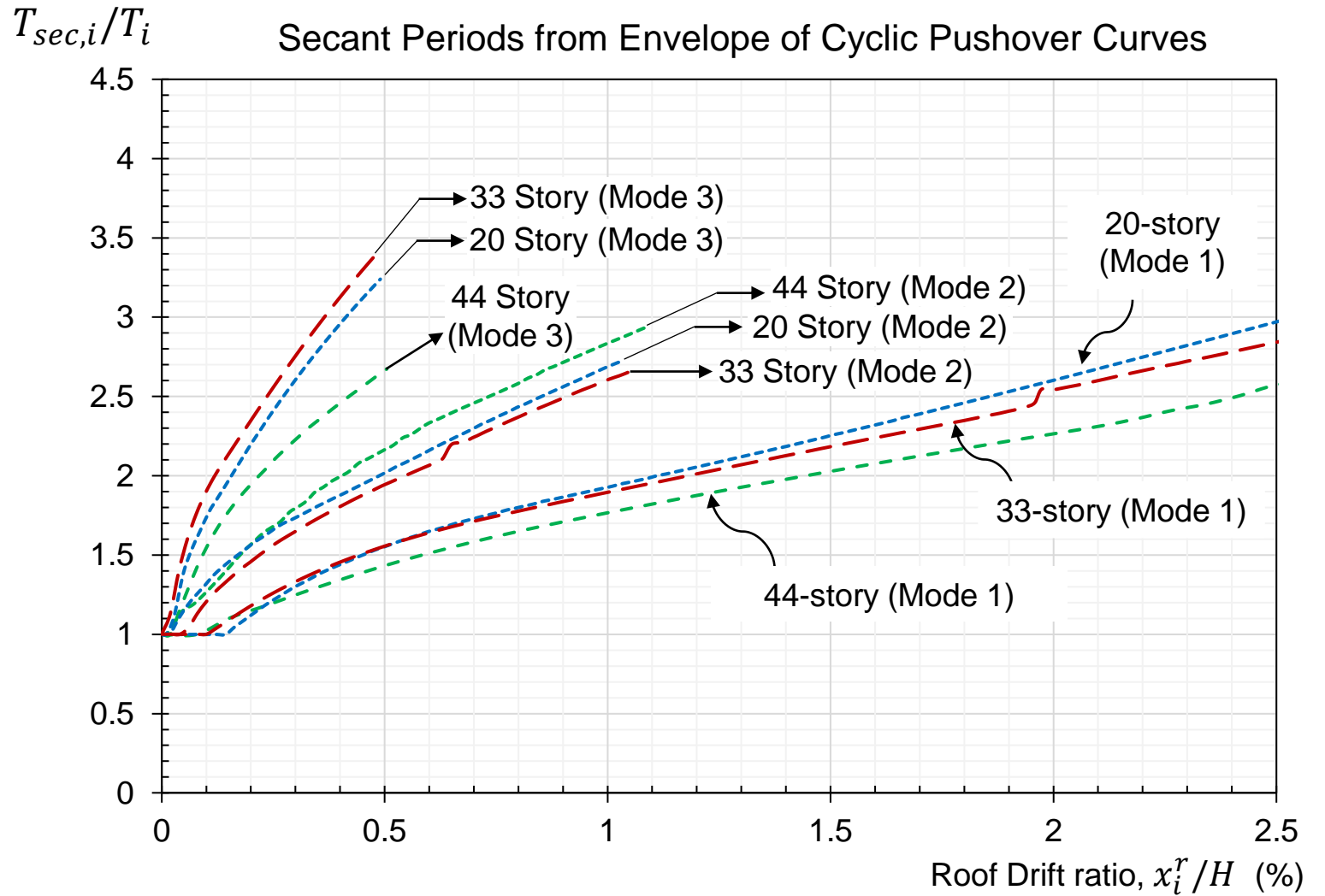
$$T_{eq} = \frac{2\pi}{\omega_{eq}}$$

$\xi_{eq}$  and  $T_{eq}$  are the “equivalent linear properties”

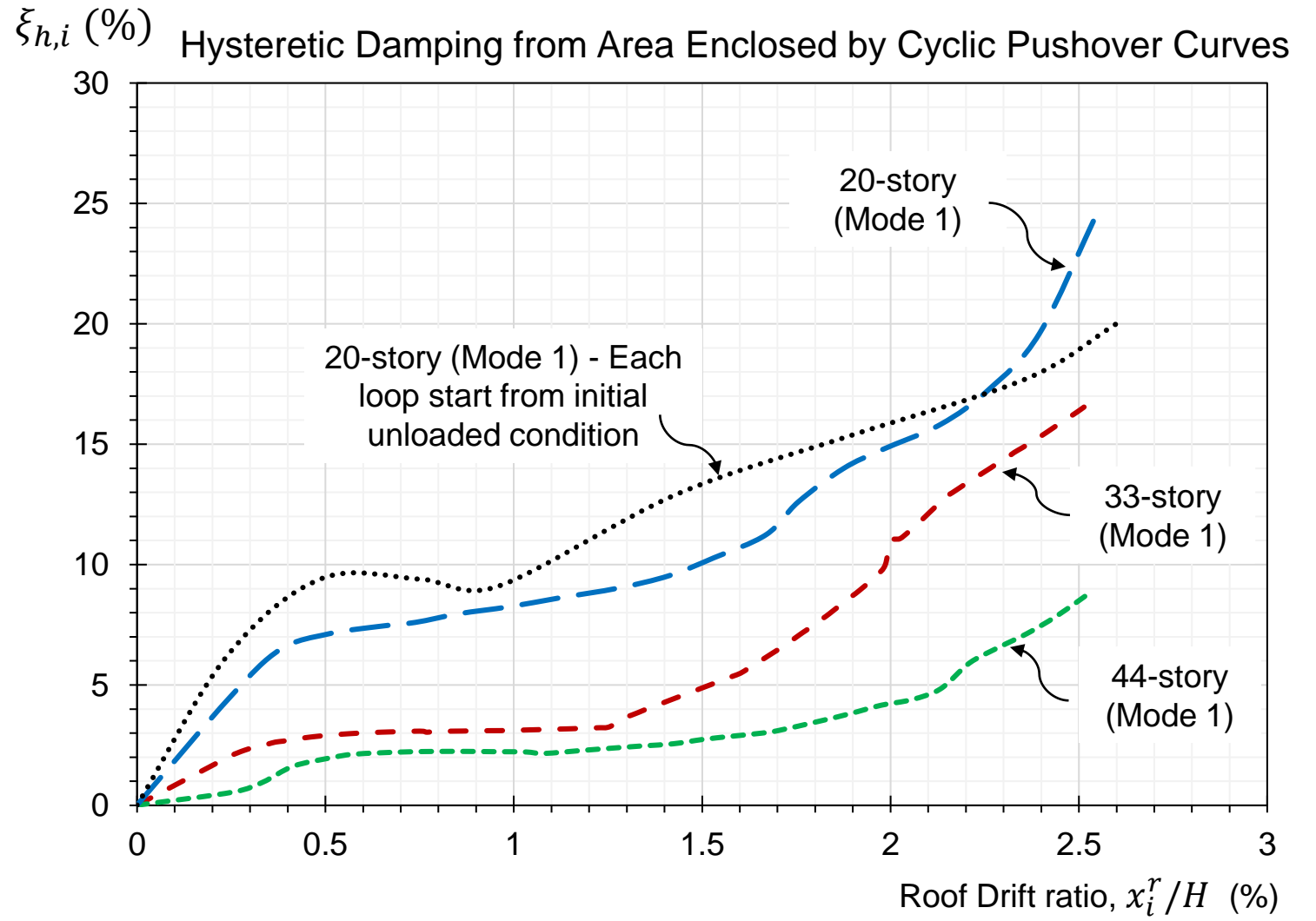
# Determination of Hysteretic Damping – Equal-energy Dissipation Rule



**“Equivalent Linear”  
Properties**



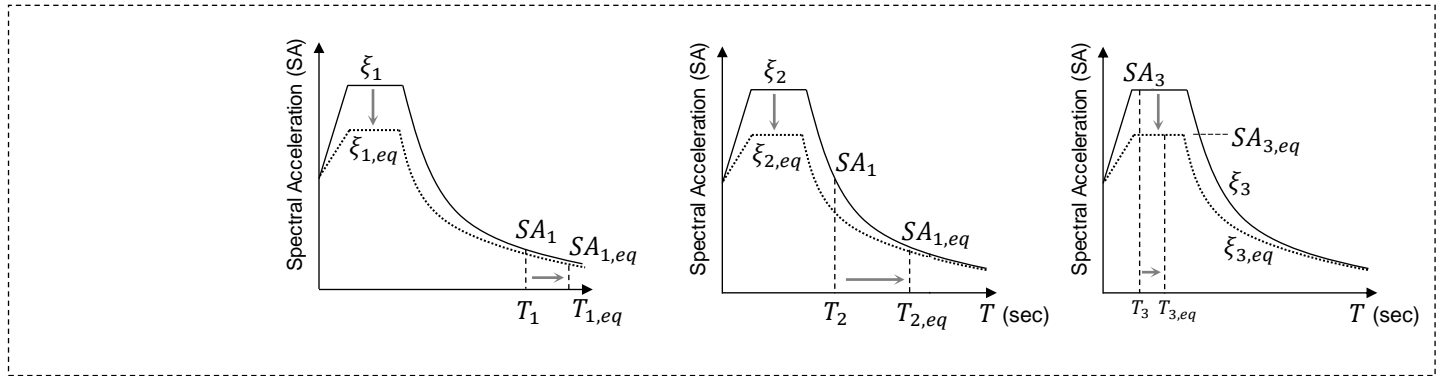
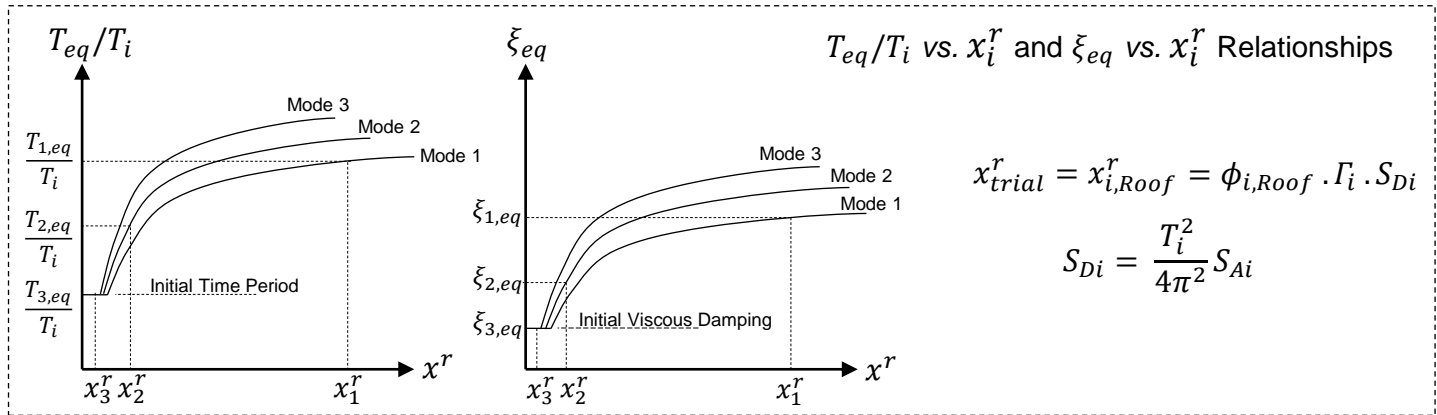
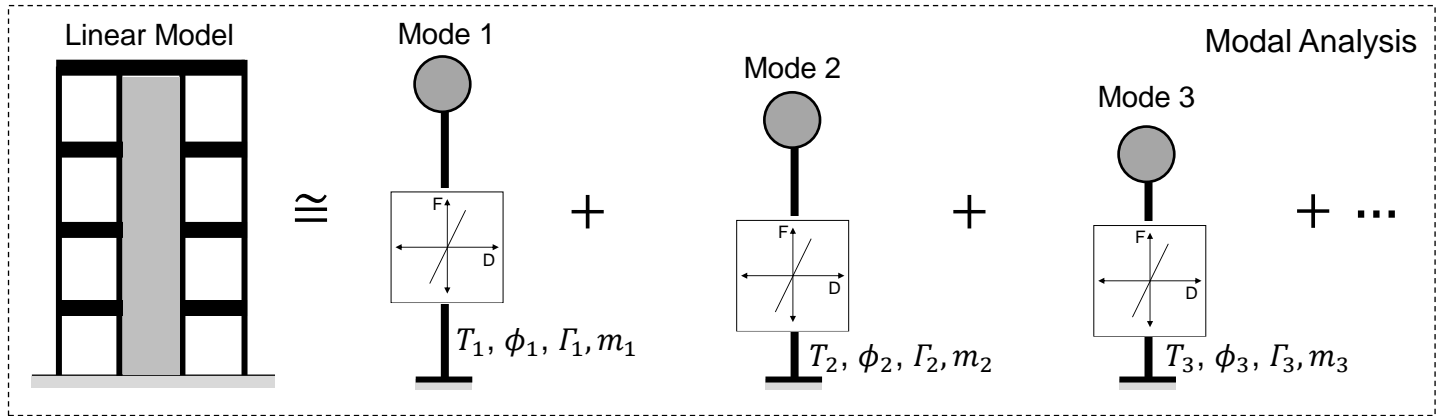
# “Equivalent Linear” Properties



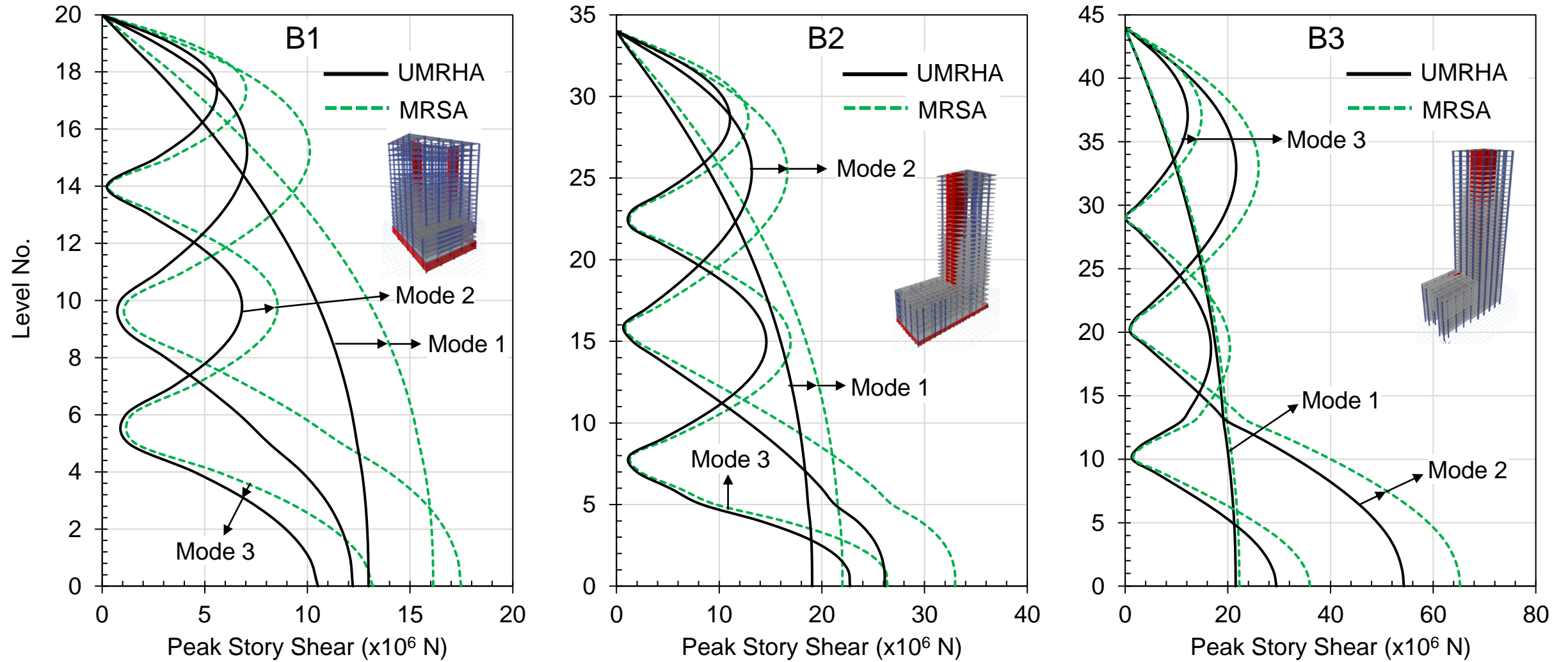
(b)  $\xi_{h,i}$  vs. Roof Drift Ratio ( $x_i^r/H$ )

# The Modified Response Spectrum Analysis (MRSA)

What is "Modified"?



# The MRSA Procedure – Individual Modal Demands – Ground Motion Set 4

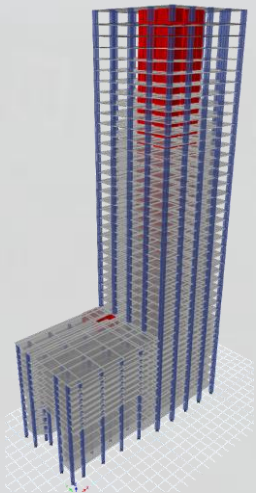




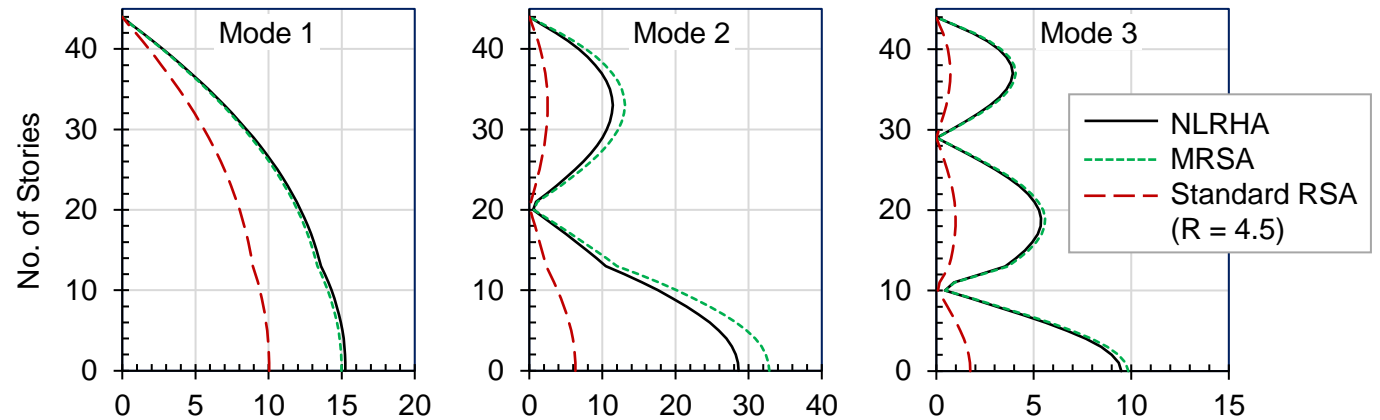
# Performance of the MRSA Procedure – Individual Modal Demands

Ground Motion Sets 1, 2 and 3

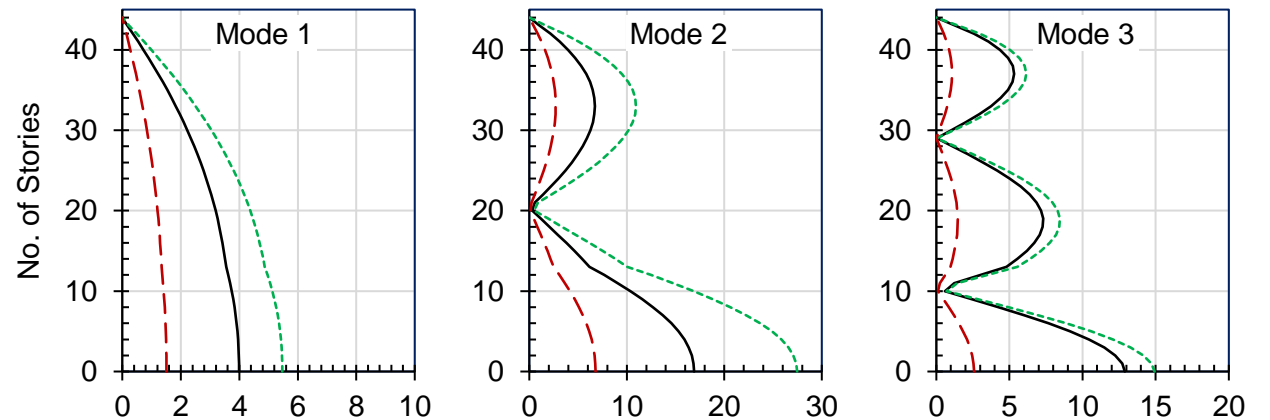
B3



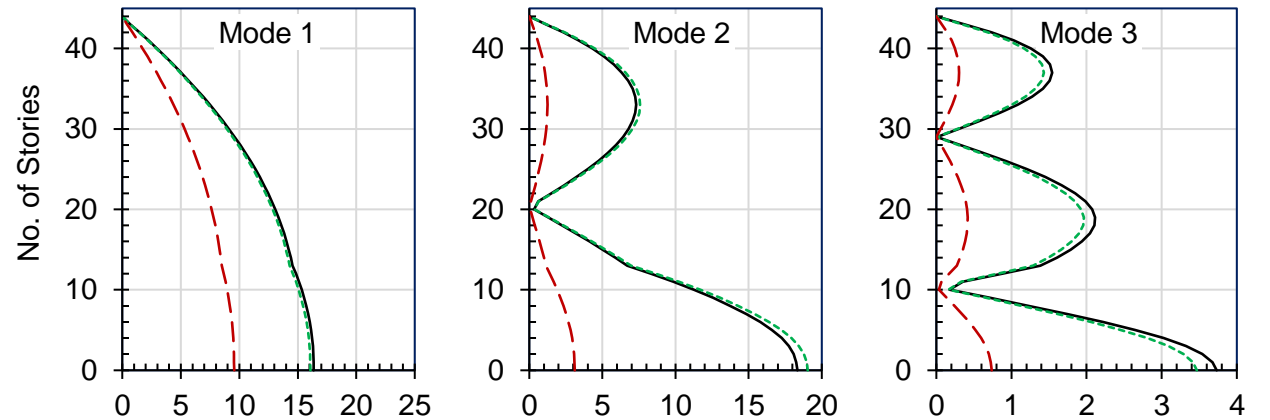
Ground Motion Set 1



Ground Motion Set 2

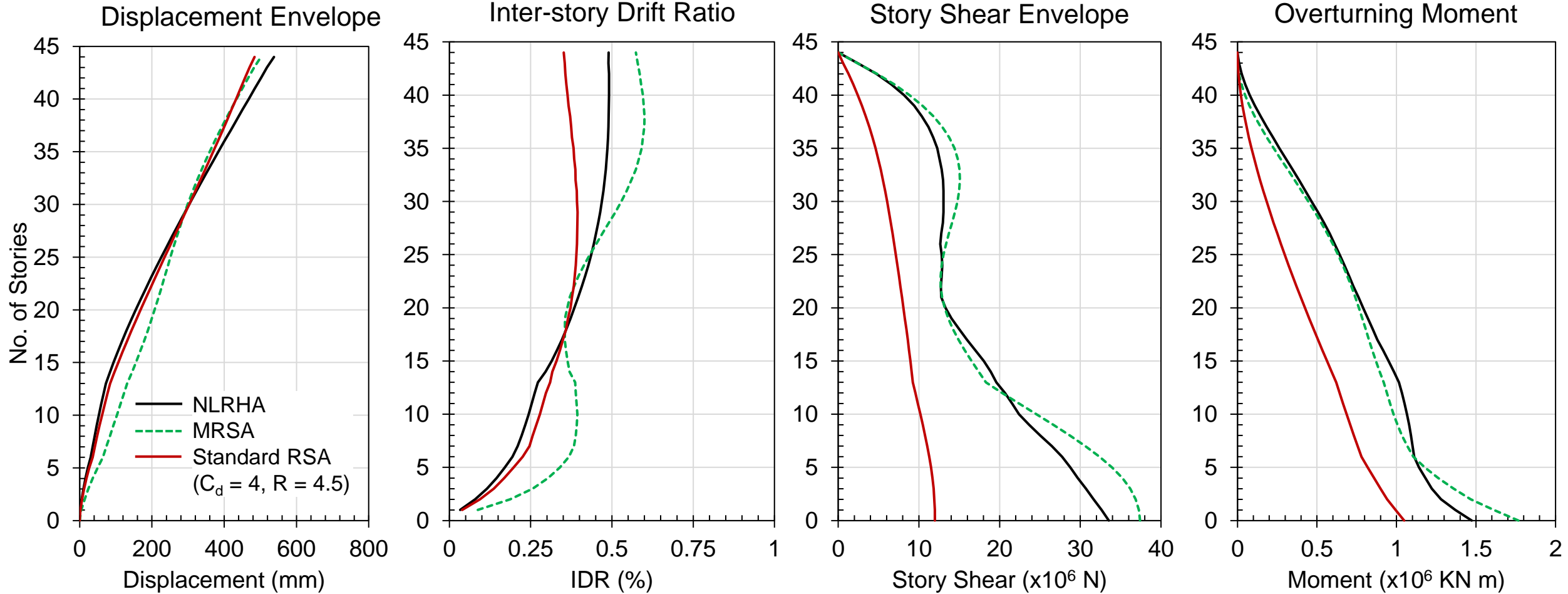


Ground Motion Set 3

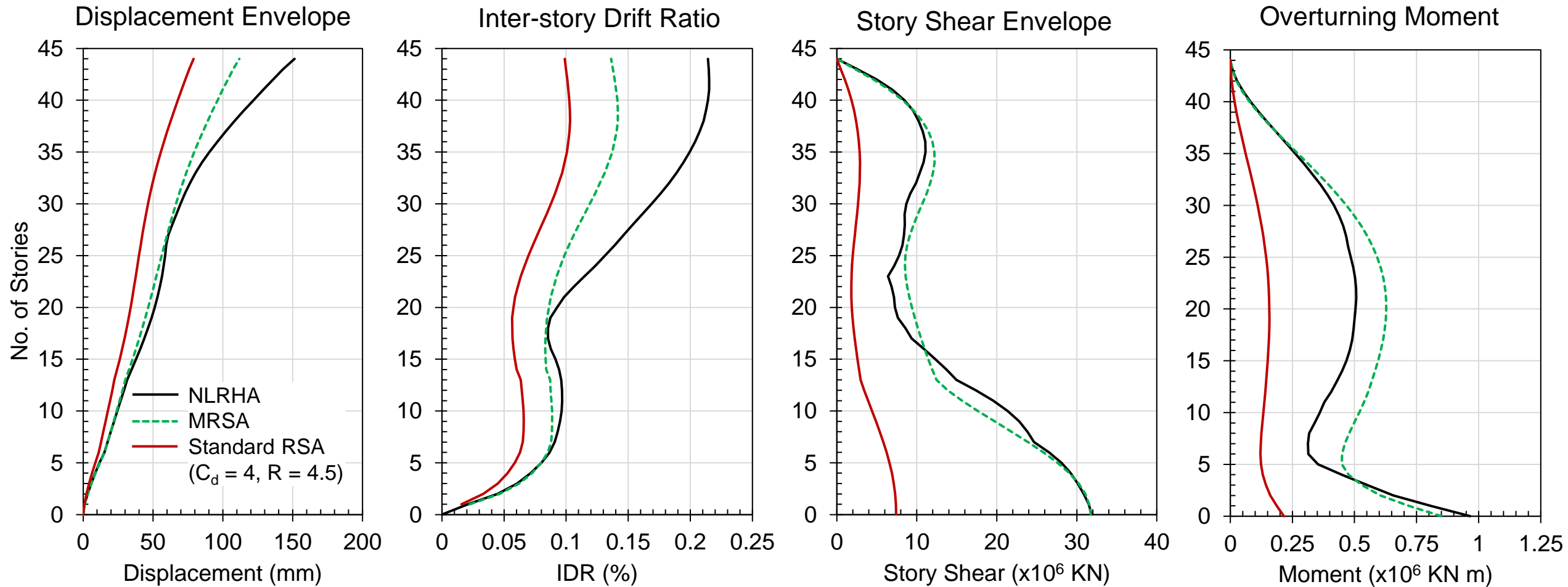


Story Shear ( $\times 10^6$  N)

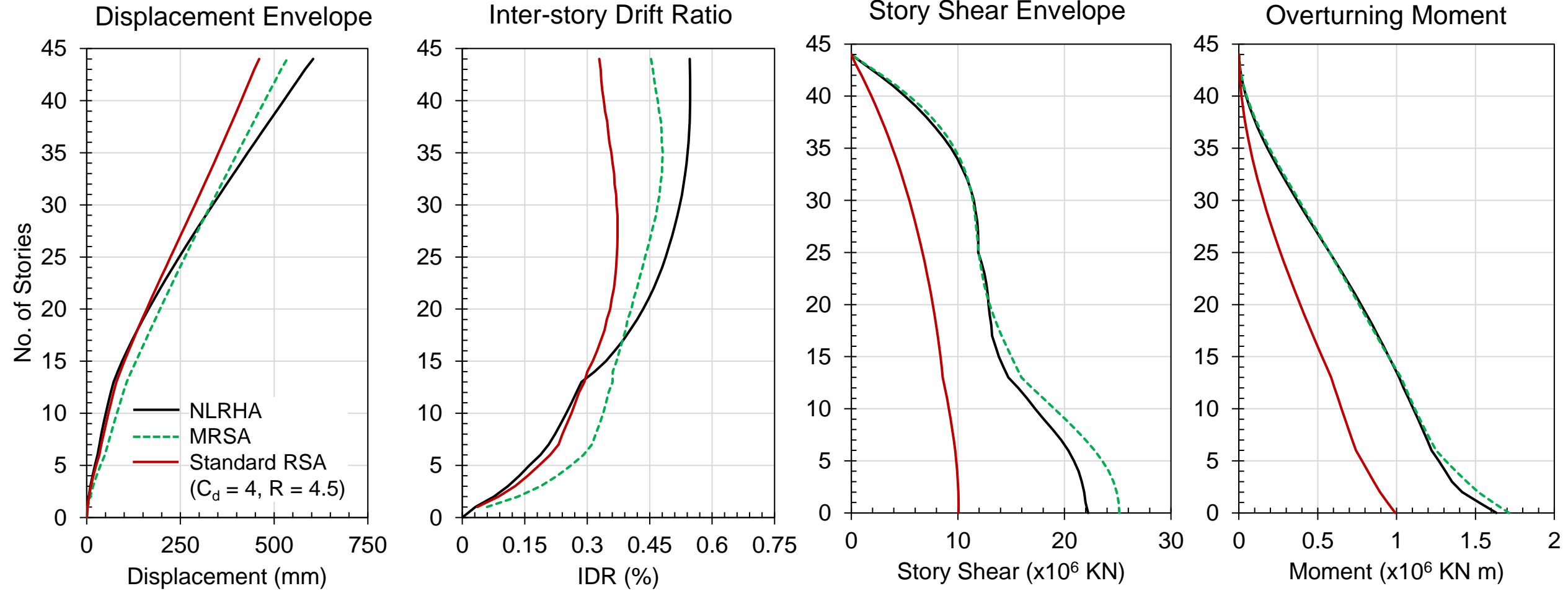
# Overall Performance of the MRSA Procedure – B3 – Ground Motion Set 1



# Overall Performance of the MRSA Procedure – B3 – Ground Motion Set 2



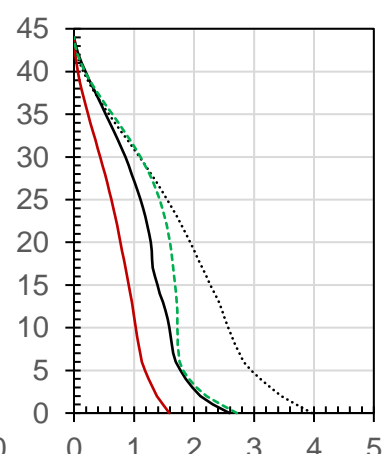
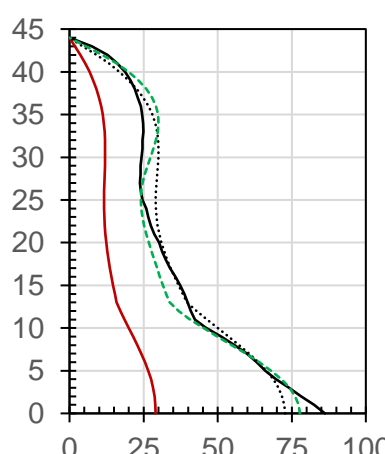
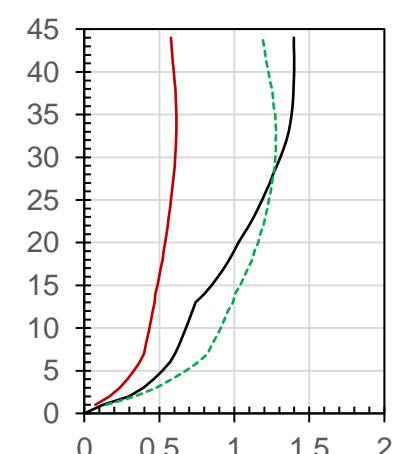
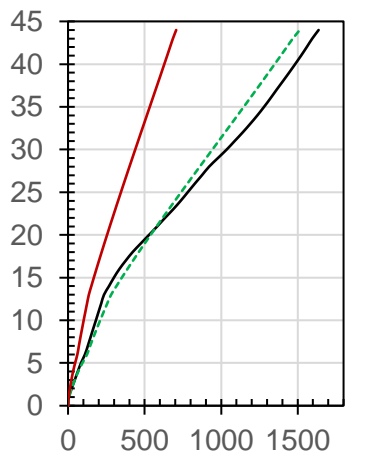
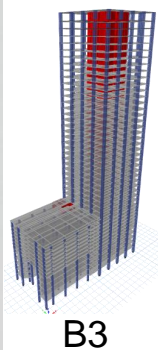
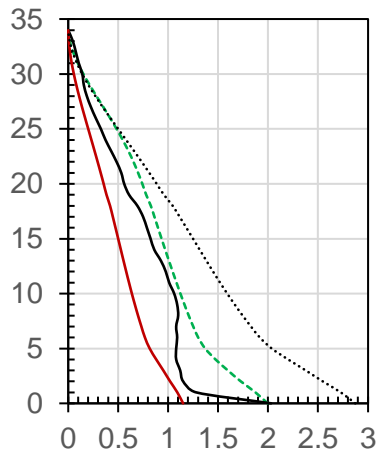
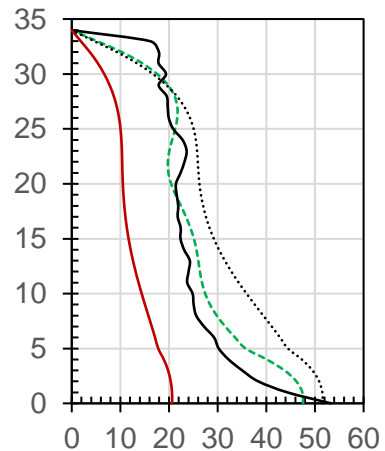
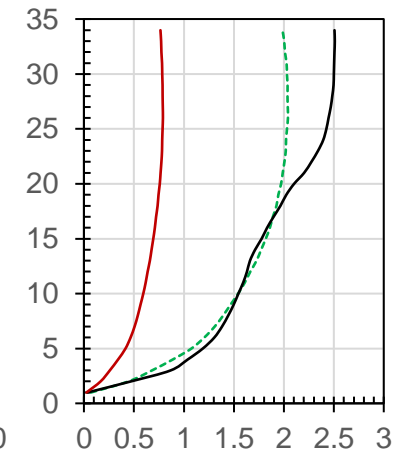
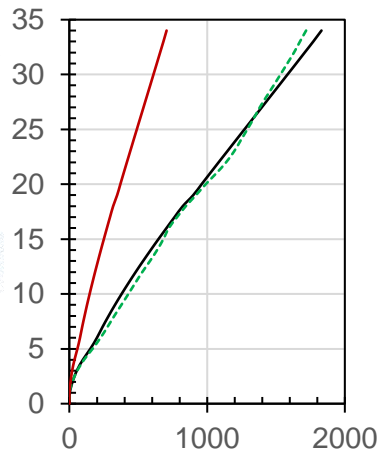
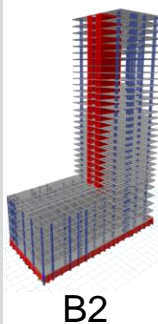
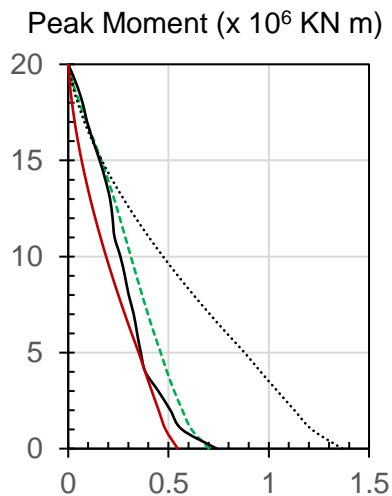
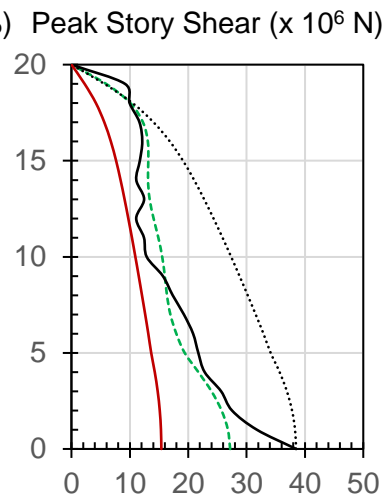
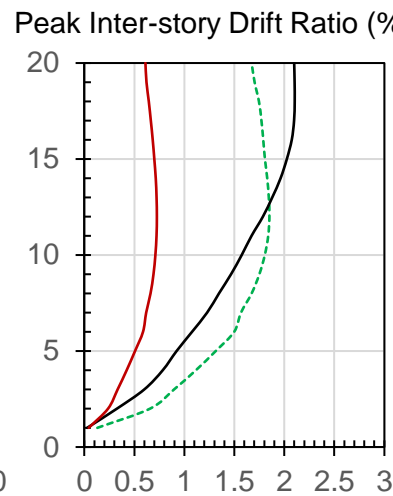
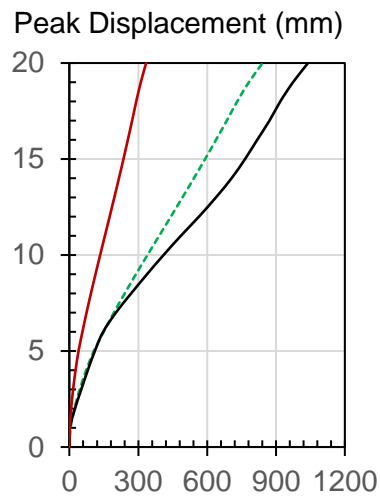
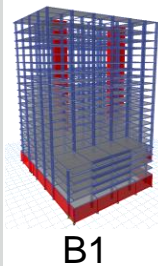
# Overall Performance of the MRSA Procedure – B3 – Ground Motion Set 3



# Overall Performance of MRSA

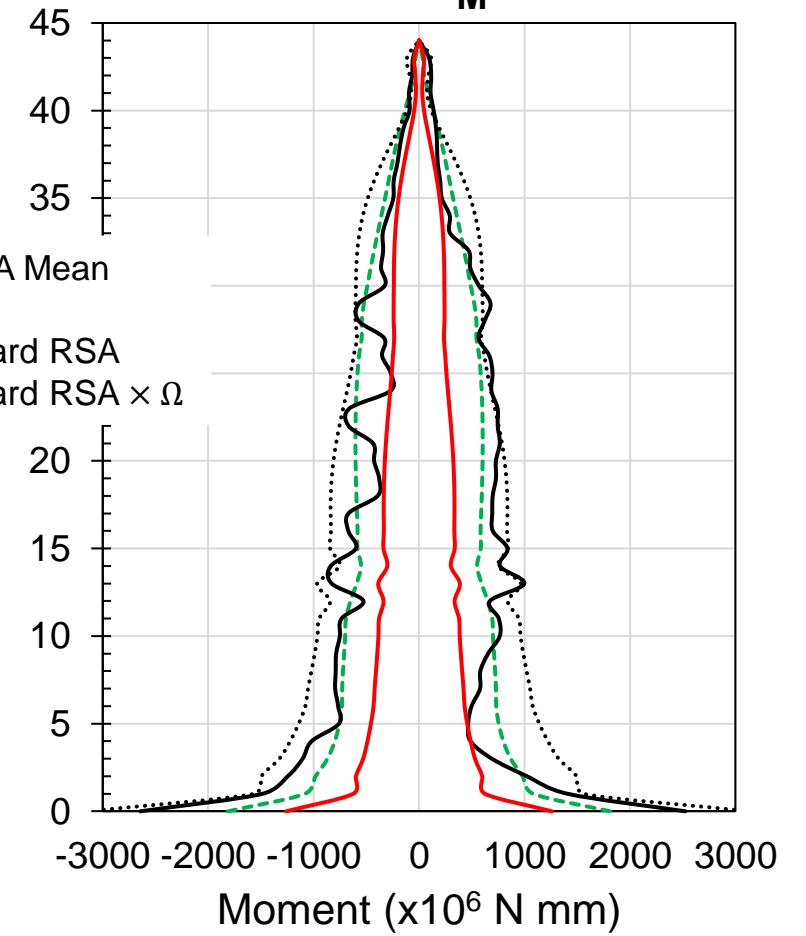
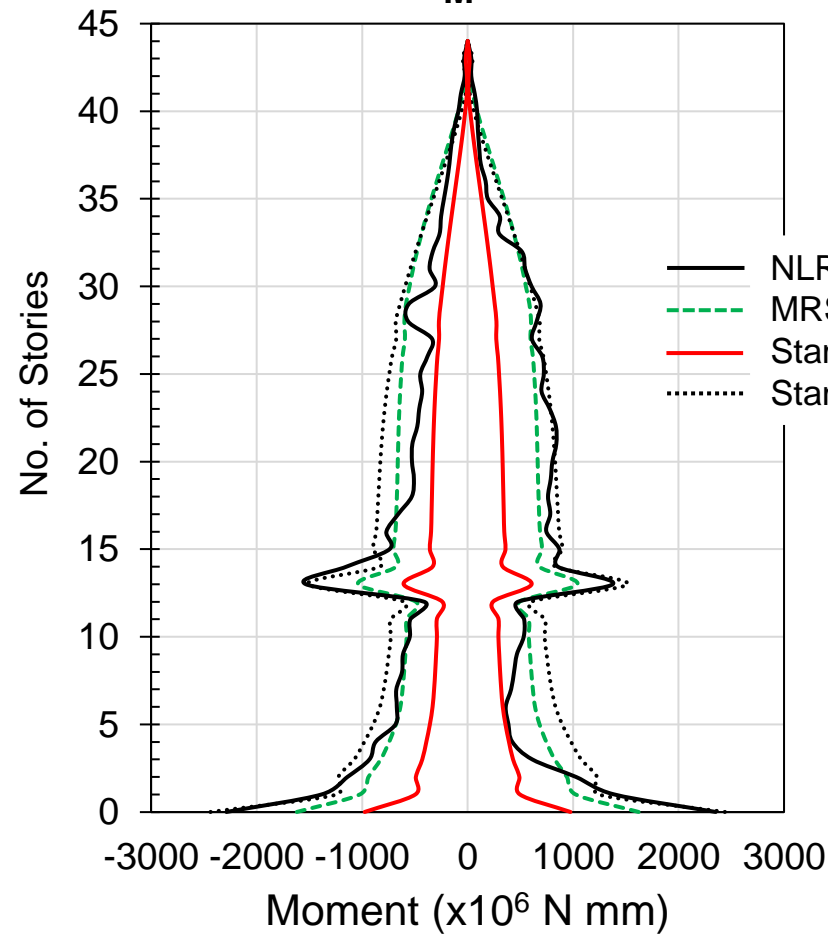
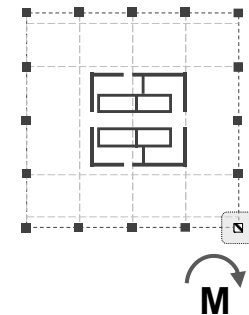
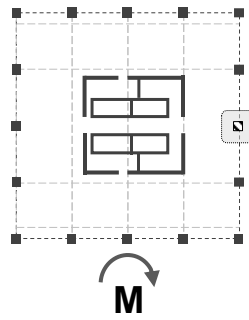
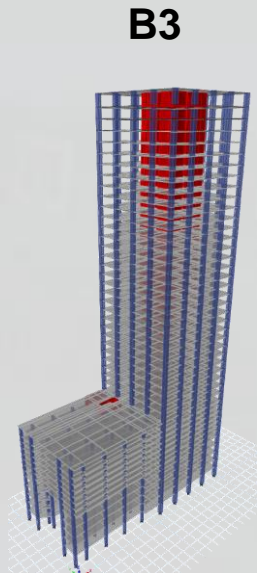
## Ground Motion Set 4

- NLRHA
- - - MRSA
- Standard RSA ( $C_d = 4, R = 4.5$ )
- ⋯ Standard RSA  $\times \Omega$



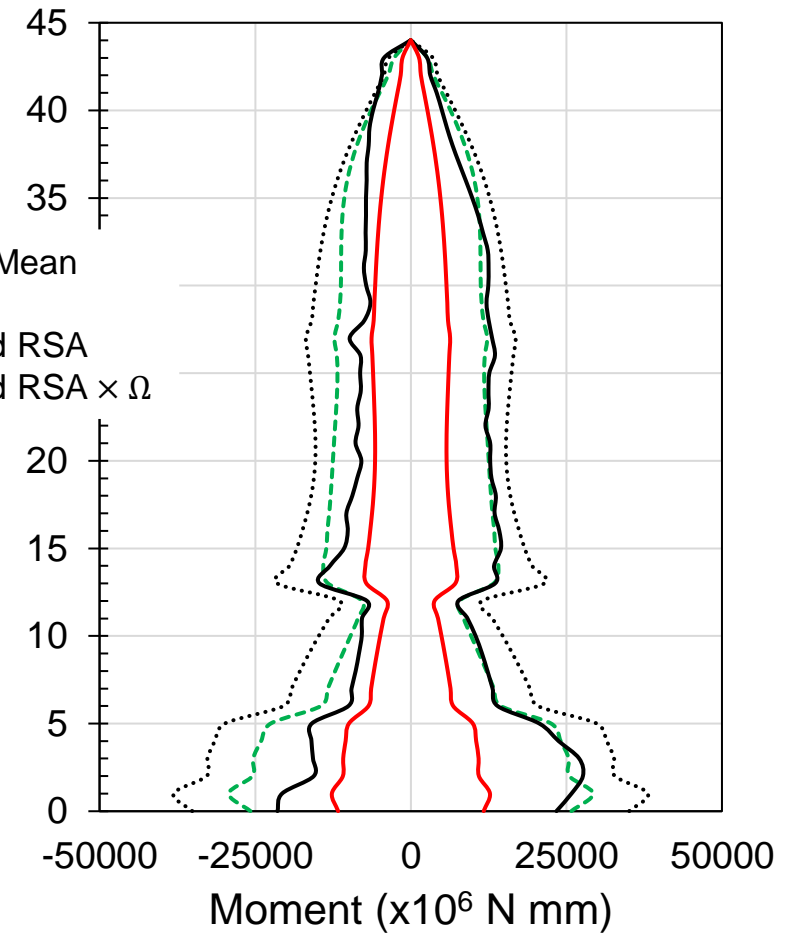
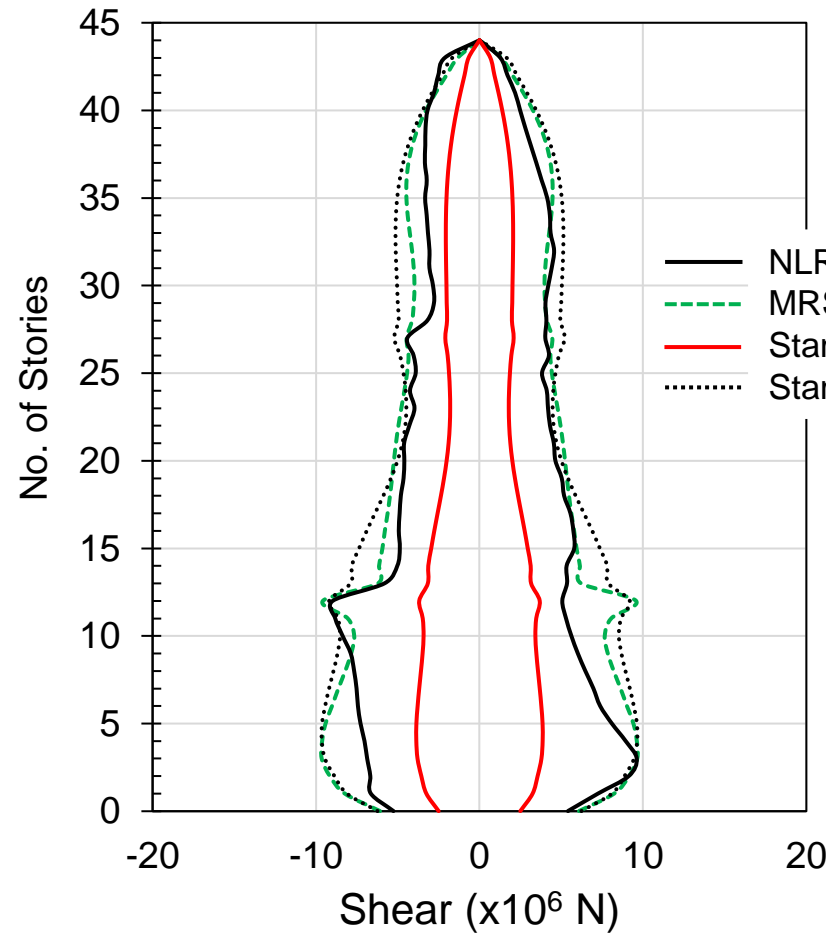
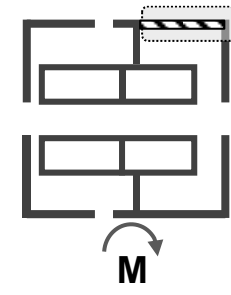
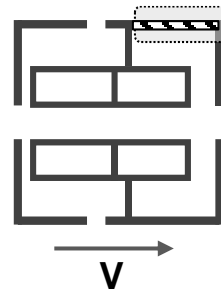
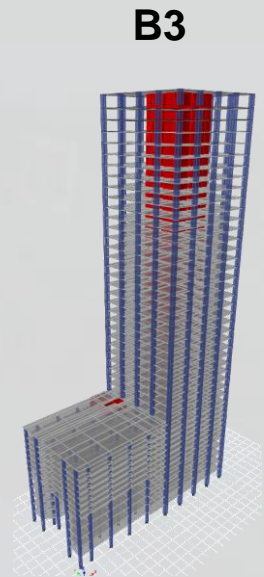
# Bending Moments in Columns

Ground Motion Set 4



# Shear Forces and Bending Moments in Shear Walls

Ground Motion Set 4



## Summary and Conclusions

- ❑ The proposed MRSA procedure works
- ❑ Requires significantly less computational time and effort compared to the detailed NLRHA procedure
- ❑ Simple, conceptually superior, provides mode-by-mode response, clear insight
- ❑ Doesn't require nonlinear modeling
- ❑ Can be considered as a convenient analysis and design option



**Thank you for your attention**