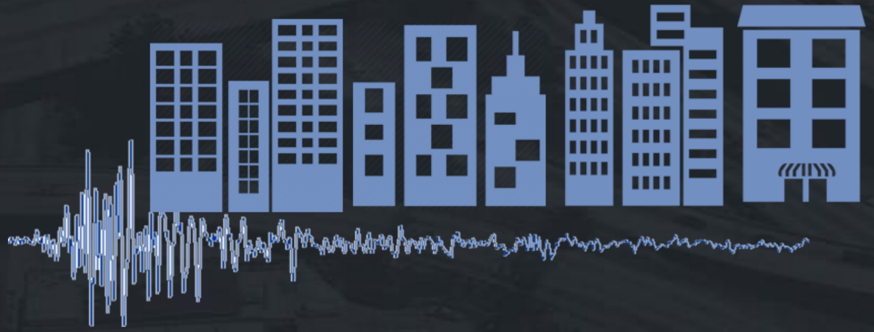
CE – 842, Credits: 3 + 0, Semester: Spring 2022

# Performance-based Seismic Design of Structures

Department of Structural Engineering

National University of Sciences and Technology (NUST)



## Seismic Analysis & Design of Buildings using IBC-2021 (BCP-2021) & ASCE 7-16

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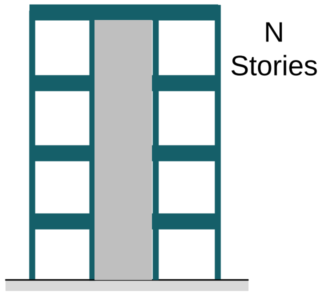
# The Standard RSA Procedure (ASCE 7-10, IBS 2012, EC 8)

Linear Elastic Model



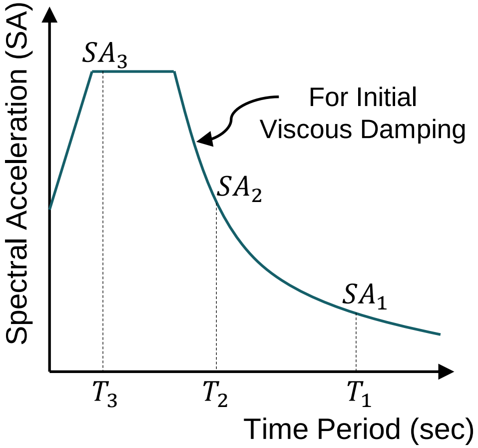
Eigen-value Analysis

$$[\mathbb{K} - \omega^2 \mathbb{M}]\Phi = \mathbb{O}$$

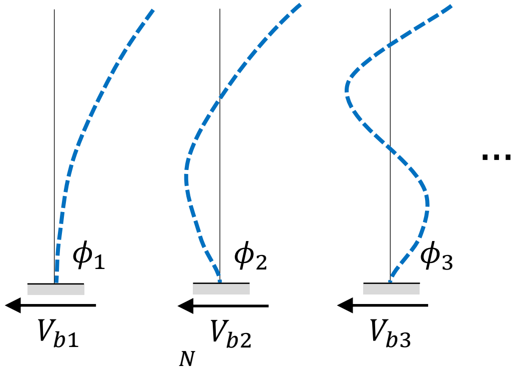
Determine Modal Properties

$$T_i,\ \phi_i,\ \Gamma_i$$

Determine Spectral Acceleration for each Significant Mode



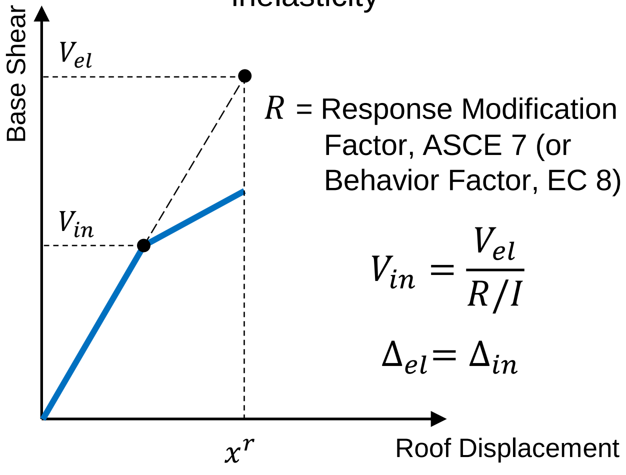
Determine Elastic Base Shears



$$V_{bn} = \sum_{i=1}^{N} \Gamma_n \,.\, m_i \,.\, \phi_{i,n} \,.\, SA_n$$

$$V_{el} = \sqrt{(V_{b1})^2 + (V_{b2})^2 + (V_{b3})^2 + \ \ldots}$$

Reduce Elastic Base Shear to account for inelasticity



$R$ = Response Modification Factor, ASCE 7 (or Behavior Factor, EC 8)

$$V_{in} = \frac{V_{el}}{R/I}$$

$$\Delta_{el} = \Delta_{in}$$

$$V_{Design} = \frac{\sqrt{(V_{b1})^2 + (V_{b2})^2 + (V_{b3})^2 + \ \ldots}}{R/I}$$

$$M_{Design} = \frac{\sqrt{(M_{b1})^2 + (M_{b2})^2 + (M_{b3})^2 + \ \ldots}}{R}$$

$$\Delta = \frac{C_d\sqrt{(\Delta_{el1})^2 + (\Delta_{el2})^2 + (\Delta_{el3})^2 + \ \ldots}}{R/I}$$

For members not desired to yield during a design earthquake $V_{Design} = \Omega \dfrac{V_{el}}{R}$

$\Omega$ = Structural Over-strength Factor

Maximum Displacement during a design earthquake $x^r_{max} = \dfrac{C_d x^r}{I}$

$C_d$ = Displacement Amplification Factor

# Response Spectrum Analysis (RSA) Procedure (IBC-2021, BCP-2021, ASCE 7-16)

**Application:**

Structure: N-story building with plan symmetric about two orthogonal axes

Seismic Loading: Earthquake ground motion along an axis of symmetry

**Step 1**

Determine the natural frequencies $\omega_n$ (natural periods $T_n = 2\,\pi/\omega_n$ ) and natural modes $\phi_n$ of vibration. Perform the Eigen-value Analysis or Ritz Analysis for this purpose.

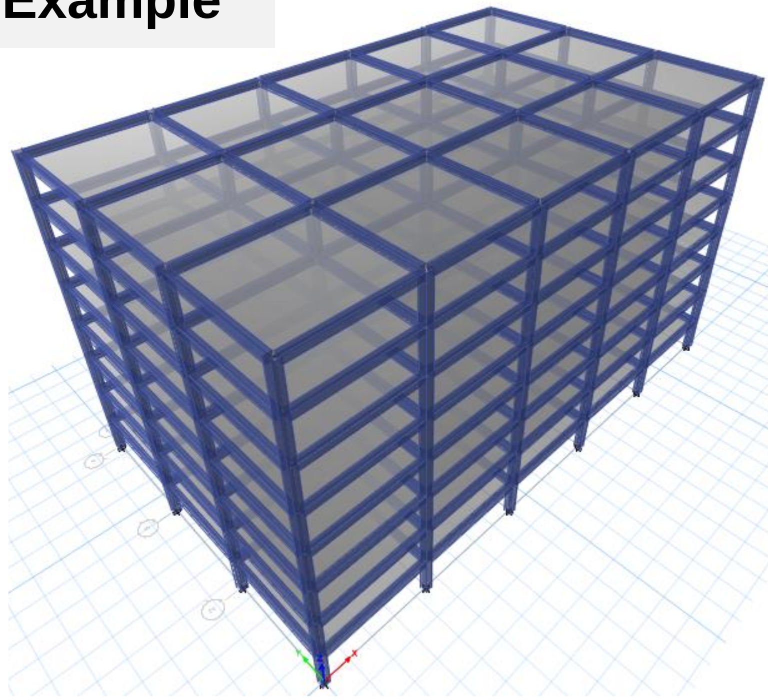
**Step 2**

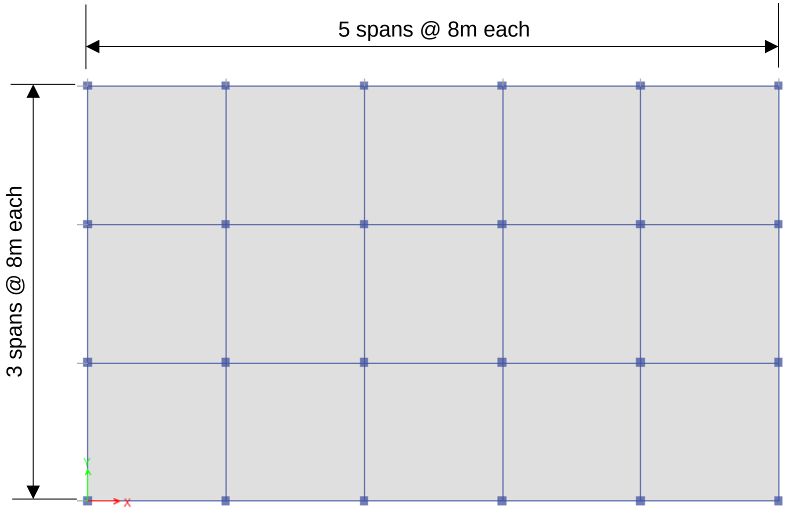
Estimate the modal damping ratios $\xi_n$ for all significantly contributing vibration modes. Generally, a 5% damping is assumed for each mode.

**Step 3**

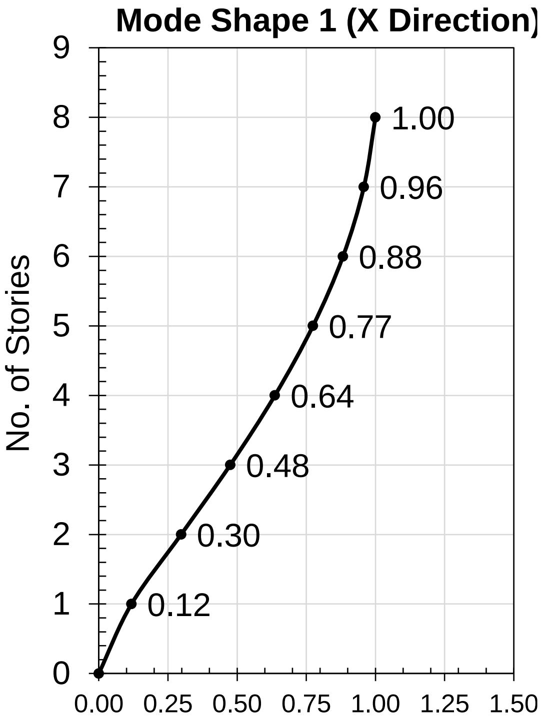
For the analysis of new buildings, construct the code-based design response spectrum using the seismic hazard parameters of the building site. Alternatively, a site-specific PSHA can also be performed to determine the design uniform hazard spectrum (UHS). If the analysis is to be performed for a past recorded earthquake, construct its response spectrum.

# Example

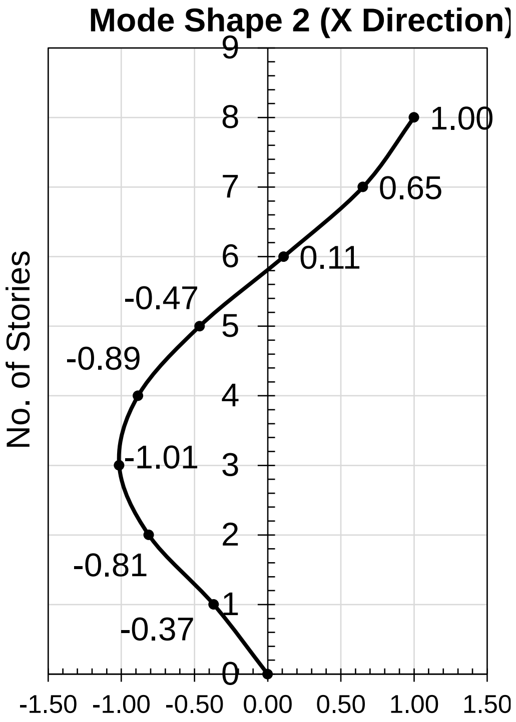




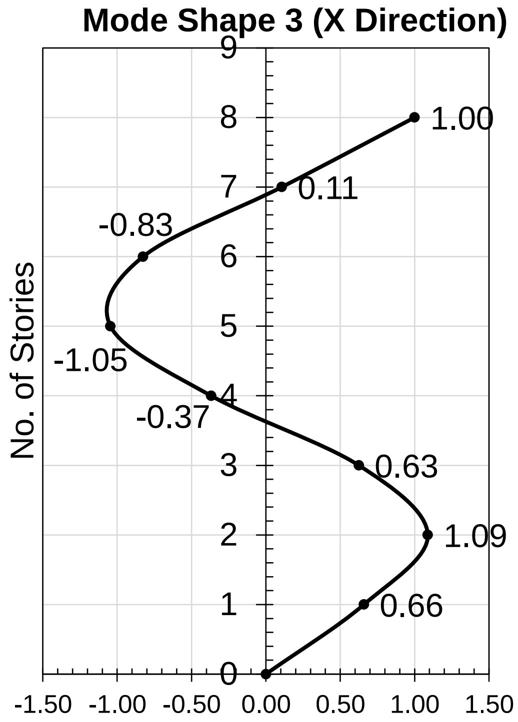
$T_1 = 1.4\ sec$



$T_2 = 0.45\ sec$



$T_3 = 0.2\ sec$



$$\boldsymbol{\phi_1} = \begin{bmatrix} 1 \\ 0.958 \\ 0.883 \\ 0.774 \\ 0.637 \\ 0.475 \\ 0.298 \\ 0.118 \\ 0 \end{bmatrix} \qquad \boldsymbol{\phi_2} = \begin{bmatrix} 1 \\ 0.651 \\ 0.109 \\ -0.465 \\ -0.886 \\ -1.015 \\ -0.813 \\ -0.369 \\ 0 \end{bmatrix} \qquad \boldsymbol{\phi_3} = \begin{bmatrix} 1 \\ 0.107 \\ -0.826 \\ -1.045 \\ -0.368 \\ 0.625 \\ 1.089 \\ 0.659 \\ 0 \end{bmatrix}$$

$$\Gamma_i = \frac{\boldsymbol{\phi_i . M . 1}}{\boldsymbol{\phi_i . M . \phi_i^T}}$$

$$\Gamma_1 = 1.3$$

$$\Gamma_2 = -0.9$$

$$\Gamma_3 = 0.4$$

# Design Response Spectrum Prescribed in ASCE 7-16

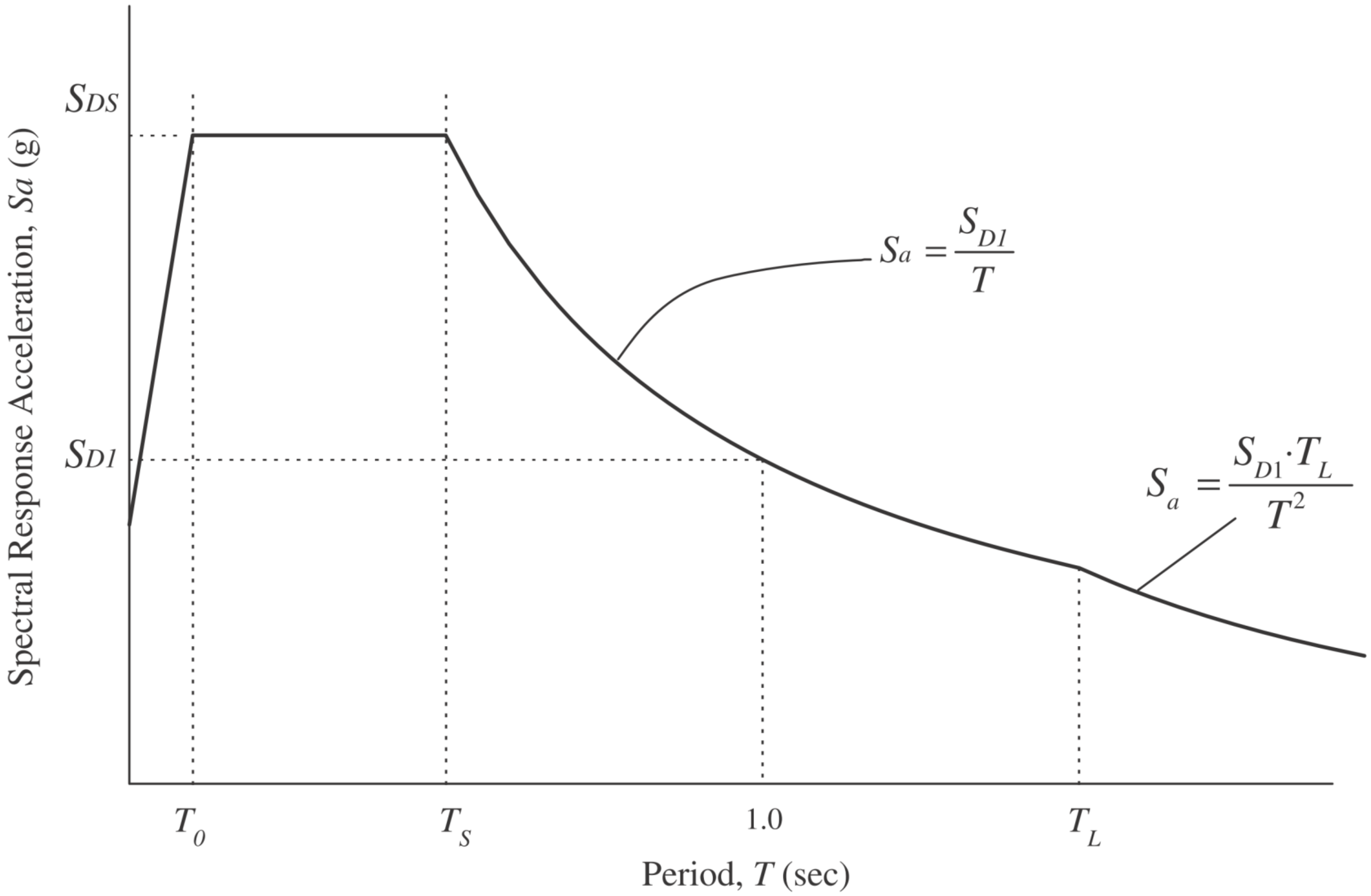
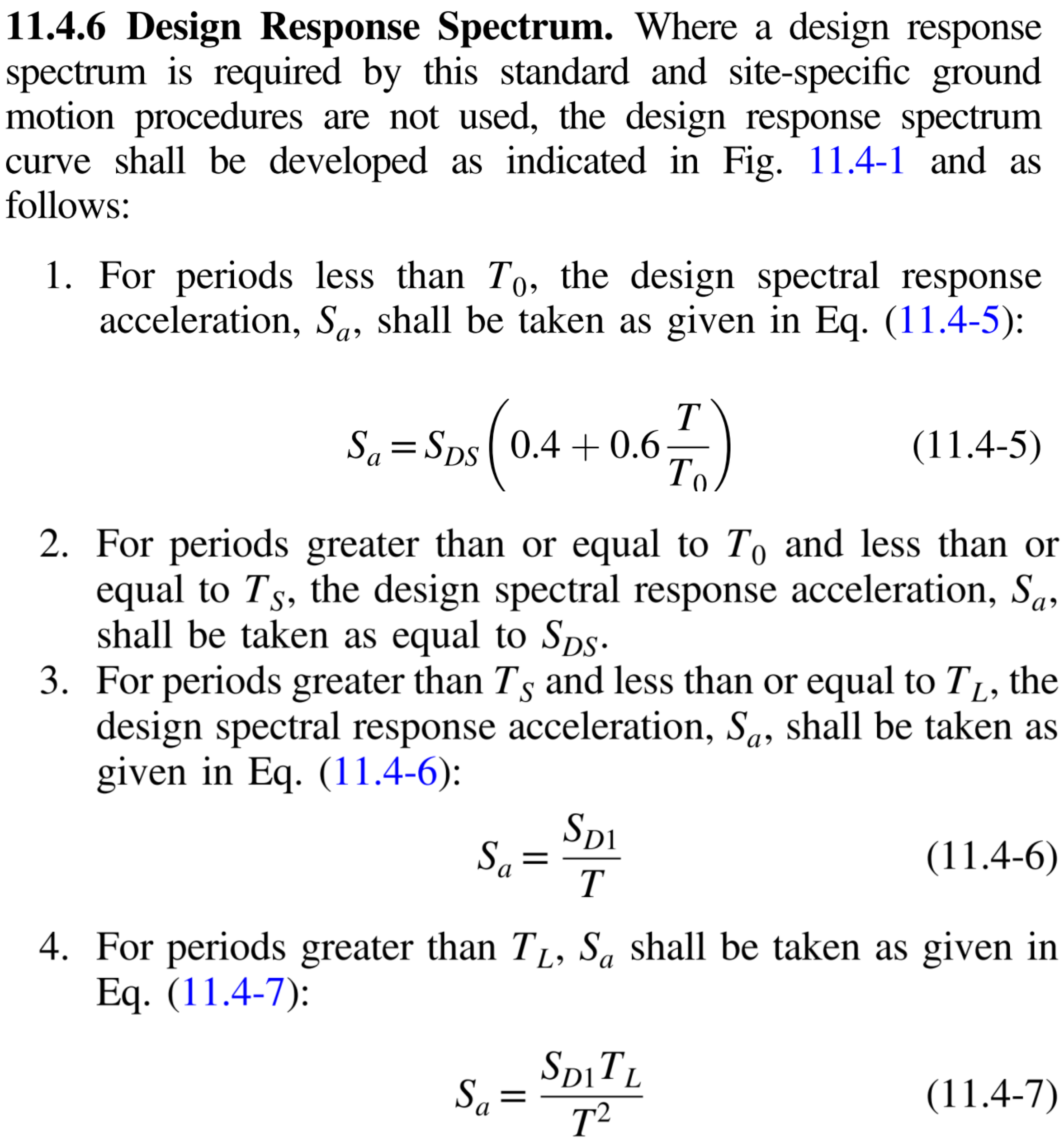


FIGURE 11.4-1 Design Response Spectrum

# Construction of Design Response Spectrum



**11.4.6 Design Response Spectrum.** Where a design response spectrum is required by this standard and site-specific ground motion procedures are not used, the design response spectrum curve shall be developed as indicated in Fig. 11.4-1 and as follows:

1. For periods less than $T_0$, the design spectral response acceleration, $S_a$, shall be taken as given in Eq. (11.4-5):

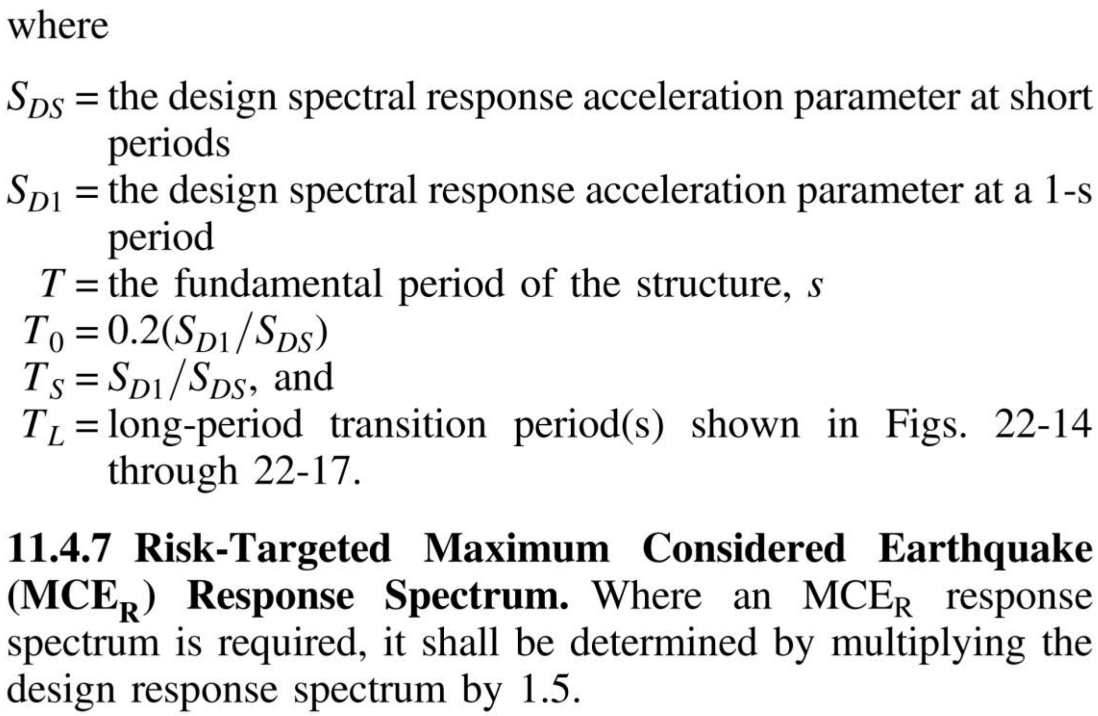
$$S_a = S_{DS}\left(0.4 + 0.6\frac{T}{T_0}\right) \qquad (11.4\text{-}5)$$

2. For periods greater than or equal to $T_0$ and less than or equal to $T_S$, the design spectral response acceleration, $S_a$, shall be taken as equal to $S_{DS}$.
3. For periods greater than $T_S$ and less than or equal to $T_L$, the design spectral response acceleration, $S_a$, shall be taken as given in Eq. (11.4-6):

$$S_a = \frac{S_{D1}}{T} \qquad (11.4\text{-}6)$$

4. For periods greater than $T_L$, $S_a$ shall be taken as given in Eq. (11.4-7):

$$S_a = \frac{S_{D1}T_L}{T^2} \qquad (11.4\text{-}7)$$



where

$S_{DS}$ = the design spectral response acceleration parameter at short periods
$S_{D1}$ = the design spectral response acceleration parameter at a 1-s period
$T$ = the fundamental period of the structure, $s$
$T_0 = 0.2(S_{D1}/S_{DS})$
$T_S = S_{D1}/S_{DS}$, and
$T_L$ = long-period transition period(s) shown in Figs. 22-14 through 22-17.

**11.4.7 Risk-Targeted Maximum Considered Earthquake ($\mathbf{MCE_R}$) Response Spectrum.** Where an $MCE_R$ response spectrum is required, it shall be determined by multiplying the design response spectrum by 1.5.

**11.4.8 Site-Specific Ground Motion Procedures.** A site response analysis shall be performed in accordance with Section 21.1 for structures on Site Class F sites, unless exempted in accordance with Section 20.3.1. A ground motion hazard analysis shall be performed in accordance with Section 21.2 for the following:

1. seismically isolated structures and structures with damping systems on sites with $S_1$ greater than or equal to 0.6,
2. structures on Site Class E sites with $S_s$ greater than or equal to 1.0, and.
3. structures on Site Class D and E sites with $S_1$ greater than or equal to 0.2.

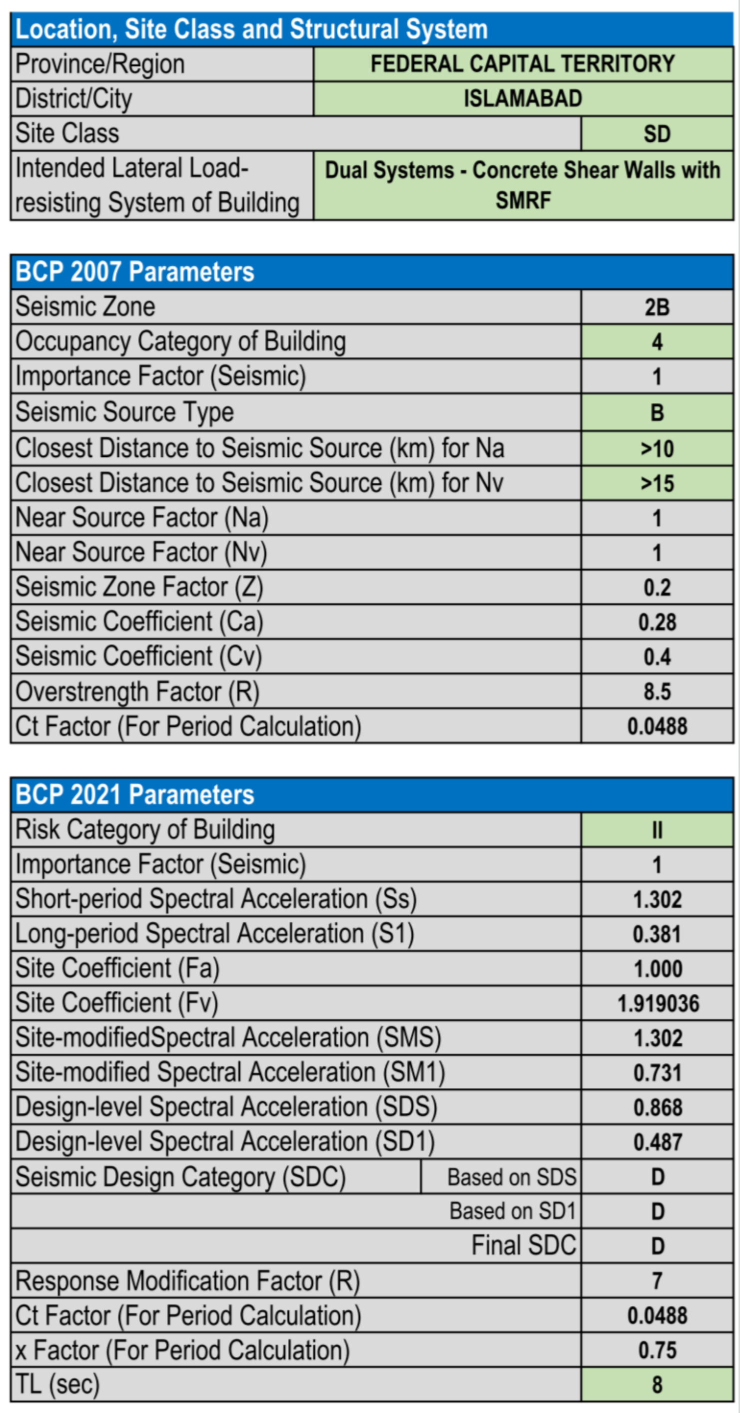
**EXCEPTION:** A ground motion hazard analysis is not required for structures other than seismically isolated structures and structures with damping systems where:

1. Structures on Site Class E sites with $S_S$ greater than or equal to 1.0, provided the site coefficient $F_a$ is taken as equal to that of Site Class C.
2. Structures on Site Class D sites with $S_1$ greater than or equal to 0.2, provided the value of the seismic response coefficient $C_s$ is determined by Eq. (12.8-2) for values of $T \leq 1.5T_s$ and taken as equal to 1.5 times the value computed in accordance with either Eq. (12.8-3) for $T_L \geq T > 1.5T_s$ or Eq. (12.8-4) for $T > T_L$.
3. Structures on Site Class E sites with $S_1$ greater than or equal to 0.2, provided that $T$ is less than or equal to $T_s$ and the equivalent static force procedure is used for design.

It shall be permitted to perform a site response analysis in accordance with Section 21.1 and/or a ground motion hazard analysis in accordance with Section 21.2 to determine ground motions for any structure.

When the procedures of either Section 21.1 or 21.2 are used, the design response spectrum shall be determined in accordance with Section 21.3, the design acceleration parameters shall be determined in accordance with Section 21.4, and, if required, the $\text{MCE}_\text{G}$ peak ground acceleration parameter shall be determined in accordance with Section 21.5.

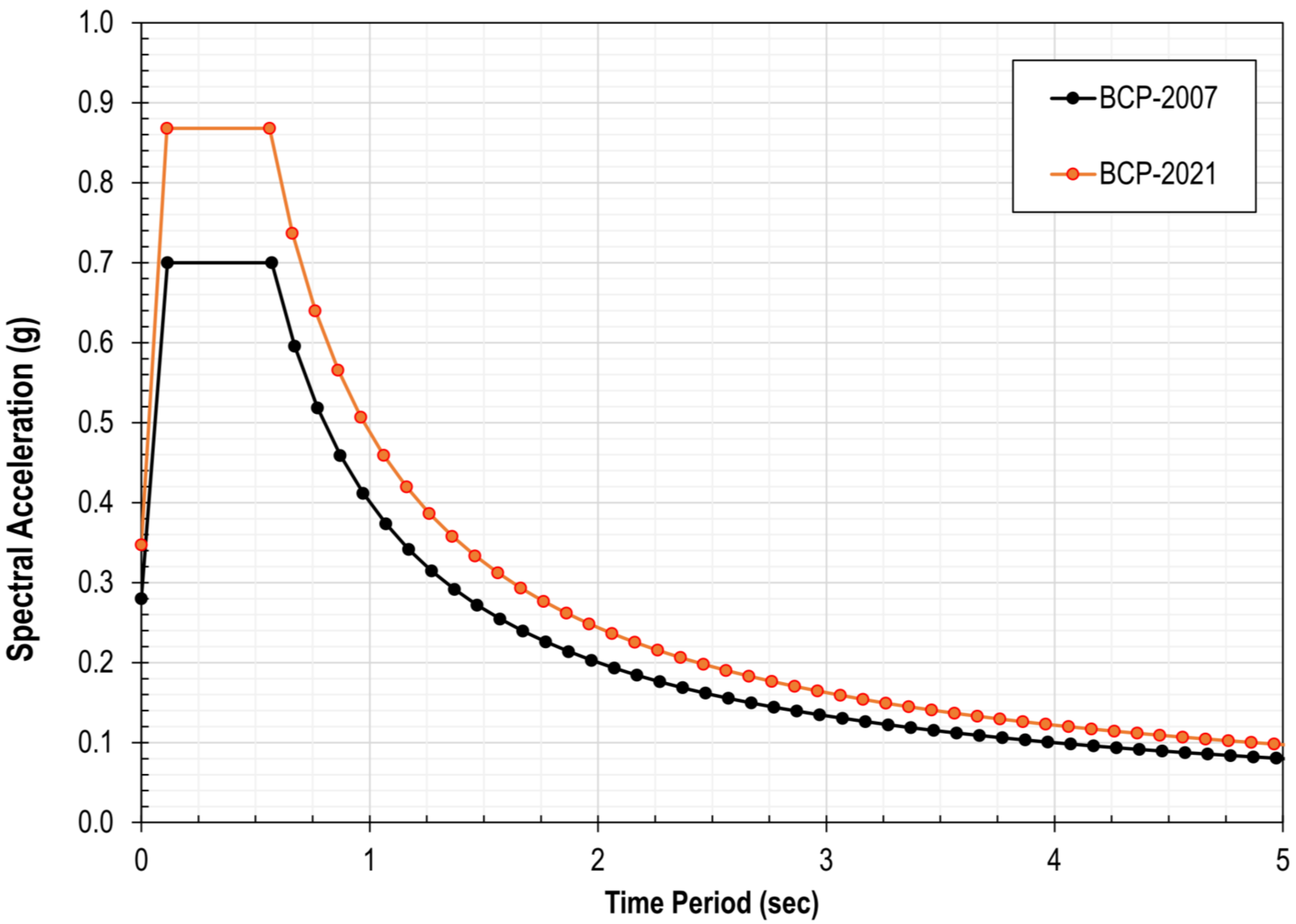
# Example



| Location, Site Class and Structural System | |
|---|---|
| Province/Region | FEDERAL CAPITAL TERRITORY |
| District/City | ISLAMABAD |
| Site Class | SD |
| Intended Lateral Load-resisting System of Building | Dual Systems - Concrete Shear Walls with SMRF |

| BCP 2007 Parameters | |
|---|---|
| Seismic Zone | 2B |
| Occupancy Category of Building | 4 |
| Importance Factor (Seismic) | 1 |
| Seismic Source Type | B |
| Closest Distance to Seismic Source (km) for Na | >10 |
| Closest Distance to Seismic Source (km) for Nv | >15 |
| Near Source Factor (Na) | 1 |
| Near Source Factor (Nv) | 1 |
| Seismic Zone Factor (Z) | 0.2 |
| Seismic Coefficient (Ca) | 0.28 |
| Seismic Coefficient (Cv) | 0.4 |
| Overstrength Factor (R) | 8.5 |
| Ct Factor (For Period Calculation) | 0.0488 |

| BCP 2021 Parameters | | |
|---|---|---|
| Risk Category of Building | | II |
| Importance Factor (Seismic) | | 1 |
| Short-period Spectral Acceleration (Ss) | | 1.302 |
| Long-period Spectral Acceleration (S1) | | 0.381 |
| Site Coefficient (Fa) | | 1.000 |
| Site Coefficient (Fv) | | 1.919036 |
| Site-modifiedSpectral Acceleration (SMS) | | 1.302 |
| Site-modified Spectral Acceleration (SM1) | | 0.731 |
| Design-level Spectral Acceleration (SDS) | | 0.868 |
| Design-level Spectral Acceleration (SD1) | | 0.487 |
| Seismic Design Category (SDC) | Based on SDS | D |
| | Based on SD1 | D |
| | Final SDC | D |
| Response Modification Factor (R) | | 7 |
| Ct Factor (For Period Calculation) | | 0.0488 |
| x Factor (For Period Calculation) | | 0.75 |
| TL (sec) | | 8 |



# Step 4

Compute the peak response in the $n_{th}$ mode by the following steps to be repeated for all modes, $n$ = 1 , 2 , . . . , $N$:

(a) Corresponding to natural period $T_n$ and damping ratio $\xi_n$ , read $D_n$ and $A_n$, the deformation and pseudo-acceleration, from the earthquake response spectrum or the design spectrum.

(b) Compute the floor displacements ($u_{jn}$) and story drifts ($\Delta_{jn}$) at any $j^{th}$ floor from the following equation.

$$u_{jn} = \Gamma_n \phi_{jn} D_n$$

$$\Delta_{jn} = \Gamma_n \left(\phi_{jn} - \phi_{j-1,n}\right) D_n$$

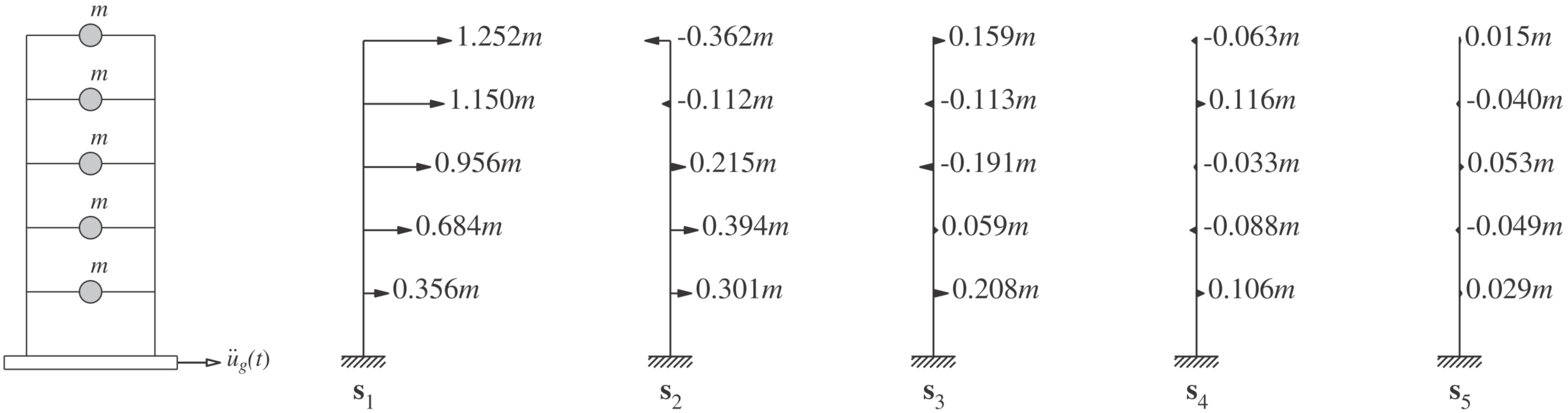
(c) Compute the equivalent static lateral forces $\boldsymbol{f_n}$ from the following equation.

$$\boldsymbol{f_n} = \boldsymbol{s_n} A_n$$

$$f_{jn} = \Gamma_n m_j \phi_{jn} A_n$$

(d) Compute the story forces—shear and overturning moment—and element forces—bending moments and shears—by static analysis of the structure subjected to lateral forces $\boldsymbol{f_n}$ .

# Equivalent Static Lateral Forces $f_n$ - An Example



## Step 5

Determine an estimate for the peak value $r_o$ of any response quantity by combining the peak modal values $r_{no}$ according to the SRSS rule as follows, if the natural frequencies are well separated.

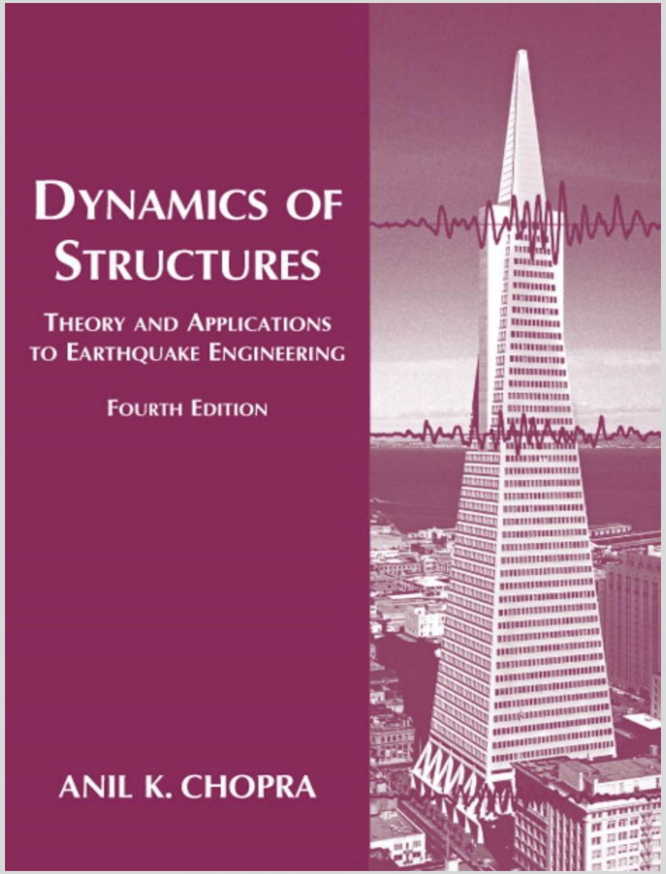
$$r_o \cong \sqrt{\sum_{n=1}^{N} r_{no}^2}$$

The CQC rule (shown below) should be used if the natural frequencies are closely spaced.

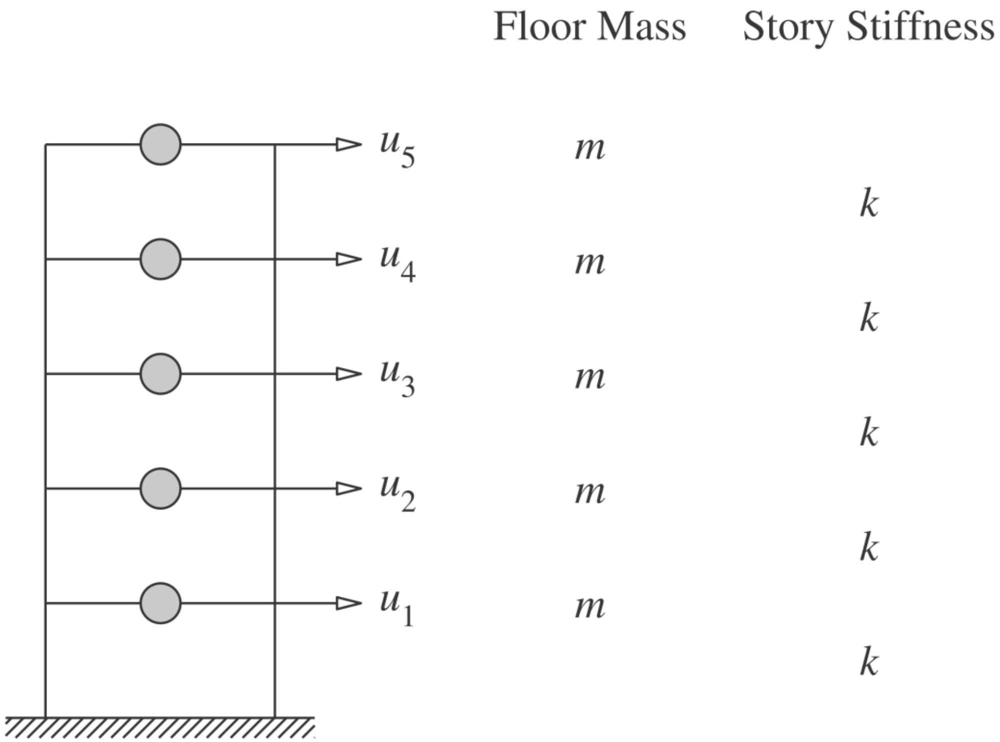
$$r_o \cong \sqrt{\sum_{i=1}^{N} \sum_{n=1}^{N} \rho_{in}\, r_{io}\, r_{no}}$$

$r_{io}$ and $r_{no}$ = Peak responses in $i^{th}$ & $n^{th}$ modes
$\rho_{in}$ = Correlation coefficient for $i^{th}$ & $n^{th}$ modes;
$\rho_{in}$ varies between 0 and 1, and $\rho_{in}$ = 1 for $i = n$.

Usually, only the lower modes contribute significantly to the response. Therefore, steps 1 to 5 need to be implemented for only these modes and the modal combinations truncated accordingly.



# Example: A Five-story Uniform Shear Building



The mass and stiffness matrices of the structure are

$$\mathbf{m} = m\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \qquad \mathbf{k} = k\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & 2 & -1 & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{bmatrix}$$

Step 1

Determined by solving the eigenvalue problem, the natural frequencies are

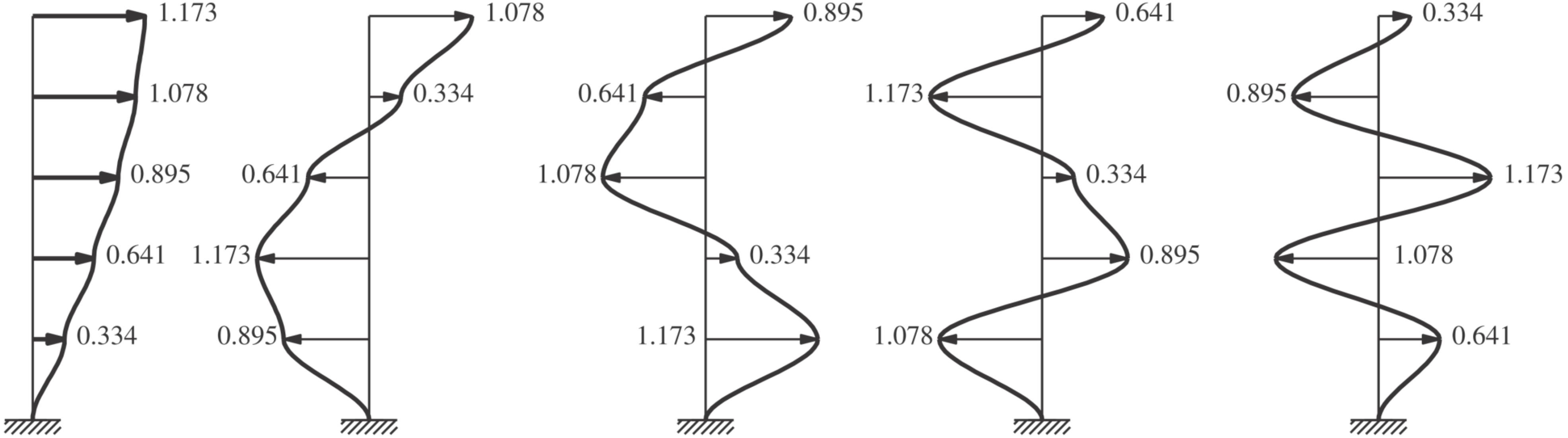
$$\omega_n = \alpha_n \left(\frac{k}{m}\right)^{1/2}$$

where $\alpha_1 = 0.285$, $\alpha_2 = 0.831$, $\alpha_3 = 1.310$, $\alpha_4 = 1.682$, and $\alpha_5 = 1.919$. For a structure with $m = 100$ kips/g, the natural vibration modes, which have been normalized to obtain $M_n = 1$, are (Fig. 12.8.2)

$$\phi_1 = \begin{Bmatrix} 0.334 \\ 0.641 \\ 0.895 \\ 1.078 \\ 1.173 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} -0.895 \\ -1.173 \\ -0.641 \\ 0.334 \\ 1.078 \end{Bmatrix} \quad \phi_3 = \begin{Bmatrix} 1.173 \\ 0.334 \\ -1.078 \\ -0.641 \\ 0.895 \end{Bmatrix} \quad \phi_4 = \begin{Bmatrix} -1.078 \\ 0.895 \\ 0.334 \\ -1.173 \\ 0.641 \end{Bmatrix} \quad \phi_5 = \begin{Bmatrix} 0.641 \\ -1.078 \\ 1.173 \\ -0.895 \\ 0.334 \end{Bmatrix}$$

$$\Gamma_n = \frac{\boldsymbol{\phi}_n^t \mathbf{M} \mathbf{1}}{\boldsymbol{\phi}_n^t \mathbf{M} \boldsymbol{\phi}_n}$$

| Mode | $\Gamma_n$ |
|---|---|
| 1 | 1.067 |
| 2 | −0.336 |
| 3 | 0.177 |
| 4 | −0.099 |
| 5 | 0.045 |



Mode 1 Mode 2 Mode 3 Mode 4 Mode 5

**Step 2**

The damping ratios are estimated as $\xi_n$ = 5%.

**Step 3**

The building needs to be analysed under the El Centro ground motion. The Response Spectra of El Centro ground motion is shown.

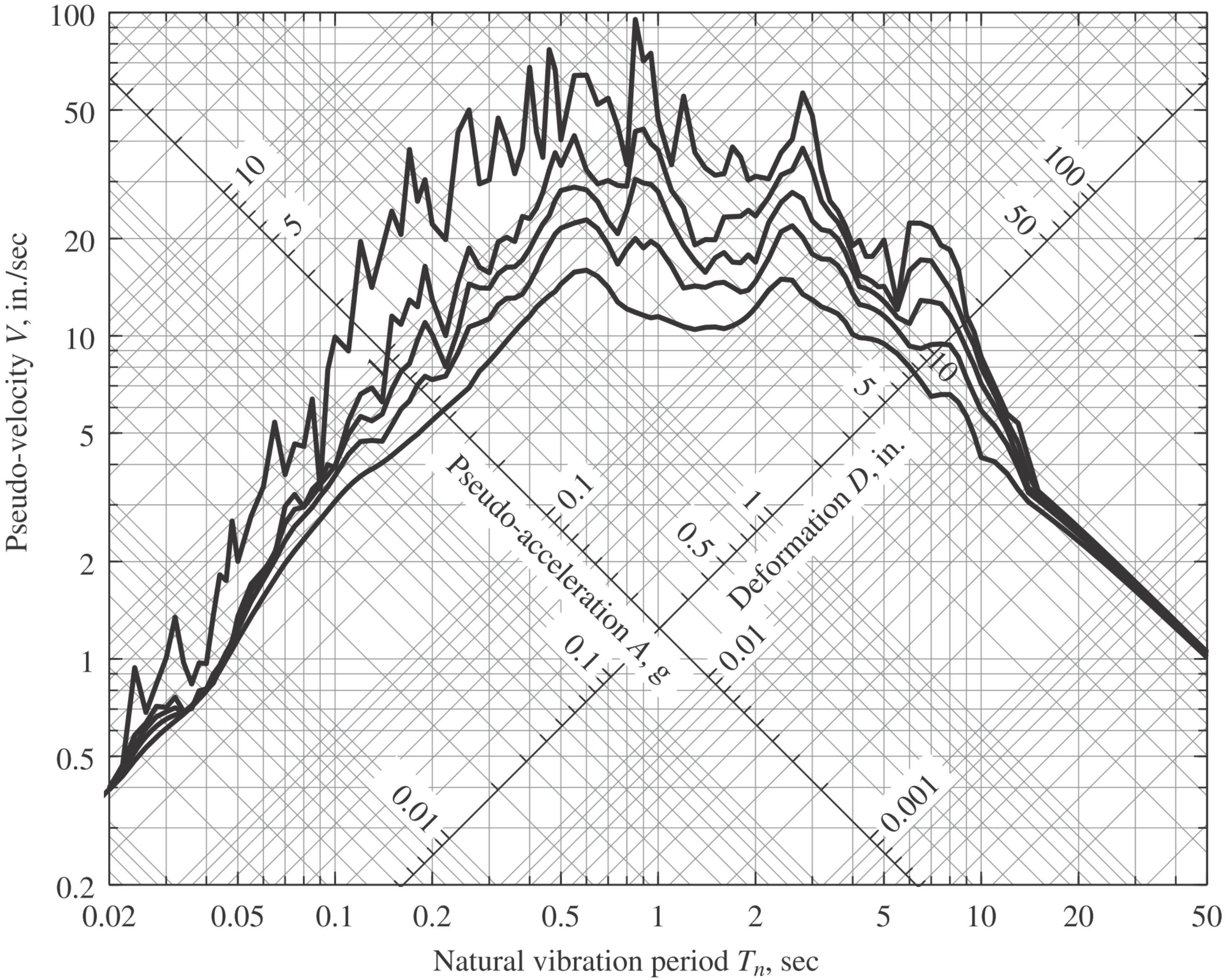
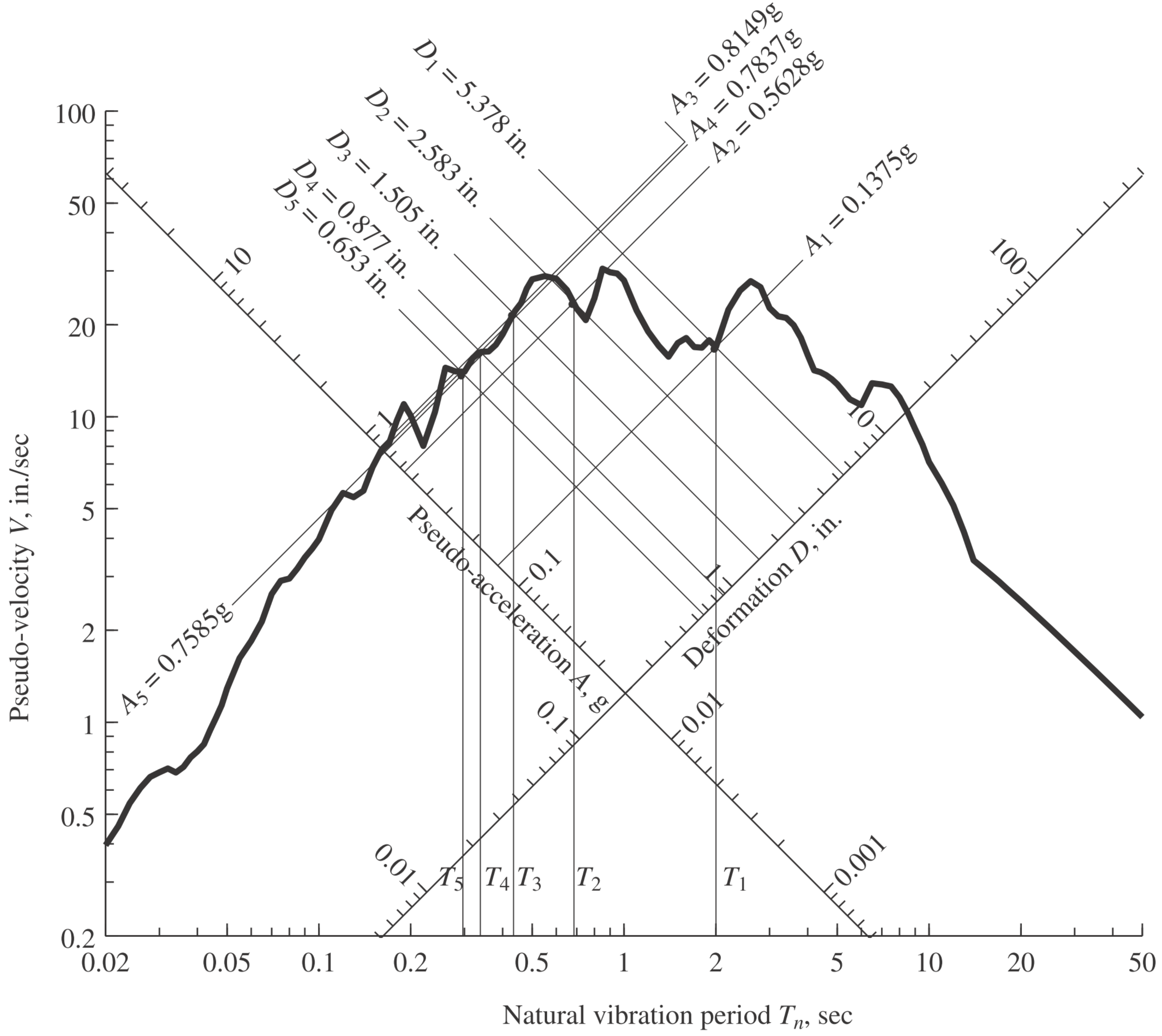


**Figure 6.6.4** Combined $D$–$V$–$A$ response spectrum for El Centro ground motion; $\zeta = 0$, 2, 5, 10, and 20%.

**Step 4 (a)**


$D_1 = 5.378$ in.
$D_2 = 2.583$ in.
$D_3 = 1.505$ in.
$D_4 = 0.877$ in.
$D_5 = 0.653$ in.
$A_3 = 0.8149$g
$A_4 = 0.7837$g
$A_2 = 0.5628$g
$A_1 = 0.1375$g
$A_5 = 0.7585$g
Pseudo-velocity $V$, in./sec
100
50
20
10
5
2
1
0.5
0.2
0.02
0.05
0.1
0.2
0.5
1
2
5
10
20
50
Natural vibration period $T_n$, sec
Pseudo-acceleration $A$, g
Deformation $D$, in.
10
1
0.1
0.01
0.001
100
10
1
0.1
0.01
$T_5$
$T_4$
$T_3$
$T_2$
$T_1$

# Step 4 (b) and (c)

$$\mathbf{u}_1 = \Gamma_1 \phi_1 D_1 = 1.067 \begin{Bmatrix} 0.334 \\ 0.641 \\ 0.895 \\ 1.078 \\ 1.173 \end{Bmatrix} 5.378 = \begin{Bmatrix} 1.916 \\ 3.677 \\ 5.139 \\ 6.188 \\ 6.731 \end{Bmatrix} \text{in.}$$

$$\mathbf{f}_1 = \mathbf{s}_1 A_1 = \begin{Bmatrix} 0.356\,m \\ 0.684\,m \\ 0.956\,m \\ 1.150\,m \\ 1.252\,m \end{Bmatrix} 0.1375\text{g} = \begin{Bmatrix} 4.899 \\ 9.401 \\ 13.141 \\ 15.817 \\ 17.211 \end{Bmatrix} \text{kips}$$

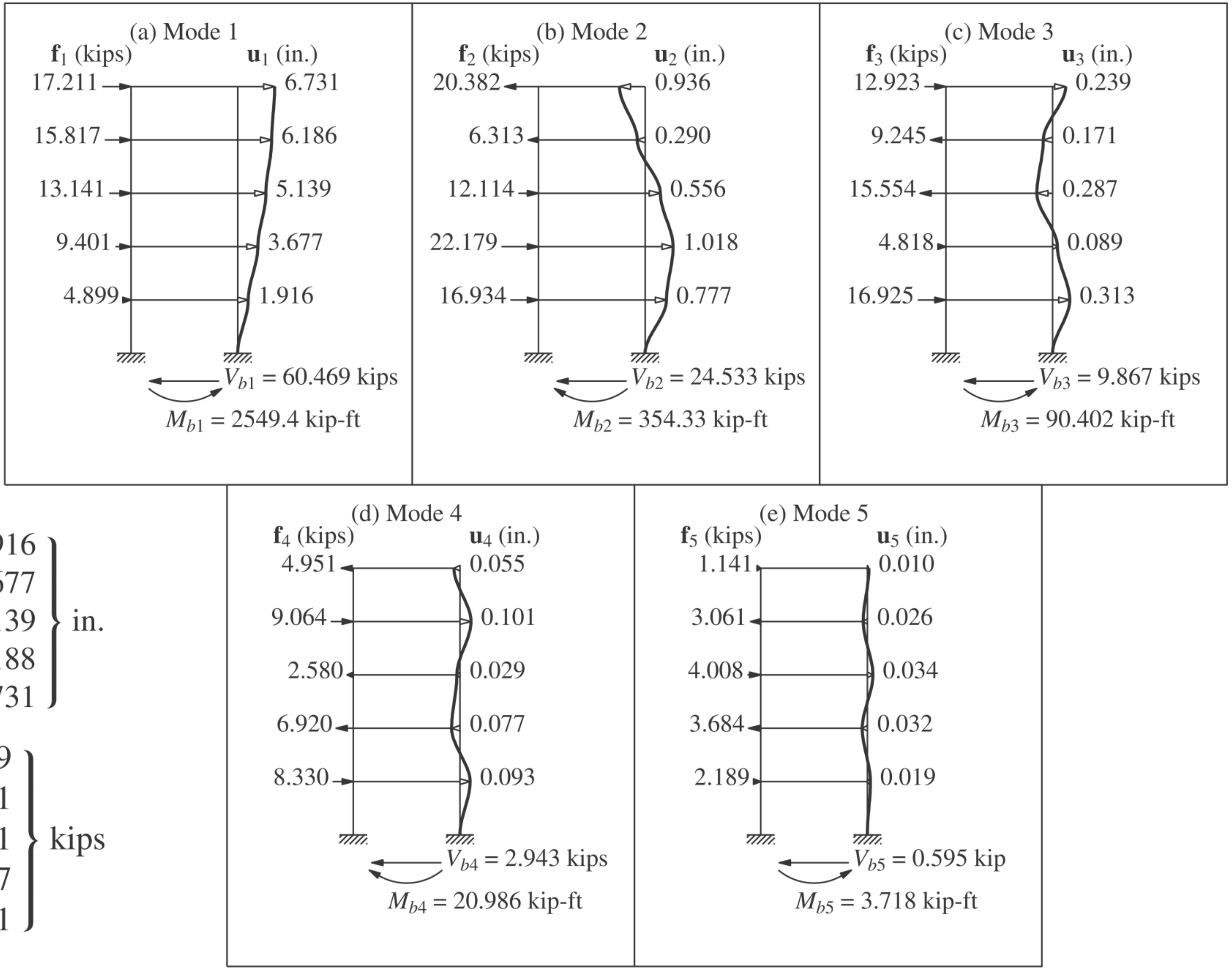


**Figure 13.8.3** Peak values of displacements and equivalent static lateral forces due to the five natural vibration modes.

## Step 4 (d)

For each mode, the peak value of any story force or element force is computed by static analysis of the structure subjected to the equivalent static lateral forces $\boldsymbol{f_n}$ .

For example, the peak values for the base shear $V_b$, top-story shear $V_5$, and base overturning moment $M_b$ are shown below. The earlier data for roof displacement $u_5$ are also included.

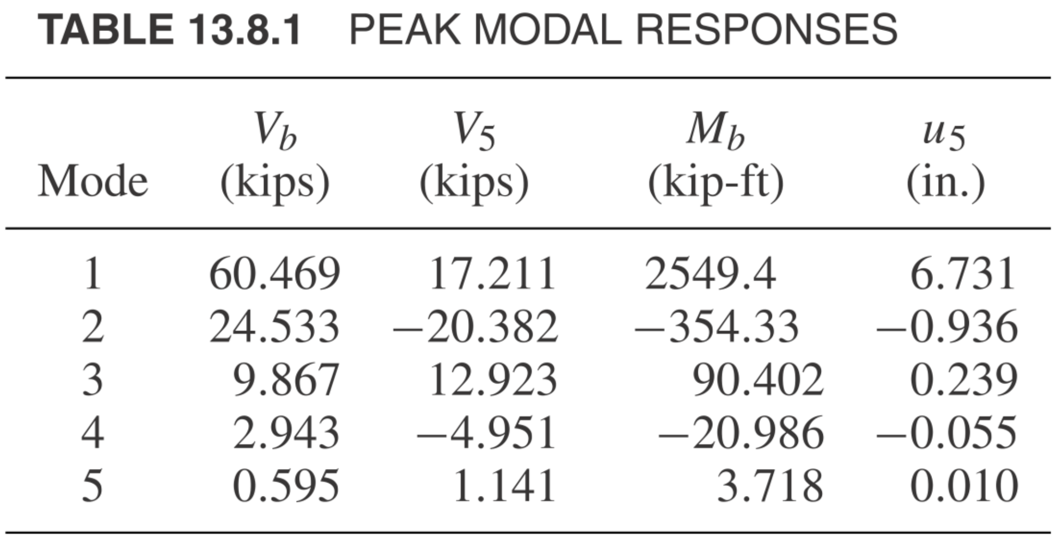


**TABLE 13.8.1** PEAK MODAL RESPONSES

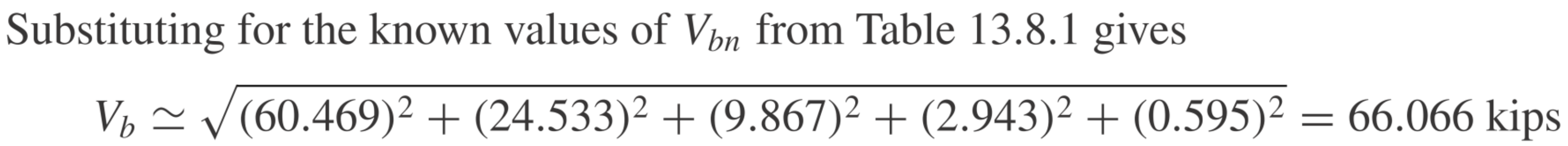
| Mode | $V_b$ (kips) | $V_5$ (kips) | $M_b$ (kip-ft) | $u_5$ (in.) |
|---|---|---|---|---|
| 1 | 60.469 | 17.211 | 2549.4 | 6.731 |
| 2 | 24.533 | −20.382 | −354.33 | −0.936 |
| 3 | 9.867 | 12.923 | 90.402 | 0.239 |
| 4 | 2.943 | −4.951 | −20.986 | −0.055 |
| 5 | 0.595 | 1.141 | 3.718 | 0.010 |

**Step 5**

The peak value $r_o$ of the total response $r(t)$ can be estimated by combining the peak modal responses according to the ABSSUM, SRSS, or CQC rules.

For example, for base shear, using SRSS rule will provide the following combined peak value.

$$V_b \cong \sqrt{\sum_{n=1}^{5} V_{bn}^2}$$



Substituting for the known values of $V_{bn}$ from Table 13.8.1 gives

$$V_b \simeq \sqrt{(60.469)^2 + (24.533)^2 + (9.867)^2 + (2.943)^2 + (0.595)^2} = 66.066 \text{ kips}$$

# 12.9 LINEAR DYNAMIC ANALYSIS

## 12.9.1 Modal Response Spectrum Analysis

**12.9.1.1 Number of Modes.** An analysis shall be conducted to determine the natural modes of vibration for the structure. The analysis shall include a sufficient number of modes to obtain a combined modal mass participation of 100% of the structure's mass. For this purpose, it shall be permitted to represent all modes with periods less than 0.05 s in a single rigid body mode that has a period of 0.05 s.

**EXCEPTION:** Alternatively, the analysis shall be permitted to include a minimum number of modes to obtain a combined modal mass participation of at least 90% of the actual mass in each orthogonal horizontal direction of response considered in the model.

**12.9.1.2 Modal Response Parameters.** The value for each force-related design parameter of interest, including story drifts, support forces, and individual member forces for each mode of response, shall be computed using the properties of each mode and the response spectra defined in either Section 11.4.6 or 21.2 divided by the quantity $R/I_e$. The value for displacement and drift quantities shall be multiplied by the quantity $C_d/I_e$.

**12.9.1.3 Combined Response Parameters.** The value for each parameter of interest calculated for the various modes shall be combined using the square root of the sum of the squares (SRSS) method, the complete quadratic combination (CQC) method, the complete quadratic combination method as modified by ASCE 4 (CQC-4), or an approved equivalent approach. The CQC or the CQC-4 method shall be used for each of the modal values where closely spaced modes have significant cross-correlation of translational and torsional response.

**12.9.1.4 Scaling Design Values of Combined Response.** A base shear ($V$) shall be calculated in each of the two orthogonal horizontal directions using the calculated fundamental period of the structure $T$ in each direction and the procedures of Section 12.8.

*12.9.1.4.1 Scaling of Forces.* Where the calculated fundamental period exceeds $C_u T_a$ in a given direction, $C_u T_a$ shall be used in lieu of $T$ in that direction. Where the combined response for the modal base shear ($V_t$) is less than 100% of the calculated base shear ($V$) using the equivalent lateral force procedure, the forces shall be multiplied by $V/V_t$ where

$V$ = the equivalent lateral force procedure base shear, calculated in accordance with this section and Section 12.8, and

$V_t$ = the base shear from the required modal combination.

*12.9.1.4.2 Scaling of Drifts.* Where the combined response for the modal base shear ($V_t$) is less than $C_s W$, and where $C_s$ is determined in accordance with Eq. (12.8-6), drifts shall be multiplied by $C_s W/V_t$.

**12.9.1.5 Horizontal Shear Distribution.** The distribution of horizontal shear shall be in accordance with Section 12.8.4, except that amplification of torsion in accordance with Section 12.8.4.3 is not required where accidental torsion effects are included in the dynamic analysis model.

**12.9.1.6 P-Delta Effects.** The P-delta effects shall be determined in accordance with Section 12.8.7. The base shear used to determine the story shears and the story drifts shall be determined in accordance with Section 12.8.6.

**12.9.1.7 Soil–Structure Interaction Reduction.** A soil–structure interaction reduction is permitted where determined using Chapter 19 or other generally accepted procedures approved by the authority having jurisdiction.

**12.9.1.8 Structural Modeling.** A mathematical model of the structure shall be constructed in accordance with Section 12.7.3, except that all structures designed in accordance with this section shall be analyzed using a 3D representation. Where the diaphragms have not been classified as rigid in accordance with Section 12.3.1, the model shall include representation of the diaphragm's stiffness characteristics and additional dynamic degrees of freedom as required to account for the participation of the diaphragm in the structure's dynamic response.

# Seismic Hazard Assessment

## Linear Time History Analysis (LTHA) Procedure (IBC-2021, BCP-2021, ASCE 7-16)

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## Development of Ground Acceleration Time Histories

| | |
|---|---|
| Step 1 | Develop a criteria for ground motion selection [seismicity level, faulting mechanism, site class, source-to-site distance, etc.] |
| Step 2 | Select of a set of suitable ground motions from any database containing past ground motion records. Perform the signal processing of acceleration histories to check ground motion parameters. |
| Step 3 | For the analysis of new buildings, construct the code-based design response spectrum using the seismic hazard parameters of the building site. Alternatively, a site-specific PSHA can also be performed to determine the design uniform hazard spectrum (UHS). |
| Step 4 | Modify (using scaling or spectral matching) the finally selected ground motions using the code-spectrum as the target spectrum. |

## Dynamic (Time History, or Response History) Analysis

Step 5

Select the analysis solution method

Option 1: Mode-superposition (Modal RHA)

Option 2: Step-by-step Direct Integration

Step 6

Compute the time histories of all responses

**Option 1:** The time histories of all responses are determined for each mode and then summed in time domain (for each instant) to determine the combined time histories. Each mode is represented mathematically by a single-degree-of-freedom system which can be solved against the given ground motion to determine time history responses of that mode.

**Option 2:** The combined time histories of all responses are determined directly by step-by-step solving the governing equations of motion using direct integration (without using the concept of mode shapes or natural time periods).

### 12.9.2 Linear Response History Analysis

**12.9.2.1 General Requirements.** Linear response history analysis shall consist of an analysis of a linear mathematical model of the structure to determine its response through methods of numerical integration, to suites of spectrally matched acceleration histories compatible with the design response spectrum for the site. The analysis shall be performed in accordance with the requirements of this section.

**12.9.2.2 General Modeling Requirements.** Three-dimensional (3D) models of the structure shall be required. Modeling the distribution of stiffness and mass throughout the structure's lateral load-resisting system and diaphragms shall be in accordance with Section 12.7.3.

*12.9.2.2.1 P-Delta Effects.* The mathematical model shall include P-delta effects. Limits on the stability coefficient, $\theta$, shall be satisfied in accordance with Section 12.8.7.

*12.9.2.2.2 Accidental Torsion.* Accidental torsion, where required by Section 12.8.4.2, shall be included by offsetting the center of mass in each direction (i.e., plus or minus) from its expected location by a distance equal to 5% of the horizontal dimension of the structure at the given floor measured perpendicular to the direction of loading. Amplification of accidental torsion in accordance with Section 12.8.4.3 is not required.

*12.9.2.2.3 Foundation Modeling.* Where foundation flexibility is included in the analysis, modeling of the foundation shall be in accordance with Section 12.13.3.

*12.9.2.2.4 Number of Modes to Include in Modal Response History Analysis.* Where the modal response history analysis procedure is used, the number of modes to include in the analysis shall be in accordance with Section 12.9.1.1.

*12.9.2.2.5 Damping.* Linear viscous damping shall not exceed 5% critical for any mode with a vibration period greater than or equal to $T_{\text{lower}}$.

**12.9.2.3 Ground Motion Selection and Modification.** Ground acceleration histories used for analysis shall consist of a suite of no fewer than three pairs of spectrally matched orthogonal components derived from artificial or recorded ground motion events. The target response spectrum for each spectrally matched set shall be developed in accordance with Sections 11.4.6 or 21.3, as applicable.

*12.9.2.3.1 Procedure for Spectrum Matching.* Each component of ground motion shall be spectrally matched over the period range $0.8T_{\text{lower}}$ to $1.2T_{\text{upper}}$. Over the same period range and in each direction of response, the average of the 5% damped pseudoacceleration ordinates computed using the spectrum-matched records shall not fall above or below the target spectrum by more than 10% in each direction of response.

**12.9.2.4 Application of Ground Acceleration Histories.** Two orthogonal directions of response, designated as $X$ and $Y$, shall be selected and used for all response history analysis. Ground motions shall be applied independently in the $X$ and $Y$ directions.

### 12.9.2.5 Modification of Response for Design

*12.9.2.5.1 Determination of Maximum Elastic and Inelastic Base Shear.* For each ground motion analyzed, a maximum elastic base shear, designated as $V_{EX}$ and $V_{EY}$ in the $X$ and $Y$ directions, respectively, shall be determined. The mathematical model used for computing the maximum elastic base shear shall not include accidental torsion.

For each ground motion analyzed, a maximum inelastic base shear, designated as $V_{IX}$ and $V_{IY}$ in the $X$ and $Y$ directions, respectively, shall be determined as follows:

$$V_{IX} = \frac{V_{EX} I_e}{R_X} \tag{12.9-1}$$

$$V_{IY} = \frac{V_{EY} I_e}{R_Y} \tag{12.9-2}$$

where $I_e$ is the Importance Factor and $R_X$ and $R_Y$ are the response modifications coefficients for the $X$ and $Y$ directions, respectively.

*12.9.2.5.2 Determination of Base Shear Scale Factor.* Design base shears, $V_X$, and $V_Y$, shall be computed in the $X$ and $Y$ directions, respectively, in accordance with Section 12.8.1. For each ground motion analyzed, base shear scale factors in each direction of response shall be determined as follows:

$$\eta_X = \frac{V_X}{V_{IX}} \geq 1.0 \tag{12.9-3}$$

$$\eta_Y = \frac{V_Y}{V_{IY}} \geq 1.0 \tag{12.9-4}$$

*12.9.2.5.3 Determination of Combined Force Response.* For each direction of response and for each ground motion analyzed, the combined force response shall be determined as follows:

a. The combined force response in the $X$ direction shall be determined as $I_e\eta_X/R_X$ times the computed elastic response in the $X$ direction using the mathematical model with accidental torsion (where required) plus $I_e\eta_Y/R_Y$ times the computed elastic response in the $Y$ direction using the mathematical model without accidental torsion.

b. The combined force response in the $Y$ direction shall be determined as $I_e\eta_Y/R_Y$ times the computed elastic response in the $Y$ direction using the mathematical model with accidental torsion (where required), plus $I_e\eta_X/R_X$ times the computed elastic response in the $X$ direction using the mathematical model without accidental torsion.

*12.9.2.5.4 Determination of Combined Displacement Response.* Response modification factors $C_{dX}$ and $C_{dY}$ shall be assigned in the $X$ and $Y$ directions, respectively. For each direction of response and for each ground motion analyzed, the combined displacement responses shall be determined as follows:

a. The combined displacement response in the $X$ direction shall be determined as $\eta_X C_{dX}/R_X$ times the computed elastic response in the $X$ direction using the mathematical model with accidental torsion (where required), plus $\eta_Y C_{dY}/R_Y$ times the computed elastic response in the $Y$ direction using the mathematical model without accidental torsion.

b. The combined displacement response in the $Y$ direction shall be determined as $\eta_Y C_{dY}/R_Y$ times the computed elastic response in the $Y$ direction using the mathematical model with accidental torsion (where required), plus $\eta_X C_{dX}/R_X$ times the computed elastic response in the $X$ direction using the mathematical model without accidental torsion.

**EXCEPTION:** Where the design base shear in the given direction is not controlled by Eq. (12.8-6), the factors $\eta_X$ or $\eta_Y$, as applicable, are permitted to be taken as 1.0 for the purpose of determining combined displacements.

**12.9.2.6 Enveloping of Force Response Quantities.** Design force response quantities shall be taken as the envelope of the combined force response quantities computed in both orthogonal directions and for all ground motions considered. Where force interaction effects are considered, demand to capacity ratios are permitted to be enveloped in lieu of individual force quantities.

**12.9.2.7 Enveloping of Displacement Response Quantities.** Story drift quantities shall be determined for each ground motion analyzed and in each direction of response using the combined displacement responses defined in Section 12.9.2.5.4. For the purpose of complying with the drift limits specified in Section 12.12, the envelope of story drifts computed in both orthogonal directions and for all ground motions analyzed shall be used.

# Thank you for your attention