NUST Institute of Civil Engineering (NUST)
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CE 809: Structural Dynamics 3 (3-0) **Semester: August 2021**

Assignment 5: Free and Forced Vibration Response of MDF systems using Modal Analysis (Modal Superposition)

Question 1:

Consider the multiple-degree-of-freedom system shown in Figure 1 below. It is a 2-dimensional frame structure idealized as lumped mass and lumped stiffness model with floors and beams assumed to be rigid. The mass of each floor is lumped at floor level and is shown in figure. Similarly, the story stiffness for each story is also given. The story height in this structure is 3 m. The structure is idealized with fixed support condition at the base.

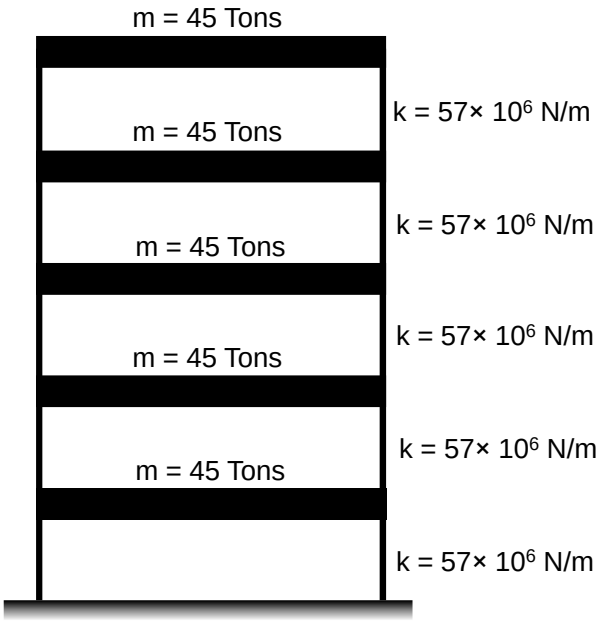
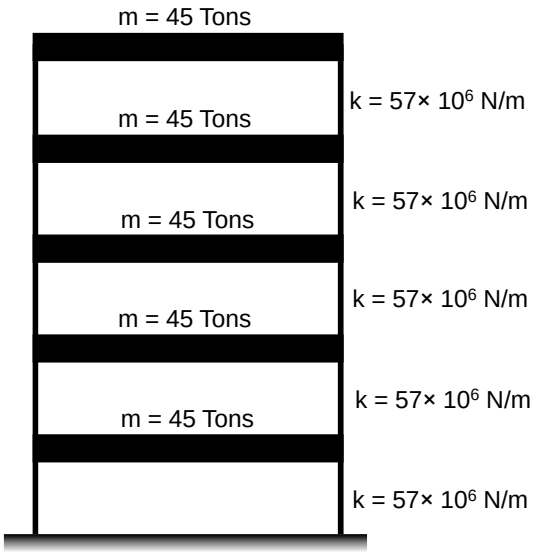


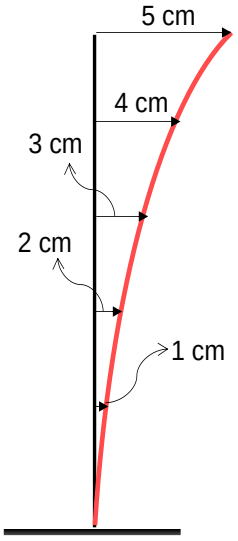
Figure 1: A multiple-degree-of-freedom (MDF) system

Task 1 (Compulsory): Formulate the K and M matrices of this structure.

Task 2 (Compulsory): Carryout the eigen-value solution for this structure and determine natural time periods and vibration mode shapes.

Task 3 (Optional): If the structure is displaced initially in the following given way (as shown in Figure 2) and let it vibrate freely, determine the free vibration response (displacements time histories) of all five stories.





$$\boldsymbol{u}(t=0)=\begin{bmatrix}u_1(t=0)\\u_2(t=0)\\u_3(t=0)\\u_4(t=0)\\u_5(t=0)\end{bmatrix}=\begin{bmatrix}5\\4\\3\\2\\1\end{bmatrix}\qquad \dot{\boldsymbol{u}}(t=0)=\begin{bmatrix}\dot{u}_1(t=0)\\\dot{u}_2(t=0)\\\dot{u}_3(t=0)\\\dot{u}_4(t=0)\\\dot{u}_5(t=0)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

Figure 2: The given initial conditions for Task 3

Task 4 (Optional): Consider that the roof of given structure is subjected to a single dynamic point load due to a rotating eccentric motor (Figure 3). It is producing the following harmonic loading on roof.

$$F(t) = 100\sin(\bar{\omega}t)$$

Where $\bar{\omega}$ = Forcing Frequency = 2 Hz

Using the classical modal analysis procedure, calculate the story displacements for each story due to this harmonic loading.

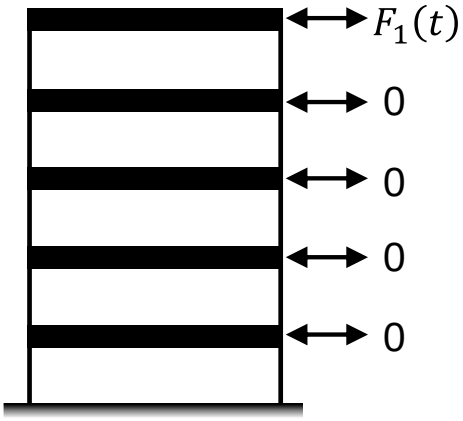


Figure 2: The given loading for Task 4

Hints for Assignment 5

Task 2:

Step 1: Formulate the governing dynamic equation of motion (matrix form).

Step 2: Solve the eigen equation to determine all ω, T and ϕ. There will be 5 ω, T and ϕ.

Task 3:

Step 1: Formulate the governing dynamic equation of motion (matrix form).

$$\boldsymbol{M}\ddot{\boldsymbol{U}}(t) + \boldsymbol{K}\boldsymbol{U}(t) = \mathbb{O}$$

Step 2: Using all 5 mode shapes ϕ determined in Task 2, formulate the modal matrix $\boldsymbol{\Phi}$).

Step 3: Convert the governing equations of motions into "uncoupled" form.

Step 4: Solve each equation separately in terms of q to determine "free vibration response of each mode $q(t)$". For this purpose, use the free vibration response solution of a SDF system directly (i.e. the right side of each equation would be zero).

Step 3: Transform back ($\boldsymbol{t}$) into $\boldsymbol{u}(\boldsymbol{t})$ using $\boldsymbol{\Phi}$ matrix to get the vector containing all story displacements.

Step 4: Plot the histories of those story displacements.

Task 4:

Step 1: Formulate the governing dynamic equation of motion (matrix form).

$$\boldsymbol{M}\ddot{\boldsymbol{U}}(t) + \boldsymbol{K}\boldsymbol{U}(t) = \boldsymbol{F}(t)$$

Step 2: Using all 5 mode shapes ϕ determined in Task 2, formulate the modal matrix $\boldsymbol{\Phi}$).

Step 3: Convert the governing equations of motions into "uncoupled" form.

Step 4: Solve each equation separately in terms of q to determine "forced vibration response of each mode (t)". For this purpose, use the harmonic vibration response solution of a SDF system directly.

Step 5: Transform back (t) into $\boldsymbol{u}(t)$ using $\boldsymbol{\Phi}$ matrix to get the vector containing all story displacements.

Step 6: Plot the histories of those story displacements.