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CE 809: Structural Dynamics 3 (3-0) **Semester: August 2021**

Assignment 1: Free Vibration Response of an SDF System

Question 1:

Consider a simple structure which is idealized as a single-degree-of-freedom (SDF) system as shown in Figure 1. The mass m of the structure is $4000\, kg$ which is supported by massless columns with combined stiffness $k = 16 \times 10^5\ N/m$.

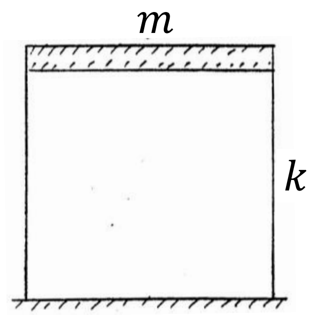


Figure 1: An undamped SDF system

Task 1: Determine the natural cyclic frequency "f", natural circular frequency "ω", and natural time period "T" of the system using analytical expressions derived in class.

Task 2: determine the change in the natural cyclic frequency "f" of the system if

Case (a): the mass m is doubled

Case (b): the stiffness k is reduced to 50% of its original value

Task 3: Suppose that we want to reduce the natural cyclic frequency "f" of this system to 85% of its actual value. For this purpose, suppose that only the mass m of this system is allowed to be changed. Determine the change in mass m which would result in the required reduction in natural cyclic frequency.

Task 4: Suppose that the real structure has some tendency to dissipate energy during the oscillations and we are interested in determining the damping coefficient c and critical damping ratio ξ of this system. For this purpose, a free vibration response test is conducted for this system and the resulting response is shown in Figure 2 below.

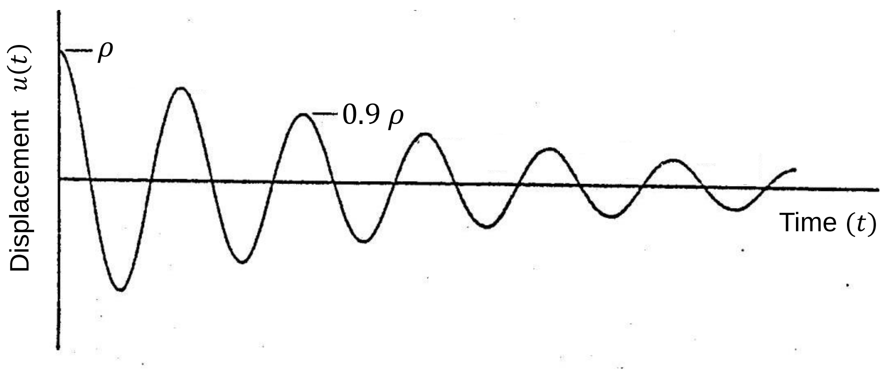


Figure 2: Free vibration response of the SDF system measured from experiment

Determine the damping coefficient c and critical damping ratio ξ of this system.

Question 2:

A simple structure is idealized as a single-degree-of-freedom (SDF) system and is shown in Figure 3 below. Its whole mass 12,000 Kg is assumed to be concentrated at top which is supported by two steel columns with hollow cross-sections. The columns are assumed massless, rigid in vertical direction, and firmly fixed to the rigid ground. Important structural dimensions and the column's cross-section are shown in Figure 3. The modulus of elasticity of steel is $2 \times 10^{11}\ N/m^2$.

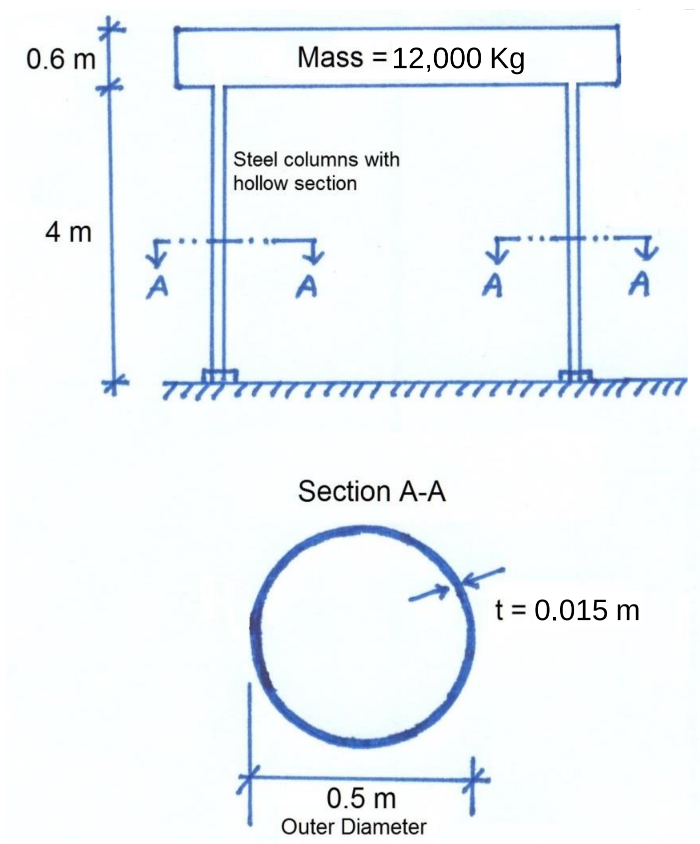


Figure 3: A simple structure supported by two columns and a lateral spring

Task 1: Construct a computer model of this simple structure using ETABS 2016 or SAP 2000. Determine the natural frequency (f), natural circular frequency (ω) and the natural time period (T) of this simple structure from computer software.

Task 2: Compare the values determined with computer program with analytical values determined using the expressions derived in class.

Task 3: Determine and plot the undamped free vibration response of the system for the following three initial conditions.

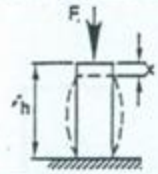
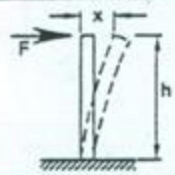
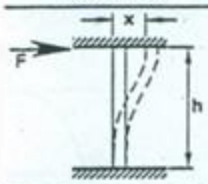
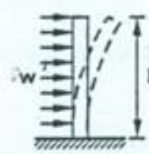
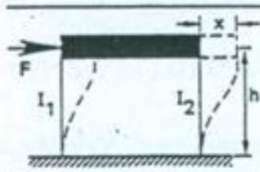
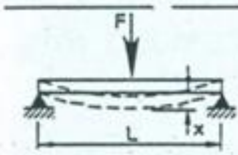
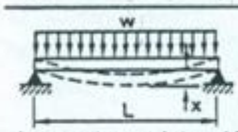
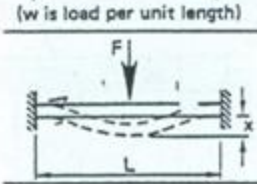
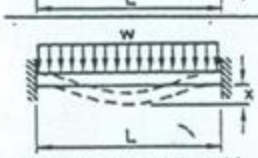
a) $u(0) = 0.03\ m, \dot{u}(0) = 0\ m/s$
b) $u(0) = 0.03\ m, \dot{u}(0) = 0.2\ m/s$
c) $u(0) = 0.03\ m, u(0.5\ sec) = -0.02\ m/s$

Overlay the plots of displacement responses for each case in the same figure.

Task 4: Consider the case when the simple structure shown in Figure 3 is modelled with an energy dissipation mechanism in the form of viscous damping. Suppose that the critical damping ratio (ξ) of this structure is 0.05. Determine and plot the damped displacement response of structure for all three sets of initial conditions given in Task 3. Compare the displacement-vs-time plots for undamped and damped cases. Overlay the plots of displacement responses for both cases in the same figure.

Elastic Stiffness

Deflection and stiffness for various systems (due to bending moment only)

System	Maximum Deflection (x)	Stiffness (k)
	$\frac{Fh}{AE}$	$\frac{AE}{h}$
	$\frac{Fh^3}{3EI}$	$\frac{3EI}{h^3}$
	$\frac{Fh^3}{12EI}$	$\frac{12EI}{h^3}$
	$\frac{wL^4}{8EI}$	$\frac{8EI}{L^3}$
	$\frac{Fh^3}{12E(I_1+I_2)}$	$\frac{12E(I_1+I_2)}{h^3}$
	$\frac{FL^3}{48EI}$	$\frac{48EI}{L^3}$
 (w is load per unit length)	$\frac{5wL^4}{384EI}$	$\frac{384EI}{5L^3}$
	$\frac{FL^3}{192EI}$	$\frac{192EI}{L^3}$
 (w is load per unit length)	$\frac{wL^4}{384EI}$	$\frac{384EI}{L^3}$